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PARAMETER ESTIMATION OF COMPOUND-GAUSSIAN CLUTTER WITH NAKAGAMI-DISTRIBUTED TEXTURE

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This paper addresses the parameter estimation of the compound-Gaussian clutter with Nakagami texture (CGNG). The CGNG distribution was recently introduced to model high-resolution sea clutter. Two estimators are proposed: the fractional-order moment estimator (FOME) and the fractional-negative-order moment estimator (FNOME). The estimation performance of the proposed estimators is assessed and compared with that of the existing higher-order moment methods (HOME) and the [*z*log(*z*)] estimator. Using both simulated and real data, estimation accuracy and modeling performance are evaluated using the chi-squared test (χ^2) and the mean square error (MSE).

1. INTRODUCTION

Parameter estimation is a crucial step in developing radar detection schemes. For radar detection and constant false alarm rate (CFAR) detectors, the statistical model of the clutter and the knowledge of its parameters are used to develop the detection scheme [1–4]. In literature, different non-Gaussian models have been introduced to describe high-resolution sea clutter, like log-normal, Weibull, compound K, compound Log-Normal (CGLNT), Pareto, compound inverse Gaussian (CIG), compound inverted exponentiated Rayleigh (CIER) [5–11]. In [12], the authors introduced the compound-Gaussian with Nakagami texture (CGNG) model, where it has been shown that the CGNG is more suitable for modeling airborne Ku-band high-resolution sea clutter data at medium/high grazing angles compared with K, Pareto, CIG, and CGLNT distributions.

Several techniques have been proposed to estimate radar clutter parameters with varying degrees of accuracy. The maximum likelihood estimator (MLE) is based on moment methods (higher order moment estimator (HOME), fractional order moment estimator (FOME), [*z*log(*z*)]) and artificial intelligence methods. In [15], the MLE method is proposed to estimate the parameters of the compound K distribution; after that, the HOME and FOME methods are developed in [16]. These estimators are designed to reduce the computational complexity of the MLE method. In [17], the authors proposed the [zlog(z)] estimator based on the logarithmic moments; this estimator provides the best estimates for the K distribution. In [18], the fractional negative order moments estimator (FNOME) is proposed to estimate the parameters of the K-distribution. In [18], the FOME and the [*z*log(*z*)] estimators are proposed to estimate the parameters of the CGLNT distribution.

This paper considers parameter estimation of the CGNG clutter. Two estimators are proposed, the FOME and the FNOME estimators. These estimators are based on the fractional positive/negative order moments. Using both simulated data and real data, the proposed estimators provide good estimation performance compared with the existing HOME and [*z*log(*z*)] methods.

2. CGNG MODEL

CGNG is a compound-Gaussian model with Nakagami-distributed texture proposed in [12]. The CGNG distribution provides a better fit to sea clutter at medium/high grazing angles. It is defined as a mixture of two components: the first represents the average local level of the clutter, called texture, and follows the Nakagami distribution, which is actually a particular form of the generalized gamma distribution [8,13,14], and the second component is the speckle that obeys the Rayleigh distribution. The global probability density function (PDF) of the CGNG model is given in [12] as follows:

$$p(x)=\frac{\pi x v^{v}}{\Gamma(v)b^{v}}\int_{0}^{+\infty}\frac{y^{2v}}{y^{3}}\exp\left(-\frac{\pi x^{2}}{4y^{2}}-\frac{v}{b}y^{2}\right)dy \qquad (1)$$

where v and b are the shape and the scale parameters, respectively.

Using the PDF in (1), the complementary cumulative distribution function (CCDF), which is the false alarm probability (PFA), where the threshold in CFAR detection is mainly based on it. The CCDF can be written as a function of the normalized threshold T as

$$\mathrm{CCDF}(T)=\int_{T}^{+\infty}\frac{\pi x v^{v}}{\Gamma(v)b^{v}}\int_{0}^{+\infty}y^{2v-3}\exp\left(-\frac{\pi x^{2}}{4y^{2}}-\frac{v}{b}y^{2}\right)dydx. \qquad (2)$$

The moment expression of the CGNG is given as [12]:

$$\langle x^{n}\rangle=\frac{\Gamma\left(v+\frac{n}{4}\right)\Gamma^{\frac{n}{2}-1}(v)\Gamma\left(1+\frac{n}{2}\right)b^{\frac{n}{2}}}{\Gamma^{\frac{n}{2}}\left(v+\frac{1}{2}\right)}, \qquad (3)$$

where n represents the order and $\Gamma(\cdot)$ denotes the gamma function.

