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ANALYSIS OF THE TIME-FREQUENCY LOCKED LOOPS USING TABLES – PART 2

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Keywords: Time-Frequency locked loop; Tracking; Digital circuit; Discrete linear system; Time-digital filter.

This article describes the universal tabular methods for the Time-Frequency Locked Loop (TFLL) analysis. TFLLs are open-loop linear discrete systems that process the input signal over time periods. Using tables, we can determine either the complete equations of the Z-transform from various phases of mathematical development or the expressions that represent the results of complex, lengthy mathematical analysis. The tabular method was developed for the TFLLs of lower order and extended to TFLL of the fifth order. It was shown how to extend the table to TFLL$_n$ of any order. The usage of the table is demonstrated in the tracking and prediction applications of the second- and third-order TFLL. However, the table approach can also be used for all other kinds of linear discrete systems. Mathematical analyses were performed by the Z-transform of the difference equations describing the system. The corresponding MATLAB application software for FIR digital filters was used for the frequency-domain analyses.

The tabular approach for determining the optimal parameters of TFLL$_n$, which enables it to track very fast changes in the input period, was developed in ref. [1]. The tabular approach described in ref. [2] allows simple determination of the conditional equations that are an integral part of the different final expressions. Using the conditional equations, we can automatically derive the complete final expressions that describe the system's properties. Both the optimal parameters and the conditional equations can be obtained after long mathematical analyses. Both tabular approaches eliminate the need for mathematical analyses that increase trigonometrically with increasing the order of TFLL$_n$. Unlike the one-dimensional mathematical sequences, well known in mathematics, both tabular methods represent two-dimensional infinite mathematical sequences.

The tabular methods described in [1,2] encouraged the idea of developing a tabular method that would be more comprehensive and that would contain a larger number of the final expressions which appear through successive mathematical procedures. This article describes the universal tabular method for analysis of TFLLs, which also represents a two-dimensional infinite mathematical sequence. Unlike the previous two tabular methods, which contain only numbers in their fields, this tabular method contains mathematical expressions in its fields. These expressions represent the transfer functions, characteristic expressions, and final forms of various Z-transforms of the output variables that typically appear in mathematical analyses. For the case when a TFLL reaches the stable state, the table also contains final values of the phase differences between the output and input for different inputs, as well as the expressions for the tracking errors. Finally, the table contains all the complete conditional equations in which the TFLL parameters are also included.

References [3–11] cover various applications of the same new field, which is applied in this article and in [1,2]. That is, the processing of the input and output pulse signal periods and the time differences between them. Time-digital filters are described in [36]. In refs. [7–11] different kinds of TFLL and time-phase locked loop (TPLL) are applied in the fields of tracking and prediction, phase shifting, time shifting, frequency multipliers, frequency synthesizers, noise rejection, frequency measurement, and others.

Realization of the systems described in [1–6] is possible only by a microprocessor due to numerous mathematical operations. The systems described in [7–11] are implemented in digital circuit technology. All mentioned references are important for this article. They illustrate how to design electronic circuits that ensure the functioning of the TFLLs according to the mathematical model and how to perform the analysis of these systems using the theory of linear discrete systems and the Z transform technique. They also illustrate how to simulate the TFLLs in the time domain and how to analyze these systems in the frequency domain using digital filter theory and application software intended for digital filter design and analysis. These articles also explain the physical meaning of the input and output variables of TFLLs, obtained in various analyses.

Since several articles by the same author are cited in the references, it is important to point out that there are no other scientific articles in the literature dealing with the processing of the periods and time differences. There are only a few other articles by the same author that are less important for this article.

Articles and books in [12–27] are used as a theoretical basis, covering the scientific disciplines on which this article relies, such as linear discrete systems, digital filters, signal processing, phase- and frequency-locked loops, applied digital electronics, and others.

