

TIME-FREQUENCY LOCKED LOOPS INTENDED FOR THE TRACKING OF THE PULSE SIGNAL PERIODS

DJURDJE M. PERIŠIĆ¹

Keywords: Time-based frequency locked loop; Tracking; Digital circuit; Discrete linear system; Time-based digital filter.

This article describes the theory of the time-based frequency-locked loop (TFLL), intended for tracking the input signal periods, which change extremely rapidly. They are based on the processing of the input signal periods. The described algorithm, when appropriately implemented, can also be applied to track and predict any physical variable in real time, provided that the pulse signal period is replaced with periodic data from the physical variable. The paper presents a mathematical procedure for determining optimal system parameters for tracking rapidly changing periods. Since this mathematical procedure is very lengthy for higher-order systems, the paper focuses on a new method of abbreviated analysis based on a tabular approach. This approach has not been described in the literature so far. Mathematical analyses in the time domain were performed using the Z transformation. Simulation of the system operation was carried out. For frequency-domain analysis, the theory of FIR digital filters and the corresponding MATLAB software were used. The system's tracking capabilities are demonstrated in the time and frequency domains.

1. INTRODUCTION

The TFLLs are based on processing the periods of the input and output signals and the time differences between them. References [1–11] describe their various applications. The refs. [1–4] describe a completely new type of digital filters based on TFLLs and intended for filtering the period of the pulse signal. References [1,2] describe the infinite impulse response (IIR) time-based digital filters, while refs. [3, 4] describe the finite impulse response (FIR) time-based digital filters.

Refs. [5,6] have already shown that some even low-order TFLLs possess outstanding capabilities for tracking and predicting the period of the input signal. In practical terms, this means that these types of FLLs will not only track changes in the period of the input signal, but they will also be able to predict the values for one or more subsequent periods of the input signal. In other words, TFLLs, as electronic circuits, will recognize the law by which the input period changes and generate a pulse signal at their output, the periods of which will be identical to the periods of the input signal. But in addition, TFLLs will be able to predict the values of one or more subsequent periods of the input signal. This article will analyze the prediction of the first period only. By a similar principle, this theory can be extended to predict more than one subsequent period of the input signal.

In refs. [5,6], the possibilities of applying low-order TFLLs in the field of tracking were only demonstrated. In this article, the general theory of the application of TFLLs in the field of tracking and prediction will be described. First, the mathematical basis of this theory will be presented, and then a tabular method will be developed that will give the same result as the mathematical analysis. The difference between these two approaches is that the mathematical method for higher-order TFLLs is extremely long, while the tabular method is simple and very short.

Some other applications of TFLLs are described in [7–11]. Although the TFLLs in [5–11] are implemented in the digital circuit technique, and they are also important for this article, although microprocessors must be used for the application of the higher-order TFLLs. They are important because they illustrate how these types of FLLs can be realized, how to perform their analysis in the time and frequency domains,

how to simulate the operation of the observed algorithms, how to use the Z transform in the analysis of TFLLs, how to understand the physical meaning of the variables in this new theory, and so on. The articles and books in [12–26] are used as a theoretical base for electronics implementations and for the development necessities.

2. MATHEMATICAL ANALYSIS

Let us first analyze low-order TFLLs mathematically to describe the procedure for finding the system parameters that provide the maximum ability of a TFLL to track the rapidly varying periods of the input signal, and which, at the same time, keep the system stable. The general time relations between the input and output variables are shown in Fig. 1.

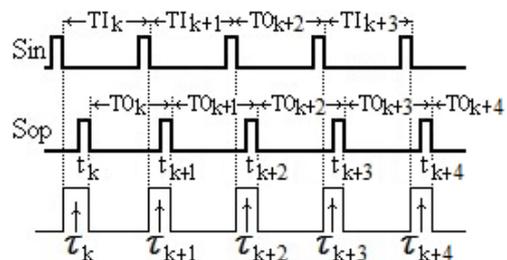


Fig. 1 – The time relations between the input and output variables of TFLLs.

Figure 1 will serve for the definition of the system equations of both the second-order TFLL₂ and the third-order TFLL₃. It represents a general case of an input signal S_{in} and an output signal S_{op} of a TFLL. The periods TI_k and TO_k , as well as the time difference τ_k , occur at discrete times t_k , t_{k+1} , t_{k+2} , t_{k+3} and t_{k+4} , which are defined by the falling edges of the pulses of S_{op} in Fig. 1. We will start with TFLL₂, which is described by eq. (1) and (2). Equation (1) represents the general difference equation that describes TFLL₂, where b_1 and b_2 are the system parameters.

$$TO_{k+2} = b_1 TI_{k+1} + b_2 TI_k. \quad (1)$$

One additional difference equation, describing the second output variable, time difference τ_k , is presented in eq. (2). The output τ_k will serve for the analysis of the transient and stable states of TFLL₂. The output τ_k represents the natural relation between the variables in Fig. 1.