In [12], the parameters of the CGNG distribution are estimated using the HOME and [*z*log(*z*)] estimators. The HOME is obtained by manipulating the first two even-order moments $\langle x^2\rangle$, $\langle x^4\rangle$ as:

$$\begin{cases} b=\langle x^{2}\rangle,\\ \dfrac{\langle x^{4}\rangle}{\langle x^{2}\rangle^{2}}=\dfrac{\Gamma(v+1)\Gamma(v)\Gamma(3)b^{2}}{\Gamma^{2}\left(v+\frac{1}{2}\right)}. \end{cases} \qquad (4)$$

The [*z*log(*z*)] estimator is obtained by the derivatives of the theoretical moment expression in (3) according to the order n, the [zlog(z)] method is given as:

$$f(v)\times h(v)=\frac{2}{N}\sum_{i=1}^{N}x_{i}^{2}\ln x_{i},$$
$$f(v)=\frac{2}{N}\sum_{i=1}^{N}\ln x_{i}+\frac{1}{2}\Psi\left(\frac{1}{v}+\frac{1}{2}\right)-\frac{1}{2}\Psi\left(\frac{1}{v}\right)+1, \qquad (5)$$
$$h(v)=\exp\left(\frac{2}{N}\sum_{i=1}^{N}\ln x_{i}-\frac{1}{2}\Psi\left(\frac{1}{v}\right)+\ln\frac{\Gamma\left(\frac{1}{v}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{v}\right)}-\Psi(1)\right).$$

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3. PROPOSED ESTIMATORS

In this section, two estimators are developed: the FOME and the FNOME estimators by manipulating the positive/negative fractional order moments. Fractional order moments exist for a wide range of probability distributions, including heavy-tailed distributions such as the CGNG distribution.

The FOME estimator is derived from the statistical ratio $\langle x^{n+1}\rangle/\langle x\rangle\langle x^n\rangle$:

$$\langle x^{n+1}\rangle = \frac{\Gamma\left(v+\frac{n+1}{4}\right)\Gamma^{\frac{n+1}{2}-1}(v)\Gamma\left(1+\frac{n+1}{2}\right)b^{\frac{n+1}{2}}}{\Gamma^{\frac{n+1}{2}}\left(v+\frac{1}{2}\right)}, \quad (6)$$

$$\langle x\rangle = \frac{\Gamma\left(v+\frac{1}{4}\right)\Gamma^{-\frac{1}{2}}(v)\Gamma\left(\frac{3}{2}\right)b^{\frac{1}{2}}}{\Gamma^{\frac{1}{2}}\left(v+\frac{1}{2}\right)}. \quad (7)$$

Dividing (6) by the multiplication of (7) and (3), the FOME is obtained as:

$$\frac{\langle x^{n+1}\rangle}{\langle x^n\rangle\langle x\rangle} = \frac{\Gamma\left(v+\frac{n+1}{4}\right)\Gamma\left(1+\frac{n+1}{2}\right)\Gamma(v)}{\Gamma\left(v+\frac{n}{4}\right)\Gamma\left(1+\frac{n}{2}\right)\Gamma\left(v+\frac{1}{4}\right)\Gamma\left(\frac{3}{2}\right)}. \quad (8)$$

The FNOME estimator is developed based on the use of fractional negative and positive orders as:

$$\langle x^{n}\rangle = \frac{\Gamma\left(v+\frac{n}{4}\right)\Gamma^{\frac{n}{2}-1}(v)\Gamma\left(1+\frac{n}{2}\right)b^{\frac{n}{2}}}{\Gamma^{\frac{n}{2}}\left(v+\frac{1}{2}\right)}, \quad (9)$$

$$\langle x^{-n}\rangle = \frac{\Gamma\left(v-\frac{n}{4}\right)\Gamma^{\frac{-n}{2}-1}(v)\Gamma\left(1-\frac{n}{2}\right)b^{\frac{-n}{2}}}{\Gamma^{\frac{-n}{2}}\left(v+\frac{1}{2}\right)}. \quad (10)$$

Multiplying (9) and (10) side by side, the scale parameter b is eliminated, and the FNOME is obtained as:

$$\langle x^{n}\rangle\langle x^{-n}\rangle = \frac{\Gamma\left(v-\frac{n}{4}\right)\Gamma\left(v+\frac{n}{4}\right)\Gamma\left(1+\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right)}{\Gamma^{2}(v)}. \quad (11)$$

The next section discusses the obtained results and compares the estimation accuracy of the proposed FOME and FNOME methods with the existing HOME and [$z\log(z)$] estimators.