2. DEVELOPMENT OF THE UNIVERSAL TABLES

Just as in [1,2], for the development of the universal tabular method intended for the analysis of TFLLs, because of simplicity, we will use mathematical analyses for TFLLs of the lowest order, *i.e.*, of the second-order TFLL$_2$ and the third-order TFLL$_3$. The general time relations between the input and output variables are shown in Fig. 1. This figure will be used to define the system equations for TFLL$_2$ and TFLL$_3$. Figure 1 shows a general case of an input signal S_{in} and an output signal S_{op} of a TFLL. The periods TI_k and TO_k, as well as the time difference τ_k, occur at discrete times t_k,

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t_{k+1}, t_{k+2}, t_{k+3}, and t_{k+4}, which are defined by the falling edges of the pulses of S_{op} in Fig. 1. Let us now analyze $TFLL_2$, whose main difference equation is described in eq. (1), in which b_1 and b_2 are the system parameters. The difference equation of the time difference τ_k is shown in eq. (2), which represents the natural relation between the variables in Fig. 1. Time difference τ_k, as the second output variable, will serve for the different analyzes of $TFLL_2$ in both transient and stable state.

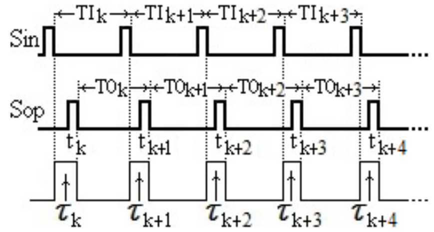

Fig. 1 – The time relations between the input and output variables.

$$TO_{k+2}=b_1 TO_{k+1} + b_2 TO_k, \quad (1)$$

$$\tau_{k+1}=\tau_k + TO_k - TI_k, \quad (2)$$

After we have mathematically defined the second-order $TFLL_2$, we will use some of the analysis for $TFLL_2$ performed in ref. [2]. The Z-transform $TO(z)=Z(TO_{k+2})$ is shown in Eq. (3). According to Eq. (3), the transfer function $H_{TO}(z)$ is presented in eq. (4). Based on eq. (4), $[H_{TO}(z)-1]$ is calculated and shown in eq. (5). The expression for $\tau(z)$ in eq. (6) is the borrowed eq. (7) from ref. [2]. Based on eqs. (6) and (5), $\tau(z)$ is calculated and shown in eq. (7). Equation (7) is valid if the first conditional equation, given in eq. (8) is satisfied. Based on eq. (7), $H\tau(z)$ is defined in eq. (9). If the second conditional equation is satisfied, which is given in eq. (12), eq. (9) will turn into eq. (10) and eq. (7) will turn into eq. (11). Equations (13)-(15) are borrowed eqs. (13)-(15) from ref. [2]. Equations (13) and (14) are the final values of the time difference τ_k and the output period TO_k when $TFLL_2$ is in the stable state. If the conditional eqs. (8) and (12) are satisfied, the expression $[H_{TO}(z)-1]$ for $TFLL_2$, shown in eq. (5), will turn into eq. (15).

$$TO(z) = TI(z)\frac{zb_1+b_2}{z^2} + TO_0, \quad (3)$$

$$H_{TO}(z)=\frac{TO(z)}{TI(z)} = \frac{zb_1+b_2}{z^2}, \quad (4)$$

$$H_{TO}(z)\text{-}1 = \frac{-z^2+zb_1+b_2}{z^2}, \quad (5)$$

$$\tau(z) = TI(z)\frac{H_{TO}(z)-1}{z-1} + \frac{z\tau_0}{z-1}, \quad (6)$$

$$\tau(z) = TI(z)\frac{-z+b_1-1}{z^2} + \frac{TO_0+z\tau_0}{z-1}, \quad (7)$$

$$b_1 + b_2 = 1 \quad or \quad b_1 + b_2 - 1 = 0, \quad (8)$$

$$H_\tau(z) = \frac{\tau(z)}{TI(z)} = \frac{-z+b_1-1}{z^2}, \quad (9)$$

$$H_\tau(z) = \frac{\tau(z)}{TI(z)} = \frac{-(z-1)}{z^2}, \quad (10)$$

$$\tau(z) = TI(z)\cdot\frac{-(z-1)}{z^2} + \frac{TO_0+z\tau_0}{z-1}, \quad (11)$$

$$b_1 = 2 \quad or \quad b_1 - 2 = 0, \quad (12)$$

$$\tau_\infty=\lim_{z\to 1}[(z\text{-}1)\tau(z)] = TI\cdot(b_1 - 2) + TO_0 + \tau_0, \quad (13)$$

$$TO_\infty = TI\cdot(b_1 + b_2), \quad (14)$$

$$H_{TO}(z)\text{-}1 = \frac{-z^2+zb_1+b_2}{z^2} = \frac{-(z-1)^2}{z^2}, \quad (15)$$

