



ANALYSIS OF THE TIME-FREQUENCY LOCKED LOOPS USING TABLES – PART 1

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Time-frequency locked loops (Time-FLL) are linear discrete systems based on the processing of the input signal periods. This article describes one tabular approach to analyzing Time-FLL. This tabular method of analysis enables the complete escaping of a very long mathematical procedure. The approach is presented in the tracking and prediction applications of Time-FLLs, but it can also be used in its other applications, for instance, in digital filtering of the input periods, noise suppression, and others. This approach can also be applied to any linear discrete system. The paper first presents a mathematical procedure for analyzing systems of the lowest order. Based on these results, the table approach was discovered and extended up to Time-FLLs of the tenth order. The table can be simply extended to the Time-FLL of any order. Mathematical analysis and time-domain simulation were carried out, as well as frequency-domain analysis. To prove the correctness of the tabular approach, the properties of the third-order Time-FLL₃ and the seventh-order Time-FLL₇ are demonstrated in the time and frequency domains.

1. INTRODUCTION

Time-FLL, Time-PLL (time-phase locked loop), and Time-digital filter, presented in [1–12], represent a completely new scientific field. They are based on the processing of the periods of the input and output impulse signals and the time differences between them. The applications of these systems are numerous. In addition to the design of Time IIR and Time FIR digital filters [1–4], they are applied in the field of tracking and prediction, phase and time shifting, frequency multipliers, frequency synthesizers, noise rejections, frequency measurement, and others [5–12]. All these systems have a common specific characteristic, which is that all their inputs and outputs are time variables. Therefore, in the analysis of these systems, we observe time variables in the function of time.

In [5], we replaced the long mathematical procedure with the simple tabular approach in finding optimal parameters of Time-FLLs for the case when Time-FLLs were used in the tracking of very fast, changeable input periods. In this article, we will describe a new tabular method and show that, apart from finding the optimal parameters, the tabular methods can be used for many other system analyses. The paper will first show how the conditional equations can be found using the mathematical procedure. These conditional equations show what relationship between parameters must exist in order for a Time-FLL to possess the desired properties. After we found the conditional equations, we discovered the relations that exist between them for Time-FLLs of different orders. Based on this, we formed a table from which the conditional equations can be determined directly from the table for Time-FLLs of any order. In this way, we can avoid very long mathematical operations, which increase trigonometrically with increasing order of the Time-FLL systems. We will see later that the solutions to the conditional equations yield the optimal parameters of a Time-FLL, as described in [5]. Apart from that, these conditional equations appear in the expressions for the Z-transforms and the final values of the output variables. This will be shown for some Time-FLLs in the tracking and predicting applications. In addition to the mentioned applications, the conditional equations can also be used for analyses in other applications of Time-FLLs. At the same time, the tabular methods can be used in the analysis of all linear discrete systems, including those applications described in [1–12].

The applications of Time-FLLs described in [6–12] are also important for this article, although they are implemented with digital circuit technology. These articles illustrate how to design electronic circuits that ensure the functioning of the Time-FLLs according to the described mathematical model and how to perform a mathematical analysis of these systems using the theory of linear discrete systems and the Z-transform technique. They also illustrate how to simulate the operation and analyze these systems in the time domain. These articles also explain the physical meaning of the input and output variables of Time-FLLs, obtained in various analyses.

It is necessary to emphasize that, since the fields Time-FLL, Time-PLL, and Time-digital filters are completely new in the literature, several articles by the same author are cited in the references, because there are no other scientific articles in the literature that cover the mentioned issue.

Articles and books in [13–28] are used as a theoretical basis, covering the scientific disciplines on which this article relies, such as linear discrete systems, digital filters, signal processing, phase and frequency locked loops, electronics, and others.

2. CONDITIONAL EQUATIONS AND TABLES

To develop the mentioned tabular method, we will first mathematically develop the conditional equations for the lowest orders of Time-FLLs, *i.e.*, for the second and third order Time-FLLs. Based on these results, we will discover the rules based on which we will define a tabular approach for finding conditional equations.

The general time relations between the input and output variables are shown in Fig. 1. Figure 1 will serve to define the system equations of the second-order Time-FLL₂ and the third-order Time-FLL₃. Figure 1 shows a general case of an input signal S_{in} and an output signal S_{op} of a Time-FLL. The periods TI_k and TO_k , as well as the time difference τ_k , occur at discrete times $t_k, t_{k+1}, t_{k+2}, t_{k+3}$, and t_{k+4} , which are defined by the falling edges of the pulses of S_{op} in Fig. 1.