¹ Faculty of Information Technologies, Slobomir P University, Str. Pavlovića put 76, 76300, Slobomir, Republic of Srpska, Bosnia and Herzegovina. E-mail: djurdje@beotel.rs.

$$\tau_{k+1} = \tau_k + TO_k - TI_k, \quad (2)$$

Let us analyse TFL₂ using Z transform. The Z transform of eq. (1) is shown in eq. (3), where TO_0 , TI_0 , represent the initial conditions of variables TO_k and TI_k . Since the process starts at $t=t_k$, all values of variables, before $t=t_k$, are equal to zero. Therefore, according to eq. (1), $TO_1=b_1 \cdot TI_0$. Entering TO_1 into eq. (3), $TO(z)$ is calculated and presented in eq. (4).

Let us suppose that the input is $TI(k) = TI = \text{constant}$. Substituting the Z transform of $TI(k)$ i.e. $TI(z) = TI \cdot z/(z-1)$ into eq. (4) and using the final value theorem, it is possible to find the final value of the output period TO_∞ , which TFL₂ reaches in the stable state. We can calculate $TO_\infty = \lim_{k \rightarrow \infty} TO(k)$ if $k \rightarrow \infty$, using $TO(z)$ as $TO_\infty = \lim_{z \rightarrow 1} [(z-1) \cdot TO(z)]$, when $z \rightarrow 1$. The result is shown in eq. (5). It comes out from eq. (5), that TFL₂ is the stable system, i.e. $TO_\infty=TI$, if eq. (6) is satisfied.

We can get a complete insight into the behavior of TFL₂ only after we analyze the second output variable τ_k . The Z transform of eq. (2) is given in eq. (7). If we enter $TO(z)$ from eq. (4), into eq. (7) we can calculate $\tau(z)$, shown in eq. (8). If we divide the denominator $(-z^2+zb_1+b_2)$ by $(z-1)$, using the condition given by the equation. (6), we can get $(-z^2+zb_1+b_2) = (z-1) \cdot (-z+b_1-1)$. If we enter this result in eq. (8), we will get the final expression for $\tau(z)$, shown in eq. (9).

Substituting now $TI(z) = TI \cdot z/(z-1)$ into eq. (9) and using the final value theorem, it is possible to find the final value of the time difference $\tau_\infty = \lim_{k \rightarrow \infty} \tau(k)$ if $k \rightarrow \infty$, using $\tau(z)$, given by eq. (10). The TFL₂ has the greatest ability to follow fast changes in the input signal if the input period does not affect the time difference $\tau(k)$, in the stable state of the system. According to eq. (10) for TFL₂, that is case when $b_1-2=0$, i.e. if $b_1=2$. If $b_1=2$, it yields from eq. (6) that $b_2=-1$. Note that for the system of the second-order TFL₂, we have two conditions, which determine the required system parameters. These conditions are $b_1+b_2=1$ and $b_1-2=0$. According to eq. (4) and (9), the Z transforms of the transfer functions $H_{TO}(z)=TO(z)/TI(z)$ and $H_\tau(z)=\tau(z)/TI(z)$ are determined and shown in eqs. (11) and (12).

$$z^2 TO(z) - z^2 TO_0 - z TO_1 = zb_1 TI(z) - zb_1 TI_0 + b_2 TI(z), \quad (3)$$

$$TO(z) = TI(z) \frac{zb_1+b_2}{z^2} + TO_0, \quad (4)$$

$$TO_\infty = \lim_{z \rightarrow 1} [(z-1) TO(z)] = TI(b_1 + b_2), \quad (5)$$

$$b_1 + b_2 = 1, \quad (6)$$

$$z\tau(z) - z\tau_0 = \tau(z) + TO(z) - TI(z), \quad (7)$$

$$\tau(z) = \frac{TI(z)}{(z-1)} \cdot \frac{-z^2+zb_1+b_2}{z^2} + \frac{TO_0+z\tau_0}{(z-1)}, \quad (8)$$

$$\tau(z) = TI(z) \cdot \frac{-z+b_1-1}{z^2} + \frac{TO_0+z\tau_0}{(z-1)}, \quad (9)$$

$$\tau_\infty = \lim_{z \rightarrow 1} [(z-1)\tau(z)] = TI(b_1 - 2) + TO_0 + \tau_0, \quad (10)$$

$$H_{TO}(z) = \frac{TO(z)}{TI(z)} = \frac{zb_1+b_2}{z^2}, \quad (11)$$

$$H_\tau(z) = \frac{\tau(z)}{TI(z)} = \frac{-z+b_1-1}{z^2}, \quad (12)$$

After we have determined the transfer functions and

optimal system parameters, the analysis of TFL₂'s ability to track rapid changes in the input period follows. Let us consider the case when the velocity input period $TI_{k+1}=10+p \cdot k$ t.u, is entered into TFL₂, where "p" is the time constant and "k" denotes an ordinal number of discrete time t_k . Z transform of TI_{k+1} is $TI_V(z)=10 \cdot z/(z-1)+p \cdot z/(z-1)^2$.