The universal table related to FLL_2 is shown in Table 1. In addition to the shaded auxiliary outer row no. 1 and the main outer row no. 2, Table 1 is bordered by thick and bold lines in which a constant sequence consisting of the constant "-1" is first entered diagonally, and then the system parameters b_1 and b_2 are entered in row no. 3. We will see practically how to use rows no. 1 and no. 2. The difference between them is only in the first field, which is omitted in row no. 1, so that row no. 1 can be omitted in practical usage of Table 1. All other fields in Table 1 are entered according to the Code next to Table 1, including the fields filled with zeros. Note that Table 1 contains the transformations only of the numerator of the expression "$H_{TO}(z)$-1", given in eq. (5), but the denominator is always z^n, where "n" is the order of $TFLL_n$. The numerator of the transfer function $H_{TO}(z)$ shown

Table 1

The universal table related to $TFLL_2$

1		z^1	z^0	
2	z^2	z^1	z^0	
3	-1	b1	b2	Con.eqs.=0
4	0	-1	b1-1	b2+b1-1
5	0	0	-1	b1-2

CODE

A	
B	X

X=A+B

in eq. (4), can be read from Table 1 by summing the products of the contents of the fields from rows no. 1 and no. 3. That is $z\cdot b_1+b_2$. Since the residuum TO_0 in eq. (3), according to ref. (5), appears in all expressions $TO(z)$ for $TFLL_n$ of any order, the expression $TO(z)$, given in eq. (3) can also be defined directly from Table 1. In the same way, the numerator of the expression "$H_{TO}(z)$-1", shown in eq. (5), can be determined using rows no. 2 and 3. That is $-z^2+zb_1+b_2$. Therefore, the expressions given by eqs. (3)-(5) can be determined directly from Table 1. Let us now compare the contents of Table 1 with the eqs. (7)-(14). After the first division $[H_{TO}(z)\text{-}1]/(z\text{-}1)]$ we got the numerator of $\tau(z)$ as "$-z$+b-1" in eq. (7) as well in eq. (9). The same result can be obtained by summing the products of the contents of the fields from rows no. 1 and no. 4. Note that eq. (7) can also be read directly from Table 1, since the residuum $(TO_0+z\tau_0)/(z\text{-}1)$ in eq. (7), according to ref. (5), is the same for $TFLL_n$ of any order. We can also see that the first conditional equation in line no. 4, given as "b_2+b_1-1=0", agrees with eq. (8), which was derived mathematically. If we divide the numerator $[-z+b_1\text{-}1]$, given in eq. (9) by $(z\text{-}1)$, we will get (-1) if the second conditional equation $[b_1\text{-}2=0]$ is satisfied. In this case $[-z+b_1\text{-}1]$ turns into $[-(z\text{-}1)]$. This result can also be obtained using rows nr. 1 and row nr. 5 in Table 1. The result $[-(z\text{-}1)]$ agrees with the numerators in eqs. (10) and (11). The second conditional equation shown in Table 1 agrees with eq. (12). The first conditional equation appears in eq. (14) and the second conditional equation appears in eq. (13) as well.

So, all eqs. (3)-(5) and (7)-(15) related to $TFLL_2$ can be derived from Table 1. Table 1 does not offer the initial conditions TO_0 and τ_0. However, since TO_0 and τ_0 appear in the same form for $TFLL_n$ of any order, they will be simply added to the expressions related to $TFLL_n$ of any order in the form in which they already exist in the expressions for $TFLL_2$. The described example of general Table 1, related to $TFLL_2$, proves the correctness of the table approach and shows how to fill in the table for $TFLL_n$ of any order.