Let us now analyze Time-FLL₂, which is described by eq. (1). It represents the general difference equation where b_1 and b_2 are the system parameters. The difference equation of the time difference τ_k is shown in eq. (2). Equation (2) shows the natural relation between the variables in Fig. 1. Time difference τ_k , as the second output variable, will serve for

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very important different analyses of Time-FLL₂ in both transient and stable states.

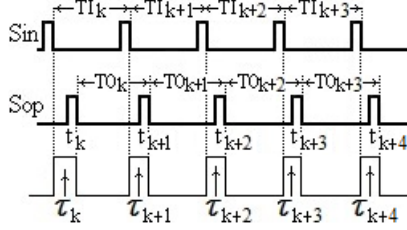


Fig. 1 – The time relations between the input and output variables of Time-FLLs.

$$TO_{k+2} = b_1 TO_{k+1} + b_2 TO_k, \quad (1)$$

$$\tau_{k+1} = \tau_k + TO_k - TI_k, \quad (2)$$

Using the Z transform technique and the theory of linear discrete systems, FLL₂ was analyzed in ref. [3]. Z transform of TO_k , shown in eq. (3), is borrowed eq. (3) from ref. [3]. TO_0 in eq. (3) is the initial value of TO_k for $k=0$. According to eq. (3), the Z transform of the transfer function $H_{TO}(z) = TO(z)/TI(z)$ is determined and shown in eq. (4). In order to discover the conditional equations, we will use the output variable time difference τ_k . The Z transform of eq. (2) is presented in eq. (5), where τ_0 is the initial condition of τ_k . Based on eq. (5), $\tau(z)$ is calculated and shown in eq. (6). Using $H_{TO}(z) = TO(z)/TI(z)$ given in eq. (4), $\tau(z)$ shown in eq. (6) is changed to eq. (7). Since $H_{TO}(z) - 1 = (-z^2 + zb_1 + b_2)/z^2$, $\tau(z)$ can be turned to eq. (8). Note that if $TI_k = \Pi = \text{constant}$, $Z(TI_k) = \Pi \cdot z/(z-1)$, if $TI_k = p \cdot k$ (p is constant), $Z(TI_k) = p \cdot z/(z-1)^2$, if $TI_k = p \cdot k^2$, $Z(TI_k) = p \cdot z(z+1)/(z-1)^3$ and so on. We can see that if TI_k is an exponential function of order "i" ($TI_k(k) = p \cdot k^i$), in the denominator of $Z[TI_k(k)]$ appears $(z-1)^{i+1}$. In order to transform $\tau(z)$ shown in eq. (8), we need first to divide the numerator $(-z^2 + zb_1 + b_2)$ by $(z-1)$. This division will give $(-z^2 + zb_1 + b_2)/(z-1) = -z + b_1 - 1$ if condition given by eq. (9) is satisfied. If eq. (9) is satisfied, $\tau(z)$ shown in eq. (8) can be changed into eq. (10). If we then divide $(-z + b_1 - 1)$ by $(z-1)$ we will get $(-z + b_1 - 1)/(z-1) = -1$ if eq. (11) is satisfied. Expressions (9) and (11) are the conditional equations of FLL₂. If they are both satisfied, $\tau(z)$ shown in eq. (10) can be changed into eq. (12). The solutions of eqs. (9) and (11), $b_1=2$ and $b_2=-1$, are the optimal parameters for FLL₂, which are described in ref. [5]. For these parameters, Time-FLL₂ possesses the maximum ability to track the rapidly changing input periods. Substituting now $TI(z) = \Pi \cdot z/(z-1)$ into eq. (10) and using the final value theorem, it is possible to find the final value of the time difference $\tau_\infty = \lim_{k \rightarrow \infty} \tau(k)$ using $\tau(z)$, given by eq. (13). Note that the second conditional equation "b₁-2" appears in eq. (13). If $b_1-2=0$, there is no effect of the input signal on the final value τ_∞ . At the same time, substituting $TI(z) = \Pi \cdot z/(z-1)$ into eq. (3) and using the final value theorem, it is possible to find the final value of the output period TO_∞ , which Time-FLL₂ reaches in the stable state. We can calculate $TO_\infty = \lim_{k \rightarrow \infty} TO(k)$ if $k \rightarrow \infty$, using $TO(z)$ as $TO_\infty = \lim_{z \rightarrow 1} [(z-1) \cdot TO(z)]$, when $z \rightarrow 1$. The result is shown in eq. (14). We can see that the first conditional equation appears in eq. (14). Only if $b_1 + b_2 = 1$, $TO_\infty = \Pi$, i.e. FLL₂ can function as the stable system.