Let us now calculate the error $K=\lim_{k \rightarrow \infty} (TO_k-TI_k)$, when TFL₂ reaches the stable state. One more suitable expression for K is $K=\lim_{k \rightarrow \infty} TI(k) \cdot [H_{TO}(k)-1]$. Using the final value theorem, K is expressed in eq. (13) by the Z transforms of $TI_V(z)$ and $H_{TO}(z)$ instead of $TI(k)$ and $H_{TO}(k)$. Note that $H_{TO}(z)$ is given by eq. (11).

Another parameter through which we will check the validity of the previous analysis is the time difference $\tau_{V\infty}$, shown in eq. (14). Note that $\tau_{V\infty}$ was calculated for $b_1=2$ and $b_2=-1$ in the same way as for the case of eq. (10), but in this case $\tau(z)=\tau_V(z)$. The expression $\tau_V(z)$ was determined by entering $TI(z)=TI_V(z)$ into eq. (9).

$$K = \lim_{z \rightarrow 1} \{(z-1) \cdot TI(z) [H_{TO}(z) - 1]\}, \quad (13)$$

$$\tau_{V\infty} = \lim_{z \rightarrow 1} [(z-1)\tau_V(z)] = -p + TO_0 + \tau_0, \quad (14)$$

Let us now simulate the functioning of TFL₂ in the time domain to check the correctness of the previous math analysis. All discrete values in simulations were merged to form continuous curves. All variables in the following diagram were presented in time units. The time unit can be μsec , msec , or any other, but assuming the same time units for all time variables TI_k , TO_k and τ_k , it was more suitable to use just "time unit" or abbreviated t.u. in the text. It was more convenient to omit the indication t.u. in the diagrams.

Using eq. (1) and (2), the simulations of TI_k , TO_k and τ_k for the velocity input $TI_k=10+2 \cdot k$ [t.u.] ($p=2$), are made and shown in Fig. 2, together with the used parameters and initial conditions. The simulations of TO_k and τ_k are made for two cases. In case nr. 1, the optimal parameters are used, namely $b_1=2$ and $b_2=-1$. If we enter $TI(z)=TI_V(z)$ for the velocity input period in the equation. (13) and use parameters $b_1=2$ and $b_2=-1$, we will get error $K=K_V=0$. We can see the same result in Fig. 2, since the output period TO_2 tracks the velocity input period without an error.

In case nr. 2 the used parameters are $b_1=0.5$ and $b_2=0.5$. If we enter $TI(z)=TI_V(z)$ for velocity input period in eq. (13) and use parameters $b_1=0.5$ and $b_2=0.5$, we will get error $K=K_V=-3$. We got the same result by simulation in Fig. 2. Note that in both cases the parameters satisfy the condition $b_1+b_2=1$, given by eq. (6), but in case nr. 2, the second condition $b_1-2=0$ is not satisfied. Consequently, in case nr. 2, the output period TO_2 tracks the input period with the constant error $K=K_V=-3$.

Let us now consider the corresponding time differences τ_1 and τ_2 in Fig. 2. If we enter $TI(z)=TI_V(z)$ for velocity input period in eq. (14), derived for the case nr. 1, we will get $\tau_{1\infty} = -p+TO_0+\tau_0 = -2+0+0 = -2$ t.u. This agrees with $\tau_{1\infty}$ generated by simulation in Fig. 2.

In case number 2, we can see in Fig. 2 that $\tau_{2\infty} \rightarrow -\infty$, which agrees with the fact that the second condition $b_1-2=0$ is not satisfied. The complete agreement between the simulations and the mathematical analysis proves the correctness of both.