To discover additional properties of the described tabular approach, let us create a universal Table 2 for the third-order $TFLL_3$. We will omit the auxiliary outer rows in Table 2. To form Table 2, we will apply the established rules for filling in Table 1. In addition to the constant sequence consisting of the constant "-1", the initial data that we have to enter in Table 2 are shown in rows no. 2 and no. 3. The main outer row no. 2 always starts with z^n and ends with z^0, where "n" is the order of $TFLL_n$. Since in this case n=3, we entered z^3, z^2, z^1, and z^0 in row no. 2. The row no. 3 always starts with "-1" and the rest

Table 2

The universal table related to $TFLL_3$

1	z^2	z^1	z^0		
2	z^3	z^2	z^1	z^0	
3	-1	b1	b2	b3	Con.eqs.=0
4	0	-1	b1-1	b2+b1-1	b3+b2+b1-1
5	0	0	-1	b1-2	b2+2b1-3
6	0	0	0	-1	b1-3

CODE

A	
B	X

X=A+B

of row no. 3 is filled with the system parameters b_1, b_2, …, b_n. Since in this particular case n=3, we entered parameters b_1, b_2, and b_3 in row no. 3. After completing rows no. 2 and no. 3, we entered zeros in the shown field. At last, we filled in the rest of Table 2, bounded by solid lines, in accordance with the Code shown next to Table 2.

Note that the conditional equations from Table 1 remain identical and in the same place in Table 2, but they are now not in the conditional equations' column of Table 2. The conditional equations column in Table 2 now applies only to $TFLL_3$. Thanks to this property, we can use the Tables intended for the higher-order $TFLL_n$ simultaneously to analyze all lower-order $TFLL_n$. The only difference between the original Table 1 and the Table 1 that is inside Table 2 is the two additions in Table 2, namely parameter b_3 in row no. 3 and "-1" in row no. 6. Both additions are considered only when Table 2 is used for the analysis of $TFLL_3$.

We previously proved that all the mathematically derived expressions related to $TFLL_2$ given by eqs. (3)-(5) and (7)-(15), can be obtained using the universal Table 1. In addition to the analysis included in the mentioned equations, the universal tables can also be used to determine the error by which the output period tracks the input period. We will show this using the example of Table 2, which refers to $TFLL_3$. In [2], we performed an error analysis of how the output period of $TFLL_3$ tracks the input period. When the input period is a constant, *i.e.*, TI_k=TI, the error obtained is K_0. If the input period *is* $TI_k=p\cdot k$ (p is the time constant), the error obtained is K_1, and if the input period is $TI_k=p\cdot k^2$, the error obtained is K_2. The expressions K_0, K_1 and K_2 are borrowed from [2] and shown in eqs. (16)-(18). According to eqs. (16)-(18), all the tracking errors K_0, K_1 and K_2 are equal to zeros if the conditional equations for $TFLL_3$, shown in Table 2, are equal to zeros. Therefore, all expressions (16)-(18) can also be obtained directly from Table 2.

$$\mathrm{K}_0 = TI \cdot (b_1 + b_2 + b_3 - 1), \tag{16}$$

$$\mathrm{K}_1 = p \cdot (2b_1 + b_2 - 3), \tag{17}$$

$$\mathrm{K}_2 = 2p \cdot (b_1 - 3), \tag{18}$$

Applying the established rules for filling Table 1 and Table 2, we created Table 3, which corresponds to FLL_5. In addition to the constant sequence consisting of the constant "-1", the basic data for the development of Table 3 are entered in rows no. 4 and no. 5. Row no. 4 was filled in accordance with the previously given instructions, *i.e.*, with z^5, z^4, z^3, z^2, z^1, and z^0. The row no. 5 is filled with -1, b_1, b_2, b_3, b_4 and b_5, where b_1 to b_5 are the system parameters of $TFLL_5$. After entering zeros in the corresponding fields, in the other fields of Table 3, i. e., in the part delimited by bold lines, all expressions have been entered in accordance with the Code shown near Table 3. In Table 3, we have also shown only the main outer rows no. 1, no. 2, and no. 3 relating to $TFFL_2$, $TFFL_3$ and $TFFL_4$ respectively. The main rows no. 1 to no. 4 will become the auxiliary rows for $TFLL_2$ to $TFLL_5$