Note that, based on the analysis of Time-FLL₂, (H_{TO2-1}) can be expressed in the form of $-(z-1)^2/z^2$ presented in eq. (15), if the conditional equations are satisfied. This will be

used later to define the expression (H_{TO_n-1}) for FLL_n of any order.

$$TO(z) = TI(z) \frac{zb_1 + b_2}{z^2} + TO_0, \quad (3)$$

$$H_{TO}(z) = \frac{TO(z)}{TI(z)} = \frac{zb_1 + b_2}{z^2}, \quad (4)$$

$$z\tau(z) - z\tau_0 = \tau(z) + TO(z) - TI(z), \quad (5)$$

$$\tau(z) = \frac{TO(z) - TI(z)}{z-1} + \frac{z\tau_0}{z-1}, \quad (6)$$

$$\tau(z) = \frac{TI(z) \cdot [H_{TO}(z) - 1]}{z-1} + \frac{z\tau_0}{z-1}, \quad (7)$$

$$\tau(z) = \frac{TI(z)}{z-1} \cdot \frac{-z^2 + zb_1 + b_2}{z^2} + \frac{TO_0 + z\tau_0}{z-1}, \quad (8)$$

$$b_1 + b_2 = 1 \quad \text{or} \quad b_1 + b_2 - 1 = 0, \quad (9)$$

$$\tau(z) = TI(z) \cdot \frac{-z + b_1 - 1}{z^2} + \frac{TO_0 + z\tau_0}{z-1}, \quad (10)$$

$$b_1 = 2 \quad \text{or} \quad b_1 - 2 = 0, \quad (11)$$

$$\tau(z) = TI(z) \cdot \frac{-(z-1)}{z^2} + \frac{TO_0 + z\tau_0}{z-1}, \quad (12)$$

$$\tau_\infty = \lim_{z \rightarrow 1} [(z-1)\tau(z)] = TI \cdot (b_1 - 2) + TO_0 + \tau_0, \quad (13)$$

$$TO_\infty = TI \cdot (b_1 + b_2), \quad (14)$$

$$H_{TO2}(z) - 1 = \frac{-(z-1)^2}{z^2}, \quad (15)$$

Any Time-FLL_n of order "n" is defined by "n" parameters $b_1, b_2, b_3, \dots, b_n$. If we want to find the conditional equations of Time-FLL_n, we have to determine the constants associated with the system parameters in the conditional equations. In order to discover a general approach using tables, instead of using very long mathematical operations, let us first show Table 1, which contains the constants of the already analyzed Time-FLL₂.

Table 1

The constants related to FLL₂

CODE		Par. Constants of		Equation 1: $b_1 + b_2 = 1$
A	B	eq. 1	eq. 2	
	b_2	1	0	Equation 2: $b_1 = 2$
	b_1	1	1	
	=	1	2	
$X = A + B$				

All constants attached to the parameters b_1 and b_2 in the conditional equations (9) and (11), as well as the constants after the equal sign are entered in Table 1. We can determine now eq. (9) from Table 1 as $1 \cdot b_1 + 1 \cdot b_2 = 1$ or $b_1 + b_2 = 1$. In the same way, we can assess eq. (11) from Table 1 as $1 \cdot b_1 = 2$ or $b_1 = 2$. Therefore, based on Table 1, we determined the conditional equations for Time-FLL₂, which are defined by eqs. (9) and (11). Note that for all Time-FLL_n systems of any order, the sum of parameters "b" must be equal to unity,

which is for Time-FLL₂ expressed by the first conditional equation next to Table 1. When the constants of the first conditional equation are entered, all other fields in the same row of Table 1 are filled according to the shown code next to Table 1. We can conclude that there is an interdependence between the contents of the fields of Table 1, expressed by the code next to Table 1. We will use this code to develop higher-order Time-FLL Tables.

If the preceding procedures and conclusions are valid, we can extend Table 1 to a Table covering all constants related to any *n*-th-order Time-FLL_{*n*}. Note that the first subsequent expansion of Table 1 will contain all FLL₃ constants, the second expansion of Table 1 will contain all Time-FLL₄ constants, and so on. We should check each new Table mathematically. But since the mathematical procedure in finding the conditional equations increases rapidly as the order of the system increases, let us first form Table 2, which covers all the constants related to Time-FLL₃. Using the code next to Table 1, Table 2 related to FLL₃ is generated by extending Table 1. Using Table 2, the conditional equations related to Time-FLL₃ are determined in the same way as for Table 1 and presented next to Table 2.