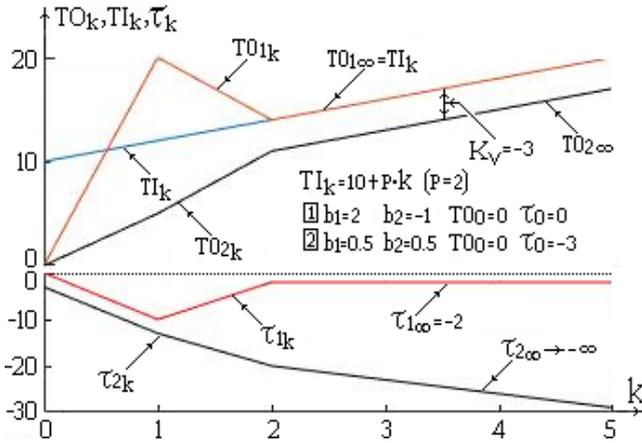


Fig. 2 – Simulation of the input and output variables of TFLL₂ for the velocity input period $TI_k=10+p \cdot k$ ($p=2$). TFLL₂ reaches the stable state in two steps.

To fully examine the abilities of TFLL₂ in tracking highly variable periods, it is necessary to study how this system behaves in the case that the input period is an accelerating function, i.e. $TI_k=10+p \cdot k^2$, where $p=2$ [t.u.].

The simulations of TO_k , TI_k and τ_k are made only for the optimal parameters $b_1=2$ and $b_2=-1$ and shown in Fig. 3. The initial conditions $TO_0=0$ and $\tau_0=0$ in Fig. 3. Z transform of $TI_k=10+p \cdot k^2$ is $TI_A(z)=10 \cdot z/(z-1) + p \cdot z(z+1)/(z-1)^3$. If we enter $TI(z)=TI_A(z)$ for the accelerated input period in eq. (13) and use parameters $b_1=2$ and $b_2=-1$, we will get error $K=K_A=-4$ t.u.

We can see in Fig. 3 that the output period TO_k tracks the input period TI_k with the constant error $K=K_A=-4$. The output τ_∞ in Fig. 3 linearly tends to $-\infty$, which agrees with the fact that TO_k tracks TI_k with a constant error K_A . We have already seen this occurrence in Fig. 2 in case number. 2. The results obtained prove once more the correctness of the complete theory and simulations, which also completely match as in the previous analysis.

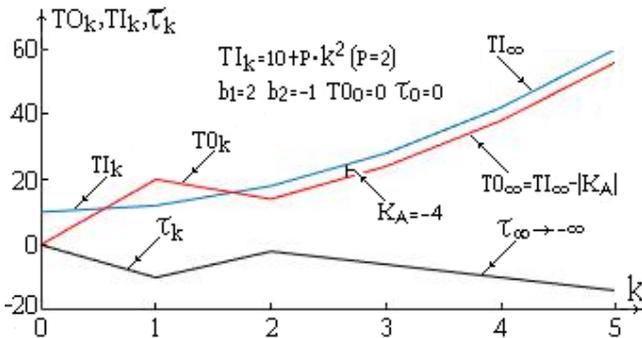


Fig. 3 – Simulation of the input and output variables of TFLL₂ for the accelerated input period $TI_k=10+p \cdot k^2$ ($p=2$). TFLL₂ reaches the stable state in two steps, even for the accelerated input period.

In the previous part, we described the complete analysis that is necessary to carry out in order to find the optimal parameters for TFLL₂ and investigate the upper limit of TFLL₂'s ability to track very fast changes in the input period. This procedure can be applied to TFLL of any order. Although TFLL₂ is a system of very low order, the mathematical procedure is long and complicated. The complexity of this procedure increases geometrically as the order of the system increases. However, to develop a shorter and simpler tabular method of this procedure, it is necessary

to perform an additional analysis of the third-order system TFLL₃. Using two or more examples, we can state the general conclusions that will be valid for TFLL_n of any order.

To shorten the procedure, we will use the results and conclusions from the reference. [3], where the system of the third order TFLL₃ was analyzed. Based on Fig. 1, its difference equation is given in eq. (15), consisting of three system parameters b_1 , b_2 and b_3 . The optimal parameters of TFLL₃, which ensure the maximum ability of the system for tracking very fast changes of the input period, are obtained in the same way as with the previously described system TFLL₂, that is, by successively dividing the expression $[H_{TO}(z)-1]$ by $(z-1)$ with the aim of obtaining the expression $[H_{TO}(z)-1]$ in the form of $-(z-1)^3$.