Table 3

The universal table related to $TFLL_5$

1	z^2	z^1	z^0	C.eqs.=0			
2	z^3	z^2	z^1	z^0	Con.eqs.=0		
3	z^4	z^3	z^2	z^1	z^0	Con.eqs.=0	
4	z^5	z^4	z^3	z^2	z^1	z^0	
5	-1	b1	b2	b3	b4	b5	Con.eqs.=0
6	0	-1	b1-1	b2+b1-1	b3+b2+b1-1	b4+b3+b2+b1-1	b5+b4+b3+b2+b1-1
7	0	0	-1	b1-2	b2+2b1-3	b3+2b2+3b1-4	b4+2b3+3b2+4b1-5
8	0	0	0	-1	b1-3	b2+3b1-6	b3+3b2+6b1-10
9	0	0	0	0	-1	b1-4	b2+4b1-10
10	0	0	0	0	0	-1	b1-5

CODE

A	
B	X

X=A+B

if we omit their first fields z^2, z^3, z^4 and z^5. The contents of Tables related to $TFLL_2$, $TFLL_3$, and $TFLL_4$ are also an integral part of Table 3. Taking in account the properties already established for Table 1 and Table 2, we can conclude that Table 3 can be used to analyze all $TFLL_2$, $TFLL_3$, $TFLL_4$ and $TFLL_5$. The explanation of how to use Table 2 as Table 1 is also valid for the case when we use Table 3 in order to analyze $TFLL_2$, $TFLL_3$, and $TFLL_4$. In these cases, we will not use all the fields in Table 3, bounded by bold lines. For instance, in the column "Conditional equations" for $TFLL_2$, there are only two conditional equations. That are $b_2+b_1-1=0$ and $b_1-2=0$, while the first and the last field of the column, containing parameter b_3 and "-1", will be used only in analysis of the higher order TFLLs, that is in analysis of $TFLL_3$, $TFLL_4$ and $TFLL_5$. The number of conditional equations is equal to the order of a $TFLL_n$.

In the same way, applying the described procedure, Table 3 can be further extended to Tables corresponding to $TFLL_n$ of any order.

3. THE OTHER APPLICATIONS OF TABLES

In the previous chapter, TFLL's applications in the field of tracking and prediction were selected as suitable for creating universal tables, because almost all kinds of different analyses are necessary to be used in these applications. As we could see, all of the analyzes are covered by the corresponding universal tables, enabling to escape very long mathematical procedures. In addition to the described applications of TFLLs, universal tables can be used to analyze the TFLLs described in [1–11] as well as in all their future applications. It was explained in [6] that the processing of the input pulse signal period is a special case of the linear discrete system, because such TFLLs are used in real-time applications. Since in this case the output period overlaps with the input period in real time, the output period TO_k cannot be expressed as a function of the input period TI_k occurring at the same discrete time t_k, because at the moment of calculating TO_k, the input period TI_k might not have ended. The output period can only be calculated using the previous input periods if a TFLL is to work in real time. This fact determined the complete theory of TFLLs, described in [1–11], which describe TFLLs that operate in real time.

However, if the TFLL does not have to operate in real time, or it operates with some time delay greater than the period of the input signal, then the difference equation describing a TFLL may take the identical form, which is used for classic digital filters. That is actually the full form of the difference equation describing any kind of linear discrete system. In this case, the theory and the MATLAB software, intended for classical digital filters, would be simpler to apply to the design and analysis of Time-digital filters and to the frequency analysis of all TFLLs. As for the universal tables, they would require certain minor changes in this case. Let's develop a universal table for this case, which refers to the second-order $TFLL_2$.