Table 2
The constants related to FLL₃

CODE	Par. 'b'	Constants of			Equation 1: b ₁ +b ₂ +b ₃ =1
		eq.1	eq.2	eq.3	
A B	b ₃	1	0	0	Equation 2: 2b ₁ +b ₂ =3
X	b ₂	1	1	0	
X=A+B	b ₁	1	2	1	Equation 3: b ₁ =3
=		1	3	3	

Let us mathematically form the conditional equations related to FLL₃ and verify that the conditional equations next to Table 2 are correct. The difference equations describing FLL₃ are presented in eq. (16) and (17). We will use the calculated $TO(z)$ given in eq. (12) from the analysis of FLL₃, performed in ref. [5]. $TO(z)$ is presented in eq. (18) in this article. Based on eq. (18), the transfer function $H_{TO3}(z)$ is determined and presented in eq. (19). The expression $[H_{TO3}(z)-1]$ was calculated and presented in eq. (20). Let's implement successive division of the numerator of $[H_{TO3}-1]$ by $(z-1)$, just like in the described case for FLL₂. After the first division, $[H_{TO3}-1]$ can be expressed as in eq. (21), provided that eq. (22) is satisfied. If we now divide the expression $[-z^2+z(b_1-1)+b_2+b_1-1]$ by $(z-1)$, $[H_{TO3}-1]$ can be expressed as in eq. (23), if eq. (24) is satisfied. Finally, after dividing the expression $(-z+b_1-2)$ by $(z-1)$, $[H_{TO3}-1]$ can be expressed as in eq. (25), provided that eq. (26) is satisfied.

$$TO_{k+3}=b_1TO_{k+2} + b_2TO_{k+1} + b_3TO_k, \quad (16)$$

$$\tau_{k+1}=\tau_k + TO_k - TI_k, \quad (17)$$

$$TO(z) = TI(z) \frac{z^2b_1+zb_2+b_3}{z^3} + TO_0, \quad (18)$$

$$H_{TO3}(z) = \frac{TO(z)}{TI(z)} = \frac{z^2b_1+zb_2+b_3}{z^3}, \quad (19)$$

$$H_{TO3}(z)-1 = \frac{-z^3+z^2b_1+zb_2+b_3}{z^3}, \quad (20)$$

$$H_{TO3}(z)-1 = \frac{(z-1)[-z^2+z(b_1-1)+b_2+b_1]}{z^3}, \quad (21)$$

$$b_1 + b_2 + b_3 = 1 \quad \text{or} \quad b_1 + b_2 + b_3 - 1 = 0, \quad (22)$$

$$H_{TO3}(z)-1 = \frac{(z-1)^2[-z+(b_1-2)]}{z^3}, \quad (23)$$

$$2b_1 + b_2 = 3 \quad \text{or} \quad 2b_1 + b_2 - 3 = 0, \quad (24)$$

$$H_{TO3}(z)-1 = \frac{-(z-1)^3}{z^3}, \quad (25)$$

$$b_1 = 3 \quad \text{or} \quad b_1 - 3 = 0, \quad (26)$$

The expressions (22), (24), and (26) are the conditional equations of Time-FLL₃. They show what kind of relations between the system parameters must exist in order for $[H_{TO3}-1]$ to be expressed as a function of $(z-1)$, which is shown in eq. (25). All of them are in complete agreement with the conditional equations next to Table 2, proving the correctness of Table 2 and, in general, the correctness of FLL analysis using the tabular approach.

The constants of the conditional equations for Time-FLL₁₀ are shown in Table 3. Note that Table 3 contains Table 1 and Table 2, as well as all tables for Time-FLL_{*n*} whose order $n < 10$. If we extend Table 3 using the same code next to Table 10, we can find out the conditional equations for any FLL_{*n*}, that is, for $n > 10$.

Table 3
The constants related to FLL₁₀

CODE	Par. 'b'	Constants of										
		eq.1	eq.2	eq.3	eq.4	eq.5	eq.6	eq.7	eq.8	eq.9	eq.10	
A B	b ₁₀	1	0	0	0	0	0	0	0	0	0	→
X	b ₉	1	1	0	0	0	0	0	0	0	0	→
X=A+B	b ₈	1	2	1	0	0	0	0	0	0	0	→
	b ₇	1	3	3	1	0	0	0	0	0	0	→
	b ₆	1	4	6	4	1	0	0	0	0	0	→
	b ₅	1	5	10	10	5	1	0	0	0	0	→
	b ₄	1	6	15	20	15	6	1	0	0	0	→
	b ₃	1	7	21	35	35	21	7	1	0	0	→
	b ₂	1	8	28	56	70	56	28	8	1	0	→
	b ₁	1	9	36	84	126	126	84	36	9	1	→
	=	1	10	45	120	210	252	210	120	45	10	→