In the same way as with the previously described system TFLL₂, by solving the equations that are obtained in the form of necessary conditions that the parameters must satisfy, the optimal parameters $b_1=3$, $b_2=-3$ and $b_3=1$ were obtained. These parameters will later be used to develop a tabular method for obtaining the optimal system parameters of any order TFLL_n.

$$TO_{k+3}=b_1TI_{k+2} + b_2TI_{k+1} + b_3TI_k, \quad (15)$$

Before that, based on the results related to TFLL₂ and TFLL₃, we can conclude some general rules, which will be very useful for the development of TFLLs intended for tracking and predictive applications. These rules are valid for TFLL_n of any order:

1. The number of system parameters is equal to the system order "n".
2. The sum of parameters "b" must be equal to one.
3. The number of the necessary divisions of $[H_{TO}(z)-1]$ by $(z-1)$, to get the optimal parameters, is equal to the system order "n". The successive division produces "n" conditional equations which define the relation between the parameters. These "n" conditional equations give the solutions for "n" optimal system parameters b_1, b_2, \dots, b_n . After "n" divisions by $(z-1)$, $[H_{TO}(z)-1]$ turns into form $-(z-1)^n$.
4. The nth-order TFLL_n is able to track changes in the input periods given as $TI_k = \pm p \cdot k^{n-1}$ without any error. The time difference τ_∞ in this case is a constant that does not depend on the input period. It depends on the initial conditions TO_0 and τ_0 only.
5. The nth-order TFLL_n can track changes in the input periods given as $TI_k = \pm p \cdot k^n$ with a constant error. The time difference τ_∞ in this case tends to $-\infty$.
6. The nth-order TFLL_n is able to track changes in the input periods given as $TI_k = \pm p \cdot k^i$ without any error if $0 < i \leq (n-1)$. In this case, the smaller "i" provides wider options in the selection of parameters to meet some additional requirements.
7. The nth-order TFLL_n is not able to track very fast changes in the input periods given as $TI_k = \pm p \cdot k^{n+1}$.
8. The number of necessary steps that TFLL_n takes to reach the stable state is equal to the order of the system "n".

To provide more data in the development of the tabular method of determining the optimal data, the values of the optimal parameters for the fourth-order system TFLL₄ were mathematically derived. The procedure of this analysis is completely identical to the one presented for TFLL₂, but it is long and does not need to be repeated.

Based on Fig. 1, the difference equation of TFLL₄ is shown in eq. (16). It is defined by 4 parameters b₁, b₂, b₃ and b₄. After the mathematical analysis, the obtained optimal parameters are b₁=4, b₂=-6, b₃=4, and b₄=-1.

$$TO_{k+4}=b_1TI_{k+3} + b_2TI_{k+2} + b_3TI_{k+1} + b_4TI_k, \quad (16)$$

3. TABULAR APPROACH TO ANALYSIS

After a long analysis of finding the optimal parameters of different TFLLs, the author noticed that there are corresponding relations between them. The mathematical approach to obtaining the optimal parameters, as we have already seen, is very long. Because of that, it was reasonable to make the additional efforts to discover this new table approach. The optimal system parameters b₁, b₂, b₃, ..., b₁₀ for TFLL₁ to TFLL₁₀ are presented in Table 1. An explanation of how to fill in any X field of Table 1 is given in the section "Code" of Table 1. Let us start from the very beginning. The orders of TFLLs are indicated on the left side of Table 1. The difference equation for the system of the first order TFLL₁ is $TO_{k+1}=b_1 \cdot TI_k$. It is obvious that b₁=1 to satisfy the rule nr. 2 of the previous Section 2.

All the other parameters for TFLL₁ do not exist. Because of that, the rest of the row is filled with zeros in Table 1. We have already found the optimal parameters for TFLL₂ b₁=2

and b₂=-1. They are entered into Table 1. The other parameters for TFLL₂ do not exist and the rest of the row is filled with zeros. The rows corresponding to TFLL₃ and TFLL₄, for which we have already found the optimal parameters mathematically, were filled in the same way like the rows for TFLL₁ and TFLL₂.

After filling the first four rows with mathematically obtained optimal parameters, we can notice that for any field inside them, the rule or pattern X=A-B is satisfied. This applies to all fields within the first four rows, including fields covered even by zeros. This means that each parameter is equal to the difference between the two parameters above. In other words, each parameter of the n-th order TFLL_n is the difference between the same parameter of TFLL_{n-1} and the neighboring lower parameter of TFLL_{n-1}.

For example, parameter b₂=-3-(3)=-6 for TFLL₄. But if we include any field covered by zero, the rule is also valid. For example, parameter b₄=0-(1)=-1 for TFLL₄. We can notice that the parameters b₁ are always equal to "n", i.e. to the order of TFLL_n. Because of that, the first column is filled with the corresponding number of "n". The rest of Table 1, i.e. the rows nr. 5 to nr. 10, were filled using the same rule. Note that, using the same rule X=A-B, Table 1 can be extended as much as we need, to cover TFLL_n of any order.