The difference equation for $TFLL_2$, intended to function in real time, is given in eq. (1). It contains only two parameters b_1 and b_2. If $TFLL_2$ is not intended to work in real time, we can also use TI_k in calculating the output period TO_k, because in that case we have the measured TI_k available at any time. Due to this fact, we have to introduce an additional parameter "b_0", just like with classic digital filters. In this case, the difference equation of $TFLL_2$ contains three parameters b_0, b_1, and b_2, as shown in eq. (19). Equation (2), referring the time difference τ_k, is valid in this case too. To determine the universal table for this case, we will first calculate the output $TO(z)$ using eq. (19). If we perform the same Z transform procedure as for eq. (1), we will get $TO(z)$, given in eq. (20). Equation (20) does not depend on the initial condition TO_0, as in eq. (3). Based on eq. (20), the transfer function $H_{TO}(z)$ is shown in eq. (21). In order to form the universal table, we need the expression $[H_{TO}(z)\text{-}1]$, which is shown in eq. (22). Based on eq. (22), we can form the corresponding universal Table 4 in the same way as we formed Table 1 based on eq. (5), which refers to the case when $TFLL_2$ is supposed to function in real time. Comparing Table 4 with Table 1, we can see certain differences. First, Table 4 operates with three system parameters while Table 1 uses two system parameters. Both of them have two conditional equations, but the second conditional equation $b_1+2b_0-2=0$ of Table 4, uses two parameters b_1 and b_0. Together with the first conditional equation $b_2+b_1+b_0-1=0$, those two equations give an infinite number of solutions for the optimal system parameters. Therefore, the choice of the optimal system parameters is very wide in this case, while the solution of the optimal parameters is unique when we use Table 1. Note also that instead of (-1) in Table 1, $(b_0\text{-}1)$ is used in Table 4.

$$TO_{k+2}=\mathrm{b}_0 TO_{k+2}+\mathrm{b}_1 TO_{k+1}+\mathrm{b}_2 TO_k, \tag{19}$$

$$TO(\mathrm{z})=TI(z)\frac{\mathrm{z}^2\mathrm{b}_0+\mathrm{z}\mathrm{b}_1+\mathrm{b}_2}{\mathrm{z}^2}, \tag{20}$$

$$\mathrm{H}_{\mathrm{TO}}(\mathrm{z})=\frac{TO(z)}{TI(z)}=\frac{\mathrm{z}^2\mathrm{b}_0+\mathrm{z}\mathrm{b}_1+\mathrm{b}_2}{\mathrm{z}^2}, \tag{21}$$

$$\mathrm{H}_{\mathrm{TO}}(\mathrm{z})\text{-}1=\frac{z^2(b_0-1)+zb_1+b_2]}{\mathrm{z}^2}, \tag{22}$$

Table 4

The universal table related to full form of $TFLL_2$

1	z^2	z^1	z^0	
2	b0-1	b1	b2	Con.eqs.=0
3	0	b0-1	b1+b0-1	b2+b1+b0-1
4	0	0	b0-1	b1+2b0-2

CODE

A	
B	X

X=A+B

We also need to investigate in which expressions the initial conditions TO_0 and τ_0 appear. We have already seen that in this case the initial condition TO_0 does not appear in $TO(z)$, eq. (20). In an identical way, as previously done for $TFLL_2$ related to Table 1, we can calculate $\tau(z)$, which is shown in eq. (23). As expected, the numerator of the fraction associated with $TI(z)$ can be read using rows no. 3 and no. 1 of Table 4. The residuum in eq. (23) contains only the initial condition τ_0. This residuum $z\tau_0/(z\text{-}1)$ will be the same for $TFLL_n$ of any order, so we can determine $\tau(z)$ directly from the corresponding universal table of any $TFLL_n$. In an identical way, as previously done for $TFLL_2$ related to Table 1, we can calculate τ_∞ for the input period TI_k=TI =constant, which is shown in eq. (24). The expression

$$\tau(\mathrm{z})=TI(z)\frac{\mathrm{z}(\mathrm{b}_0-1)+\mathrm{b}_1+\mathrm{b}_0-1}{z^2}+\frac{z\tau_0}{z-1}, \tag{23}$$

$$\tau_\infty=\lim_{z\to 1}[(\mathrm{z}\text{-}1)\tau(z)]=TI\cdot(\mathrm{b}_1+2\mathrm{b}_0-2)+\tau_0, \tag{24}$$

$(\mathrm{b}_1+2\mathrm{b}_0-2)$ associated with TI is the conditional equation presented in row no. 4 of Table 4. The time difference τ_∞ that any $TFLL_n$ reaches in the stable state can be determined directly from the corresponding universal table since the constants TI and τ_0 in eq. (24) will be the same for $TFLL_n$ of any order. This is just a check of the correctness of Table 4. The other expressions in Table 1 can also be obtained from Table 4.