3. USAGE OF THE CONDITIONAL EQUATIONS

Based on eq. (15) related to Time-FLL₂ and eq. (25) related to Time-FLL₃, we can conclude that any $[H_{TO_n}-1]$ for Time-FLL_{*n*} of the *n*-th order can be expressed as in eq. (27), providing that all of "n" conditional equations are satisfied.

$$H_{TO_n}(z)-1 = \frac{-(z-1)^n}{z^n}, \quad (27)$$

The solutions of eq. (22), (24) and (26) are $b_1=3$, $b_2=-3$ and $b_3=1$. Note that these values of the parameters are the optimal system parameters for Time-FLL₃, which provide the highest ability of Time-FLL₃ for the tracking of the rapid changes of the input periods. These solutions are in agreement with optimal system parameters for Time-FLL₃, described in ref. [5]. Besides that, we have already seen in the analysis of

Time-FLL₂ that the conditional eq. (1) and (2) appeared in the final values TO_∞ and τ_∞ , eqs. (14) and (13), corresponding to the stable state of Time-FLL₂. In the application of Time-FLLs, depending on the requirements, it is not necessary that all of the conditional equations are equal to zero. In this way, we get the opportunity for a wider selection of the system parameters, and thus the opportunity to create Time-FLL features that meet some other system requirements, depending on the kind of applications. Because of simplicity, we will illustrate this through various analyses of still low-order Time-FLL₃ and confirm that the conditional equations are integral parts in the various finite expressions. Let us consider the tracking error of Time-FLL₃ in case that the input period is in the form of $TI_i(k)=\text{TI} \cdot k^i$. Using the final value theorem, the tracking error $K_n = \lim_{k \rightarrow \infty} [TO(k) - TI_i(k)] = \lim_{z \rightarrow 1} \{(z-1) \cdot [TO(z) - TI_i(z)]\}$. This expression can be presented in another form, shown in eq. (28). Let us first consider the case $i=0$, that is $TI_0(k) = \text{TI} = \text{constant}$, corresponding to the step function. Z transform of TI_k is $Z[TI_0(k)] = \text{TI} \cdot z/(z-1)$. Note that we previously produced several forms of $[TO(z)-1]$ related to FLL₃, depending on which ones of the conditional equations are satisfied. These forms are shown in eqs. (20), (21), (23) and (25). If we enter into eq. (28) $TI_{i=0}(z) = \text{TI} \cdot z/(z-1)$ and $H_{TO}(z)$, given by eq. (20), we will get K_0 , shown in eq. (29). The output period will track the input period without an error only if $b_1+b_2+b_3=1$, i.e., if the conditional equation number 1, shown in eq. (22), is satisfied. Obviously, an enormous number of parameters satisfy the conditional equation number 1. We are free to select parameters that meet additional system requirements.

$$K_i = \lim_{z \rightarrow 1} [(z-1)\tau(z) \cdot TI_i(z)[H_{TO3}(z)-1], \quad (28)$$

$$K_0 = \text{TI} \cdot (b_1 + b_2 + b_3 - 1), \quad (29)$$

Let us suppose now that the input period $TI_i(k)=p \cdot k^i$, where “p” is a time constant. If $i=1$, $TI_1(k)=p \cdot k$ and $TI_1(z) = Z[TI_1(k)] = Z[p \cdot k] = p \cdot z/(z-1)^2$. If we enter in eq. (28) $TI_1(z)=TI_1(z)$ and $[H_{TO}(z)-1]$ given in eq. (21), we will get the tracking error K_1 , shown in eq. (30). Based on eq. (30), it follows that the output period will track the input period $TI_1(k)$ without an error, only if $2b_1+b_2=3$ and $b_1+b_2+b_3=1$, i.e., if conditional eq. (2) and (1) are satisfied. The number of parameters that satisfy the conditional equation. (2) and (1) are decreased, but we still have a lot of space to meet the rest of the system requirements, choosing the corresponding parameters. If $i=2$, $TI_2(k)=p \cdot k^2$ and $TI_2(z)=Z[TI_2(k)] = Z[p \cdot k^2] = p \cdot z(z+1)/(z-1)^3$. If we enter in eq. (28) $TI_2(z)=TI_2(z)$ and $[H_{TO3}(z)-1]$ given in eq. (23), we will get the tracking error K_2 , shown in eq. (31). Based on eq. (31), it yields that the output period will track the input period $TI_2(k)$ without an error, only if $2b_1+b_2=3$, $b_1+b_2+b_3=1$, and $b_1=3$, i.e., if the conditional equation. (3), (2), and (1), shown in eqs. (26), (24), and (22) are satisfied. Since $b_1=3$, it follows that $b_2=-3$ and $b_3=1$. In this case, there is no possibility for the other choice of parameters. At last, it would be of interest to study what would happen if $i=3$, i.e., if $TI_3(k)=p \cdot k^3$. In this case $Z[TI_3(k)] = Z[TI_3(z)] = Z[p \cdot k^3] = p \cdot z(z^2+4z+1)/(z-1)^4$. If we enter in eq. (28) $TI_3(z)=TI_3(z)$ and $[H_{TO3}(z)-1]$ given in eq. (25), we will get the tracking error K_3 , shown in eq. (32). Based on eqs. (30), (31) and (32), we can conclude that the output period TO will track the input period TI_3 with an error K_3 , if all conditional equations nr. 1 to nr. 3, next to Table 2, are satisfied.