Table 1

The optimal system parameters and Code X=A-B, showing how to fill in any field "X" of the Table 1

		Optimal system parameters									
FLL's order		b1	b2	b3	b4	b5	b6	b7	b8	b9	b10
1	1	1	0	0	0	0	0	0	0	0	0
2	2	2	-1	0	0	0	0	0	0	0	0
3	3	3	-3	1	0	0	0	0	0	0	0
4	4	4	-6	4	-1	0	0	0	0	0	0
5	5	5	-10	10	-5	1	0	0	0	0	0
6	6	6	-15	20	-15	6	-1	0	0	0	0
7	7	7	-21	35	-35	21	-7	1	0	0	0
8	8	8	-28	56	-70	56	-28	8	-1	0	0
9	9	9	-36	84	-126	126	-84	36	-9	1	0
10	10	10	-45	120	-210	252	-210	120	-45	10	-1

B	A
	X

X=A-B

Let us now check the accuracy of Table 1. As it was stated, the first four rows of Table 1 were filled by mathematically obtained optimal parameters, but the fifth row, concerning the system of the fifth order TFLL₅, was filled using the rule X=A-B. The best check is to simulate the difference equation of TFLL₅ using the optimal parameters from Table 1 and see if the obtained results match the expectations, defined by the rules in the Section 2. The difference equation of TFLL₅ is given by eq. (17). It is defined by five parameters b₁, b₂, b₃, b₄ and b₅. If we now, using Table 1, define b₁=5, b₂=-10, b₃=10, b₄=-5 and b₅=1, eq. (17) will turn into eq. (18). Using

eqs. (18) and (2), the simulations of TI_k , TO_k and τ_k for the input $TI_k=10+2 \cdot k^4$ [t.u.] (p=2), are made and shown in Fig. 4,

$$TO_{k+5}=b_1TI_{k+4} + b_2TI_{k+3} + b_3TI_{k+2} + b_4TI_{k+1} + b_5TI_k, \quad (17)$$

$$TO_{k+5}=5TI_{k+4} - 10TI_{k+3} + 10TI_{k+2} - 5TI_{k+1} + TI_k, \quad (18)$$

together with the used parameters and initial conditions.

The simulations of TO_k and τ_k are made for two cases. In

case number 1, the optimal parameters were used. We can see in Fig. 4 that TO_{1k} tracks TI_k without an error, after TFLL₅ reaches the stable state. In case number 2, the parameters used are $b_1=4$, $b_2=-9$, $b_3=10$, $b_4=-5$, and $b_5=1$. The parameters b_1 and b_2 were changed, but the sum of all parameters is still equal to one. However, they are not optimal, and we can see that TO_2 is not able to track TI_k . Time difference in case nr. 1 $\tau_{1\infty} = \text{constant}$ and $\tau_{2\infty}$ in case nr. 2 tends to negative infinity. That agrees with the general rules stated in Section 2.

To prove that TFLL₅, using the optimal parameters, can track $TI_k = p \cdot k^5$ ($p=2$) but with a constant error, Fig. 5 is shown. The constant error K_5 can be better seen in the enlarged part of the TO_k output around $k=5$, when TFLL₅ reaches the stable state. As it was expected, the corresponding τ_{∞} in Fig. 5, tends to minus infinity.

The obtained results presented in Fig. 4 and Fig. 5 confirm the accuracy of Table 1, but at the same time they prove the correctness of the presented theory.

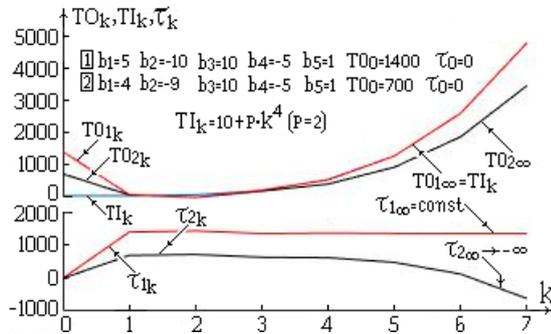


Fig. 4 – Simulation of the input and output variables of TFLL₅ for the input period $TI_k = 10 + p \cdot k^4$ ($p=2$).

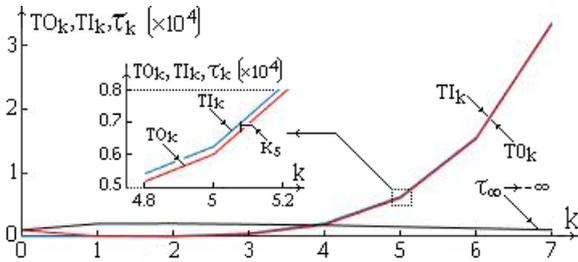


Fig. 5 – Simulation of the input and outputs of TFLL₅ for the input period $TI_k = 10 + p \cdot k^5$ ($p=2$) and for the optimal system parameters.