We can conclude that if a TFLL does not operate in real time, its difference equation can possess the full form, just like classical digital filters. Using the example of the second-order $TFLL_2$, we showed how we can create the universal table for $TFLL_n$ of any order, avoiding the mathematical operations. For instance, if we want to create the universal table for the third-order $TFLL_3$, we will first enter diagonally the constant sequence consisting of the constants "b_0-1". After that we will simply enter z^3, z^2, z^1 and z^0 in the row no. 1 as well as b_1, b_2 and b_3 after $(b_0\text{-}1)$ in the row no. 2 of the Table 4. The other fields of such a table will be filled using the same code and the same rules previously defined. This means that the tables of the higher-order systems contain all the tables of the lower-order systems. Therefore, the analysis of all lower-order $TFLL_i$ can be performed using the universal table of a higher-order $TFLL_n$, where $n > i$.

Finally, one very important conclusion is that the universal tables can be used to analyse any linear discrete system without using very long mathematical procedures. Note that some expressions may contain initial conditions that are not included in the tables, but they appear in an identical form for $TFLL_n$ of any order. Therefore, it is not difficult to take them into account when determining the expressions in which the initial conditions figure.

4. FREQUENCY RESPONSES OF $TFLL_2$

In [1–11], the difference equations were chosen to enable TFLLs to work in real time. For the first time, in this article we consider a TFLL which is not intended to operate in real time. The description of this case was considered for the second-order $TFLL_2$, which, instead of two, uses three parameters, giving the possibility of a wider choice of $TFLL_2$ performances. It is of interest to see how the third parameter b_0 affects the frequency responses of the full version $TFLL_2$. We will limit the frequency analysis only to the cases when optimal system parameters are used, that is, when both conditional equations from Table 4 are equal to zero.

Let us determine the frequency response of the transfer function $H_{TO}(z)$, which is shown in eq. (21). Based on the transfer function $H_{TO}(z)$, the corresponding "b" vector is $b_{TO}=[b_0\ b_1\ b_2]$. According to two conditional equations from Table 4, if we choose $b_0=0$, we will get $b_1=2$ and $b_2=-1$. Note that with this choice of parameters, we have reduced full version $TFLL_2$ defined by three parameters, to $TFLL_2$ defined by two parameters, eq. (1). This approach will allow us to discover the influence of the parameter b_0 on the frequency response of the system. Using the command "freqz (b_{TO}, 1, 1024, fs)", the frequency response of $H_{TO}(z)$ is determined and presented in Fig. 2a, for half of the sample rate f_s=20 kHz. We can see in Fig. 2a that the first part of the magnitude, expressed in [dB], is equal to zero for lower frequencies. We can also see in Fig. 2a that the system provides a zero phase difference between the output and input signals for lower frequencies. These two parts of the magnitude and phase, close to a frequency of 0 Hz, are labeled "0 dB" and "0°" in Fig. 2a. However, the second part of the magnitude in Fig. 2a increases significantly with increasing frequency. We have also seen this property in the analysis of $TFLL_3$, defined to provide the tracking of the very fast changes in the periods of the input signals [1].

Fig. 2b shows the changes of the magnitudes in dependence on parameter b_0 for $b_0=1$, $b_0=0$, $b_0=-1$ and $b_0=-3$. Using the conditional equations from Table 4, we can calculate that if $b_0=1$, $b_1=b_2=0$, and according to eq. (19) $TO_{k+2}=TI_{k+2}$. It means that, in this case, the magnitude and phase of the frequency response are zero, *i.e.*, $TFLL_2$ does not make any change to the input signal at its output. In accordance with this conclusion, it can be seen in Fig. 2b that the magnitude is equal to zero for $b_0=1$. However, if $b_0\neq1$, the magnitude and phase of the frequency response will differ from zero. Fig. 2b shows the cases when the used values of b_0 decrease in comparison to $b_0=1$. We can conclude that the magnitudes increase as the difference between the currently used b_0 and $b_0=1$ increases. In other words, the amplification of the input period increases with the increase in the difference between the used b_0 and $b_0=1$. In Fig. 2b, the case $b_0=0$ refers to $TFLL_2$ defined by two parameters, while the cases $b_0=-1$ and $b_0=-3$ refer to $TFLL_2$ defined by three parameters. We can conclude that the additional parameter b_0 has an enormous effect on the increase in magnitude.