4. ESTIMATION PERFORMANCE ANALYSIS

The performance analysis of the proposed FOME and FNOME estimators is conducted by means of a comparison with the existing HOME and [$z\log(z)$] estimators, and the estimation accuracy is evaluated by the mean square error (MSE) and the chi-squared test (χ^2) criteria. The data used in this section are both simulated CGNG and the IPIX high-resolution sea clutter database [20].

First, based on simulated data generated from the CGNG distribution, the estimation accuracy is evaluated using the MSE criterion across L = 10000 independent trials. The sample number M is 1000, and the value of the fractional order moment n is 0.01 for both FOME and FNOME. The value of order n is set based on the results obtained for different values of n using the FOME shown in Fig. 1. Lower values suggest better estimation accuracy. Figure 2 shows the MSE curves against the shape parameter. From this figure, the FNOME provides the best estimation performance, especially when the clutter spikiness increases, while the FOME offers approximately similar performance to the [$z\log(z)$] estimator.

On the other hand, the IPIX sea clutter database is also used to confirm the simulation results. The empirical PDF and CCDF of the real data will be compared with the theoretical PDFs and CCDFs of the CGNG obtained using the estimated parameters using HOME, [$z\log(z)$], FOME, and FNOME estimators. The MSE and the χ^2 are also calculated to show the estimation accuracy. Table 1 represents the obtained values of the MSE and the χ^2.

The PDFs and CCDFs curves plotted in Fig. 3 are obtained using HH polarization, 27th range cell, and a resolution of 3 m. The results show the ability of fitting to the real data of the proposed estimators, which is clear from Table 1, where the FNOME estimator offers the lowest MSE for both PDFs and CCDFs, followed by the FOME estimator. For the χ^2, the FNOME provides the best result for the CCDFs. For VV polarization, the 1st range cell and the resolution of 15 m. The curves of the PDFs and the CCDFs are plotted in Fig. 4. Here, the FOME method offers a better fit to the empirical PDF according to the minimum MSE obtained, and the FNOME method offers better values of MSE and χ^2 for the CCDF curves, as shown in Table 1. It is noteworthy that the PDF and CCDF of CGNG diverge from the real PDF and CCDF in the tail area. Previous modeling studies [10,20] showed that it is difficult to find a specific clutter model that can fit various situations and datasets of sea clutter. For more scenarios, Fig. 5 is obtained by using data for a resolution of 30m, with VV polarization and 21st range cell. The results confirm the performance of the proposed estimators.

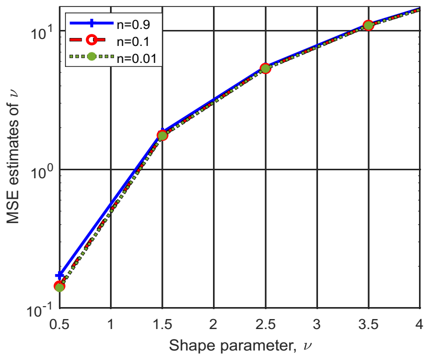


Fig. 1 – Impact of the fractional order n on quality estimation of the FOME method.

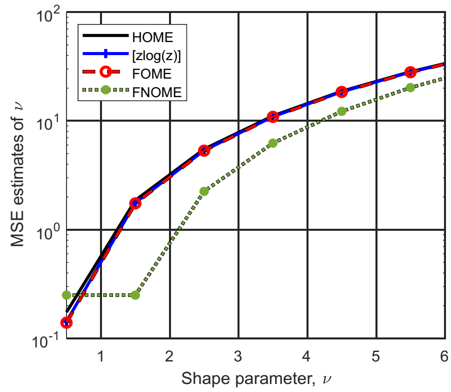
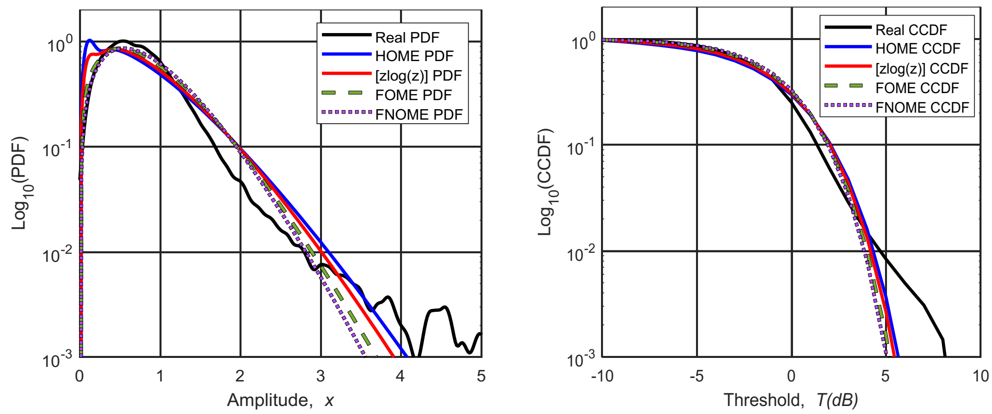


Fig. 2 – MSE curves of the shape parameter v by HOME, [$z\log(z)$], FOME, and FNOME methods.