$$K_1 = p \cdot (2b_1 + b_2 - 3), \quad (30)$$

$$K_2 = 2p \cdot (b_1 - 3), \quad (31)$$

$$K_3 = -6p, \quad (32)$$

Let's analyze in which way the conditional equations appear in the expressions for the time difference τ_k . Z transform of eq. (17) is shown in eq. (33). If we enter $TO(z)$ given by eq. (18) into eq. (33) and use the conditional equation 1 ($b_1 + b_2 + b_3 = 1$), next to Table 2, we can calculate $\tau(z)$, shown in eq. (34). In order to get expressions of $\tau(z)$ for the different input $TI(z)$, we added suffix “i” to $\tau(z)$ and $TI(z)$ in eq. (34). The procedure to find out the final values of $\tau_{i\infty}$ for the different values of “i” is the same as the previous one for K_i , given in eq. (28). If $i=0$, $TI_i(k)=\text{TI}=\text{constant}$, we will calculate the final $\tau_{0\infty}$, shown in eq. (35). If $i=1$, we will calculate the final $\tau_{1\infty}$, shown in eq. (36). If $i=2$, we will calculate the final $\tau_{2\infty}$, shown in eq. (37). We can see that the conditional eq. (2) and (3) appeared respectively in eq. (35) and (36). If we implement Time-FLL₃, adapt the initial conditions $TO_0=\tau_0=0$, and if we choose the parameters to satisfy the appropriate conditional equations in eq. (35) and (36), we can reduce $\tau_{0\infty}$ and $\tau_{1\infty}$ to zero. That means that the output period of FLL₃ will ensure tracking of the input period without time delay. However, if $TI_i = p \cdot k^2$, according to eq. (37), the time delay of the output period will be a constant, which depends on “p”, TO_0 , and τ_0 . According to eq. (31), if $b_1=3$, $K_2=0$. It means the output period will still track the input period without any error in the stable state of FLL₃.

$$z\tau(z) - z\tau_0 = \tau(z) + TO(z) - TI(z), \quad (33)$$

$$\tau_i(z) = TI_i(z) \frac{-z^2+z(b_1-1)+b_1+b_2-1}{z^3} + \frac{TO_0+z\tau_0}{z-1}, \quad (34)$$

$$\tau_{0\infty} = \text{TI} \cdot (2b_1 + b_2 - 3) + TO_0 + \tau_0, \quad (35)$$

$$\tau_{1\infty} = p \cdot (b_1 - 3) + TO_0 + \tau_0, \quad (36)$$

$$\tau_{2\infty} = -2p + TO_0 + \tau_0, \quad (37)$$

In order to confirm the important role of conditional equations in tabular analysis of FLL₃, let's simulate a suitable example in the time domain. All discrete values in simulations were merged to form continuous curves. All variables in the following diagram were presented in time units. The time unit can be μsec , msec , or any other, but assuming the same time units for all time variables TI_k , TO_k , K_2 , and τ_2 , it was more suitable to use just “time unit” or abbreviated t.u. in the text. It was more convenient to omit the indication t.u. in the diagrams.