4. ANALYSIS OF FLLs IN THE FREQUENCY DOMAIN

We can see in [5–11] that the optimal choice of parameters for one kind of TFLL application degrades its characteristics for another application. Therefore, a good knowledge of the filtering characteristics of TFLLs, which use the optimal system parameters in different tracking applications, is required. For this purpose, we will use the MATLAB application software intended for the development of FIR digital filters, as described in [3, 4]. To analyze TFLL₅, we need the appropriate vector "b₅", which is defined by the corresponding transfer function.

The transfer function of TFLL₅ is determined by eq. (13) and the vector "b₅" by eq. (23) in ref. [3]. Vector "b₅" is borrowed from ref. (3) and shown in eq. (19) with entered values of optimal parameters for TFLL₅, defined in Table 1. Based on this vector and using MATLAB command freqz (b_5 , 1024, f_s), the frequency response of TFLL₅ is

determined and presented in Fig. 6 for half of the sample rate, $f_s = 10000$ Hz. The magnitude is equal to zero dB, and the phase is equal to zero degrees up to about 1100 Hz. This means that TFLL₅, at its output, does not introduce any change in magnitude or phase in any component up to 1100 Hz belonging to the period of the input signal.

This part of the magnitude is the same as with a low-pass digital filter that has a cut-off frequency of about 1100 Hz. However, the second part of the magnitude between 1100 Hz and 5000 Hz increases significantly with increasing frequency. Unlike a low-pass digital filter that attenuates the input signal after the cut-off frequency, the TFLL₅ amplifies the input period at the output up to about 25 dB. Due to this characteristic, TFLL₅ can follow very fast changes in the input period as shown in Fig. 4 and Fig. 5. It is also necessary to see how the magnitudes change in dependence of TFLL's order.

$$b_5 = [0 \ b_1 \ b_2 \ b_3 \ b_4 \ b_5] = [0 \ 5 \ -10 \ 10 \ -5 \ 1], \quad (19)$$

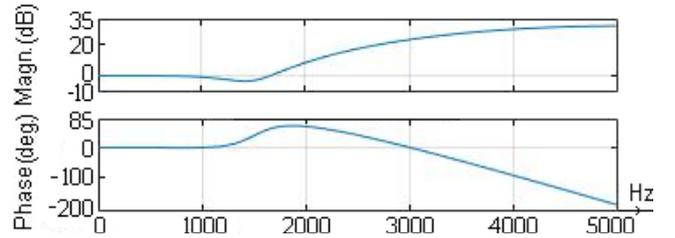


Fig. 6 – Magnitude and phase of TFLL₅ frequency response.

Figure 7 shows the magnitudes of TFLL₃, TFLL₅, TFLL₇, and TFLL₉. It can be seen in Fig. 7 that the magnitudes of TFLL's frequency responses increase with increasing of TFLL's order. In other words the amplification of the input period increases with the increase of TFLL's order, which agrees with the previous analysis in the time domain.

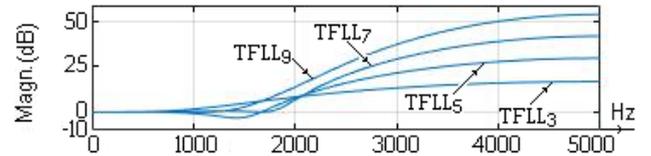


Fig. 7 – Magnitudes of the frequency responses of TFLL₃, TFLL₅, TFLL₇, and TFLL₉.

6. CONCLUSION

This article describes a general theory of the time TFLLs intended for the tracking and predicting applications. In this article the theory is expressed in case when a TFLL, as an electronic circuit, is used to generate the output periods which are equal to the input periods, no matter how fast changes of the input periods are. The faster changes require a TFLL of higher order. Any of TFLL is a predictor at the same time, because any next output period can be determined in the real time by using the previous input periods. There are many applications of the TFLL, as an electronic circuit, in tracking and predicting of the signal. For instance, the conversion of a physical variable into the period of an impulse signal is a very common case in electronics. By tracking the period, we measure the values of a physical variable.

In addition, this theory can be applied in tracking and predicting of any physical variable or phenomenon using the mathematical form of a TFLL algorithm. All that is required

is to measure the variable periodically and to feed the measured values into the appropriate algorithm instead of the input periods. In such an application, the properties of that physical variable must be previously studied. In accordance with its properties, a suitable algorithm should be developed, and a suitable frequency of sampling should be determined. The algorithm will recognize the regularity of the occurrence of the variable and predict each subsequent value, regardless of the speed of its change.