Therefore, the full version of $TFLL_2$, which is defined with three parameters, possesses significantly greater ability to track very rapid changes in the input signal period than $TFLL_2$ defined by two parameters. In addition, this version of $TFLL_2$ also has the ability to provide a very wide range of frequency responses, enabling adjustment of the abilities of $TFLL_2$ to track the input period according to the needs.

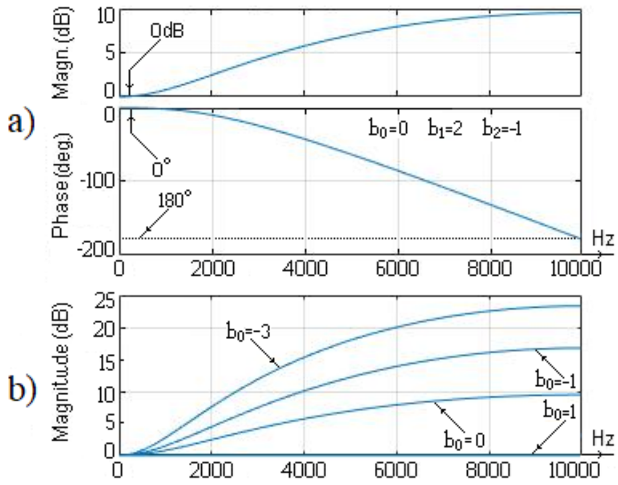

Fig. 2 – a) The frequency response of $H_{TO}(z)$ for $b_0=0$, $b_1=2$ and $b_2=-1$, **b**) The magnitudes of $H_{TO}(z)$ as a function of b_0.

5. CONCLUSION

This article describes the development and application of the universal tables which are used for analyses of TFLLs instead of the long mathematical analyses. The main part of the article is devoted to the universal table related to TFLLs, intended to operate in real time [1–11]. It is characteristic of this group of TFLLs that the period of the output signal can be calculated only based on the previous periods of the input signal, which can be measured before the calculation of the output period starts.

However, universal tables can be used instead of lengthy mathematical analyses to analyze other discrete linear systems. This is demonstrated in the article through the development of the universal table of $TFLL_2$ in which the output period is calculated with a time delay. In such cases, the simultaneous input period can be taken into account to calculate the output period. Such TFLLs are described by the full form of the difference equations, just as in the case with the classical FIR digital filters. Such TFLLs use the modified universal tables and possess different system characteristics in comparison to TFLLs intended to operate in real time.

Usage of universal tables for the analysis of linear discrete systems, instead of long mathematical procedures, is not described in the literature up to now. Unlike the one-dimensional mathematical sequences, well known in mathematics, these tabular methods represent the combination of two-dimensional infinite mathematical sequences, which are formed using the special code. Let us mention the well-known one-dimensional Fibonacci sequence, in which each number of the sequence is the sum of the two previous numbers. Universal tables are two-dimensional sequences, whose fields are filled based on neighboring fields. They expand triangularly to infinity as the order of the system increases. It is important to point out that a special kind of universal tables is also described in [1,2]. Only numbers appear in them. They serve to determine the optimal parameters of the system and the coefficients of the conditional equations, while the universal tables described in this article contain complete mathematical expressions, based on which we can determine the final equations that are obtained after the complete system mathematical analyses. Note that the universal tables of the higher-order systems can also be used for the analysis of the lower-order systems.

To demonstrate the application of universal tables in the article, applications of TFLLs in the field of tracking and predicting very rapidly changing periods of the input signal were used. These applications include very complex, extensive, and comprehensive analyses that include almost all-important system analyses. Because of that, they are very suitable for the development and checking of the correctness of universal tables. However, universal tables can be used in all other applications of TFLLs and in the analysis of all other linear discrete systems.

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