Using eq. (16) and (17), the simulations of TI_k , TO_k , K_2 , and τ_k for the accelerated input $TI_i(k)=10+2 \cdot k^2$ t.u., are made and shown in Fig. 2, together with the used parameters and initial conditions. The simulations are made for two cases. In the case nr. 1 the optimal parameters are used, namely $b_1=3$, $b_2=-3$ and $b_3=1$. For this case the output period tracks the input period without any error, i.e. $K_2=0$. Using eq. (31), we can calculate the same result for this case, that is $K_2=2p \cdot (b_1-$

$3)=2 \cdot 2 \cdot (3-3)=0$ t.u. At the same time, according to eq. (37), time difference $\tau_{21\infty}=-2p+TO_0+\tau_0=-2 \cdot 2+0+0=-4$ t.u. For this case, the same result $\tau_{21\infty}=-4$ t.u. can be seen in Fig. 2. In the case number 2, the used parameters are $b_1=1$, $b_2=1$, and $b_3=-1$. Note that these parameters satisfy conditional eq. (1) and (2), but they do not satisfy eq. nr. 3, i.e., $b_1 \neq 3$. According to eq. (31), for this case, we can calculate the tracking error $K_{22}=2p \cdot (b_1-3)=2 \cdot 2 \cdot (1-3)=-8$ t.u. The same result we can see in Fig. 2. Time difference $\tau_{22\infty}$ for this case tends to infinity in Fig. 2 because of the conditional eq. (3) is not satisfied.

The agreement between the mathematical expressions and the simulation of FLL₃ is proof that both the mathematical analysis and the simulation are correct. Note that we can all final values K_0 , K_1 , K_2 , $\tau_{0\infty}$, and $\tau_{1\infty}$, related to Time-FLL₃, determine from Table 2, since the affection of TO_0 and τ_0 is always the same and the affection of “p” can be defined in the function of power “i”. Using Tables, we can completely escape the long mathematical analysis for Time-FLL_n of any order.

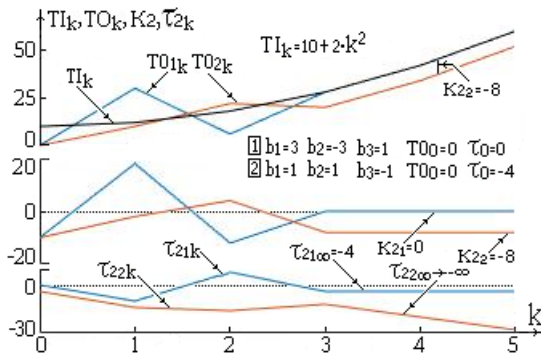


Fig. 2 – Simulation of the input and output variables of FLL3 for the accelerated input period $Tl(k)=10+2 \cdot k^2$ t.u.

4. ANALYSIS OF FLL_s IN THE FREQUENCY DOMAIN

A more precise insight into the operation of Time-FLLs can be obtained if, in addition to the previous time analysis, we also use the frequency analysis. We will limit the frequency analysis only to the cases when optimal system parameters are used, that is, when all conditional equations are satisfied. The frequency responses of FLLs are shown in Fig. 3 for three cases. Figure 3a presents FLL₇ at half the sampling rate, $f_s = 10000$ Hz. Fig. 3b presents FLL₃ for half of the sample rate, $f_s=10000$ Hz. Figure 3c presents FLL₃ for half of the sample rate, $f_s=20000$ Hz. We can see that for all three cases, the first part of the magnitude, for lower frequencies, is equal to zero, expressed in [dB]. It means that, for that part of the frequency range, there is no change in the output period compared to the input period. Approximately in the same frequency range, the system provides a zero phase difference between the output and input periods. These frequency ranges are marked with FR (Frequency range) in Fig. 3. Comparing Fig. 3a and Fig. 3b, we can conclude that, for the same sample rate $f_s=10000$ Hz, FLL₇ provides FR~1.6 kHz, which is approximately twice in comparison to FR~0.8 kHz related to FLL₃. Comparing Fig. 3b and Fig. 3c, we can conclude that FLL₃, in the case that the sample rate is $f_s=20000$ Hz, provides FR~1.6 kHz, which is twice that of FR~0.8 kHz related to FLL₃ in the case of $f_s=10000$ Hz. From here follows the useful conclusion that the frequency range FR, in which the system provides the described

properties, can be extended either by increasing the order of the FLL system or by increasing the sampling rate. Why is the frequency range FR important? We can notice that the finite values of time difference $\tau_{0\infty}$, $\tau_{1\infty}$, and $\tau_{2\infty}$, given by eqs. (35), (36) and (37), consist of two characterized parts.