Table 1

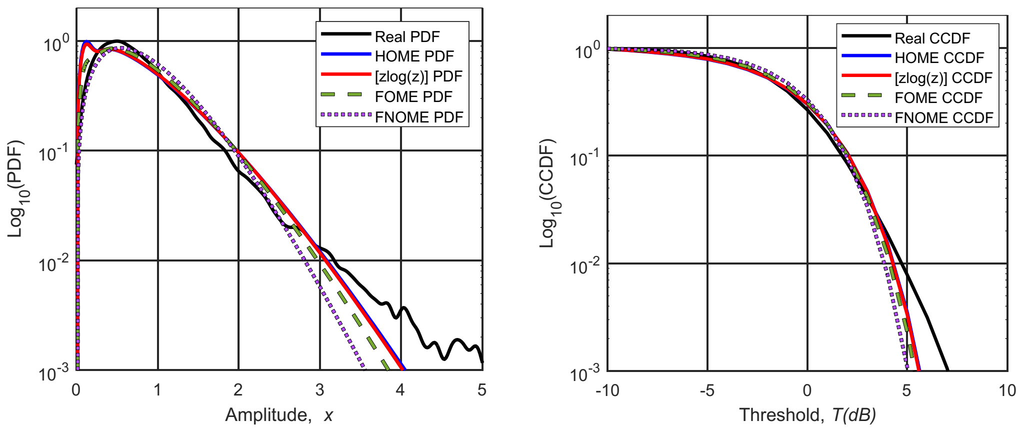
MSE and χ^2 criteria.

IPIX data			HOME	[zlog(z)]	FOME	FNOME
27th cell range of resolution 3m, polarization HH	PDF	MSE	0.0136	0.0062	0.0024	0.0020
		χ^2	0.0487	0.0229	0.0109	0.0100
	CCDF	MSE	0.0012	2.9002 x 10-04	1.8621 x 10-05	1.7574 x 10-05
		χ^2	0.0013	3.7670 x 10-04	1.0257 x 10-04	1.0677 x 10-04
1st cell range of resolution 15m, polarization VV	PDF	MSE	0.0078	0.0065	0.0021	0.0023
		χ2	0.0223	0.0188	0.0064	0.0066
	CCDF	MSE	9.8594e-04	7.9562 x 10-04	1.2374 x 10-04	2.3249 x 10-05
		χ^2	0.0010	8.2871 x 10-04	1.5327 x 10-04	7.5594 x 10-05
21st cell range of resolution 30m, polarization VV	PDF	MSE	0.0434	0.0246	0.0135	0.0129
		χ^2	0.0948	0.0561	0.0334	0.0353
	CCDF	MSE	0.0047	0.0036	0.0019	1.3420e-04
		χ^2	0.0048	0.0038	0.0020	4.1888e-04



(a) (b)

Fig. 3 – PDFs (a) and CCDFs (b) of CGNG obtained by HOME, [*z*log(*z*)], FOME, and FNOME methods using IPIX data of the 27th cell range, resolution 30 m, and polarization HH.



(a) (b)

Fig. 4 – PDFs (a) and CCDFs (b) of CGNG obtained by HOME, [*z*log(*z*)], FOME, and FNOME using IPIX data of the 1st cell range, resolution 15 m and polarization VV.

5. CONCLUSIONS

In this paper, parameter estimation for the CGNG has been considered, and two estimators are proposed: the FOME and the FNOME. Their performance has been assessed and validated using both CGNG-simulated data and real high-resolution sea clutter data from the IPIX database. Based on the results, the proposed FOME and FNOME estimators demonstrate superior estimation accuracy, achieving the lowest MSE and χ^2 values among existing methods. On the other hand, the results show that the CGNG distribution can fit IPIX data.