In addition to the mentioned possibilities of application, the tabular way of analyzing a TFLL is described for the first time in the literature. This approach has completely replaced long mathematical analysis. The article shows a table for TFLL of the tenth order, but it can be easily extended to TFLL_n of any order. Since any TFLL is a linear discrete system, it means that the tabular approach of analysis can be applied to any linear discrete system. The tabular approach to analysis of the linear discrete systems is not described in the literature so far.

The described method of tabular determination of the optimal parameters of any TFLL, intended for the tracking and predicting very fast changes of input periods, indicates the probability that some other analyses of TFLLs will be able to be performed using tables. That will be the most likely direction of further research.

ACKNOWLEDGEMENTS

This article was supported by the Ministry of Science and Technology of the Republic of Serbia within the project TR 32047.

Received on 20 December 2024

REFERENCES

- Dj.M. Perišić, *Generalization of the time infinite impulse response digital filters*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **69**, 3, pp. 327–332 (2024).
- Dj.M. Perišić, *New kind of IIR digital filters intended for pulse period filtering*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **69**, 1, pp. 61–66 (2024).
- Dj.M. Perišić, *Digital filters intended for pulse signal periods*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **67**, 2, pp. 161–166 (2022).
- Dj.M. Perišić, *Frequency locked loops of the third and higher order*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **66**, 4, pp. 261–266 (2021).
- Dj.M. Perišić, M. Perišić, D. Mitić, M. Vasić, *Time recursive frequency locked loop for the tracking applications*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **62**, 2, pp. 195–203 (2015).
- Dj.M. Perišić, A. Zorić, M. Perišić, V. Arsenović, Lj. Lazić, *Recursive PLL based on the measurement and processing of time*, Electronics and Electrical Engineering, **20**, 5, pp. 33–36 (2014).
- Dj.M. Perišić, A. Zorić, Ž. Gavrić, N. Danilović, *Digital circuit for the averaging of the pulse periods*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **63**, 3, pp. 300–305 (2018).
- Dj.M. Perišić, A. Zorić, M. Perišić, D. Mitić, *Analysis and application of FLL based on the processing of the input and output periods*, Automatika, **57**, 1, pp. 230–238 (2016).
- Dj.M. Perišić, M. Bojović, *Multipurpose time recursive PLL*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **61**, 3, pp. 283–288 (2016).
- Dj.M. Perišić, M. Perišić, S. Rankov, *Phase shifter based on a recursive phase locked loop of the second order*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **59**, 4, pp. 391–400 (2014).
- Dj.M. Perišić, A. Zorić, Dj. Babić, Dj.Dj. Perišić, *Decoding and prediction of energy state in consumption control*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **58**, 3, pp. 263–272 (2013).
- D. Jovčić, *Phase locked loop system for FACTS*, IEEE Transactions on Power Systems, **18**, pp. 2185–2192 (2003).
- A.S.N. Mokhtar, B.B.I. Reaz, M. Maruffuzaman, M.A.M. Ali, *Inverse park transformation using CORDIC and phase-locked loop*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **57**, 4, pp. 422–431 (2012).
- C.C. Chung, *An all-digital phase-locked loop for high-speed clock generation*, IEEE Journal of Solid-State Circuits, **38**, 2, pp. 347–359 (2003).
- F. Amrane, A. Chaiba, B.E. Babes, S. Mekhilef, *Design and implementation of high-performance field-oriented control for grid-connected doubly fed induction generator via hysteresis rotor current controller*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **61**, 4, pp. 319–324 (2016).
- M. Büyüç, M. İnci, M. Tümay, *Performance comparison of voltage sag/swell detection methods implemented in custom power devices*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **62**, 2, pp. 129–133 (2017).
- L. Joonsuk, B. Kim, *A low noise fast-lock phase-locked loop with adaptive bandwidth control*, IEEE Journal of Solid-State Circuits, **35**, 8, pp. 1137–1145 (2000).
- D. Abramovitch, *Phase-locked loops: a control-centric tutorial*, American Control Conference, vol. 1, pp. 1–15 (2002).
- R. Vich, *Z transform theory and application*, 1st ed., Springer (1987).
- S.W. Smith, *Digital signal processing*, 2nd ed., California Technical Publishing (1999).
- G. Bianchi, *Phase-locked loop synthesizer simulation*, McGraw-Hill, New York (2005).
- W.F. Egan, *Phase-lock basics*, 2nd ed., John Wiley & Sons (2008).
- B.D. Talbot, *Frequency acquisition techniques for PLL*, Wiley-IEEE Press (2012).
- C.B. Fledderman, *Introduction to electrical and computer engineering*, Prentice Hall (2002).
- M. Gardner, *Phase lock techniques*, Wiley-Interscience, Hoboken (2005).
- S. Winder, *Analog and digital filter design*, 2nd ed., Elsevier (2002).