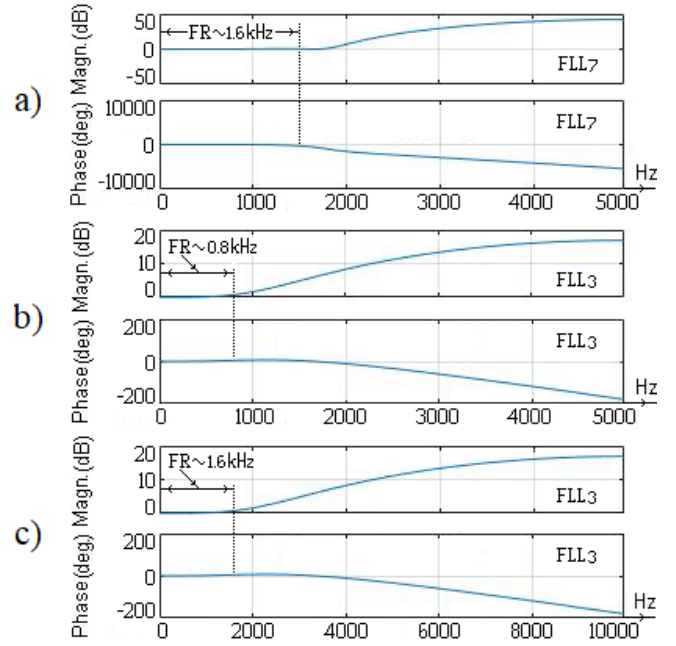


Fig. 3 – a) Frequency response of FLL₇ for $f_s/2=5000$ Hz; b) Frequency response of FLL₃ for $f_s/2=5000$ Hz; c) Frequency response of FLL₃ for $f_s/2=10000$ Hz.

The first part presents the effect of the input period and parameters. The second part depends only on the initial conditions TO_0 and τ_0 . We can always adopt in the realization of any Time-FLL that $TO_0=\tau_0=0$. As shown, we can also choose the optimal parameters of any Time-FLL and eliminate the effect of both the input period and the parameters. In this case, we will get, according to eqs. (35), (36) and (37) that $\tau_{0\infty}=0$, $\tau_{1\infty}=0$, and $\tau_{2\infty}=-2p$. Note that any τ_{∞} is the time difference between the output and input period when Time-FLL is in the stable state. Therefore, the phase difference is $Ph=(\tau_{\infty}/TO_{\infty}) \cdot 360^\circ$. If $\tau_{\infty}=0$, the phase difference $Ph=0$. It yields that if $\tau_{0\infty}=0$, Time-FLL₃ will, for the constant input period, provide the output period which will be in phase with the input period. If $\tau_{1\infty}=0$, it means that FLL₃ can provide the output period in phase with the input period, even in the case when the input period is a velocity function ($Tl_k=p \cdot k$). At last, $\tau_{2\infty}=-2p$ means that FLL₃ is not able to generate the output period in phase with the input period ($Tl_k=p \cdot k^2$). This case is simulated in Fig. 2. In this case, the output period tracks the input period without an error ($K_{21}=0$), but the time difference $\tau_{21\infty}=-2p=-4$ t.u.=constant.

Although Time-FLL_n is an open system without feedback, it possesses some properties of a PLL when using optimal system parameters. In this case, if the input period $Tl_k=p \cdot k^i$, the system can generate an output period that is in phase with the input period, provided that the order of FLL_n is $n \geq i+1$. Could we call this specific kind of open-loop PLL semi-PLL or dynamic-PLL?

5. CONCLUSION

This paper describes a tabular approach to the mathematical analysis of the discrete linear system Time-FLL_n of any order. The mathematical procedure consists of

the successive rearrangement of the transfer function of the observed system into a form that is very suitable for various mathematical analyses. In order for the new forms of the transfer function to be valid, the conditional equations are generated during the successive rearrangement steps. They define relationships between system parameters, which must be satisfied. The article shows how this long mathematical procedure can be avoided by creating tables from which all conditional equations can be obtained directly. For the different rapidly changing input periods, it was shown that conditional equations are the main integral parts of the expressions for the tracking errors and phase differences between the input and output periods. However, if we know the conditional equations, all final results can be determined automatically, without mathematical analysis. That was shown in the usage of Time-FLL₃ for the tracking and predicting applications, but analysis by the tabular approach can be used for all other applications of Time-FLLs as well.

The described approach can be applied to all linear discrete systems. This table approach in the analysis of a discrete linear system has not been described in the literature so far.

It was shown in the article that when Time-FLLs use optimal parameters, the system has some properties of a phase-locked loop, because it eliminates the influence of the input signal and system parameters on the phase difference between the input and output periods, which in this case is reduced to a constant. This constant can be equal to zero or any other value, which can be changed according to the current requirements in the realized device. This property enables the use of Time-FLLs in phase shifters and other applications.

This article indicates that the tabular way of analyzing Time-FLLs can be further expanded and improved in order to extremely simplify the analysis and application of not only these systems but all discrete linear systems.

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