CREDIT AUTHORSHIP CONTRIBUTION STATEMENT

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REFERENCES

1. J. Ai, X. Yang, J. Song, Z. Dong, L. Jia, F. Zhou, *An adaptively truncated clutter-statistics-based two-parameter CFAR detector in SAR imagery*, IEEE Journal of Oceanic Engineering, **43**, *1*, pp. 267–279 (2017).

2. F.D.A. García, A.C.F. Rodriguez, G. Fraidenraich, J.C.S. Santos Filho, *CA-CFAR detection performance in homogeneous Weibull clutter*, IEEE Geoscience and Remote Sensing Letters, **16**, *6*, pp. 887–891 (2018).
3. I. Chalabi, *Application of CFAR detection to multiple pulses for gamma distributed clutter*, Remote Sensing Letters, **13**, *10*, pp. 1011–1019 (2022).
4. L. Asalomia, G. Samoilescu, M. Mihăilescu, *Un système avancé basé sur l'IA pour la surveillance intelligente des alarmes de pont sur les navires maritimes*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **70**, *2*, pp. 223–228 (2024).
5. A. Ballard, *Detection of radar signals in log-normal sea clutter*, TRW Systems Document, **7425**, *8509-T0*, pp. 1–10 (1966).
6. D.C. Schleher, *Radar detection in Weibull clutter*, IEEE Transactions on Aerospace and Electronic Systems, **6**, pp. 736–743 (1976).
7. E. Jakeman, P. Pusey, *A model for non-Rayleigh sea echo*, IEEE Transactions on Antennas and Propagation, **24**, *6*, pp. 806–814 (1976).
8. J. Carretero-Moya, J. Gismero-Menoyo, Á. Blanco-del-Campo, A. Asensio-Lopez, *Statistical analysis of a high-resolution sea-clutter database*, IEEE Transactions on Geoscience and Remote Sensing, **48**, *4*, pp. 2024–2037 (2009).
9. G.V. Weinberg, *Assessing Pareto fit to high-resolution high-grazing-angle sea clutter*, Electronics Letters, **47**, *8*, pp. 516–517 (2011).
10. A. Mezache, F. Soltani, M. Sahed, I. Chalabi, *Model for non-Rayleigh clutter amplitudes using compound inverse Gaussian distribution: An experimental analysis*, IEEE Transactions on Aerospace and Electronic Systems, **51**, *1*, pp. 142–153 (2015).
11. I. Chalabi, *High-resolution sea clutter modeling using compound inverted exponentiated Rayleigh distribution*, Remote Sensing Letters, **14**, *5*, pp. 433–441 (2023).
12. G. Yang, X. Zhang, P. Zou, P. Shui, *Compound-Gaussian model with Nakagami-distributed textures for high-resolution sea clutter at medium-high grazing angles*, Remote Sensing, **16**, *1*, pp. 1–10 (2024).
13. F. Gini, M. Greco, M. Diani, L. Verrazzani, *Performance analysis of two adaptive radar detectors against non-Gaussian real sea clutter data*, IEEE Transactions on Aerospace and Electronic Systems, **36**, *4*, pp. 1429–1439 (2000).
14. M. Greco, B. Federica, F. Gini, *X-band sea-clutter nonstationarity: Influence of long waves*, IEEE Journal of Oceanic Engineering, **29**, *2*, pp. 269–283 (2004).
15. I.R. Joughin, D.B. Percival, D.P. Winebrenner, *Maximum likelihood estimation of K distribution parameters for SAR data*, IEEE Transactions on Geoscience and Remote Sensing, **31**, *5*, pp. 989–999 (1993).
16. D.R. Iskander, A.M. Zoubir, *Estimation of the parameters of the K-distribution using higher order and fractional moments*, IEEE Transactions on Aerospace and Electronic Systems, **35**, *4*, pp. 1453–1457 (1999).
17. D. Blacknell, R.J.A. Tough, *Parameter estimation for the K-distribution based on [z log (z)]*, IEE Proceedings – Radar, Sonar and Navigation, **148**, *6*, pp. 309–312 (2001).
18. I. Chalabi, A. Mezache, *Estimating the K-distribution parameters based on fractional negative moments*, IEEE 12th International Multi-Conference on Systems, Signals & Devices (SSD15), Tunisia, pp. 1–6 (2015).
19. I. Chalabi, A. Mezache, *Estimators of compound Gaussian clutter with log-normal texture*, Remote Sensing Letters, **10**, *7*, pp. 709–716 (2019).
20. M. Greco, F. Gini, M. Rangaswamy, *Statistical analysis of measured polarimetric clutter data at different range resolutions*, IEEE Proceedings – Radar, Sonar and Navigation, **153**, *6*, pp. 473–481 (2006).