Rev. Roum. Sci. Techn.– Électrotechn. et Énerg.
Vol. **70**, *3*, pp. 361–366, Bucarest, 2025

USING THE SMITH DIAGRAM IN THE ANALYSIS OF HIGH-FREQUENCY ANALOG CIRCUITS

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Keywords: Smith diagram; Analog circuits; Reflection coefficient; Voltage standing wave ratio.

The main objectives of this paper are to learn how to generate impedances on the Smith chart. The Smith diagram is a commonly utilized graphical technique for circuit applications with high frequencies, providing an innovative method for visualizing complex functions in the complex plane, where impedances are represented, as well as in the polar plane, where reflection coefficients (phase and amplitude) are described. Additionally, this allows for the representation of circles with a constant Q (quality factor), continuous standing wave ratio (SWR), and stability circles. Through examples given below, we've shown how the admittances and impedances can be graphically analyzed using the Smith chart.

1. INTRODUCTION

The Smith diagram, created by Phillip Smith while working at Bell Labs, was published in January 1939 in Electronics Magazine.

Although numerous mathematical instruments are available for use by microwave engineers, the Smith diagram is the most commonly used tool for describing transmission lines. The Smith diagram enables the quick solution of transmission line problems; however, it can also enhance intuitive thinking. For analytical solutions of the issues involving transmission lines, in addition to the printed Smith diagram, we will need a compass and a protractor. Every point on the diagram corresponds to a complex impedance [1–7].

The Smith diagram has been successfully utilized by multiple generations of radio frequency designers to perform circuit synthesis and analysis, as well as to represent impedances. The graphical representation substantially reduces calculation effort and allows a built-in impedance visualization.

Today, the Smith diagram is generated using network analyzers or computer programs. The use of calculation programs has considerably simplified the design process using the Smith diagram. However, those who utilize this diagram should have a good understanding of impedance, admittance, and reflection coefficient, as well as a profound knowledge of these representation methods. For this study, the Smith chart program, created by Fritz Dellsperger – HB9AJY, and the MATLAB RF Toolbox have been used.

For those having less experience in this field, the use of the Smith diagram seems to have a "black magic" allure. Nowadays, things aren't so complex that today various non-specialists in radio can correctly interpret a Smith diagram without specialized training. On this topic, numerous articles of popularization appeared in domestic or foreign magazines published for radio amateurs. This chapter aims to provide an easy-to-understand explanation of this diagram, along with a few practical examples, to facilitate a better understanding of utilization and data interpretation.

From a mathematical viewpoint, regarding coordinates described by the reflection complex coefficient, the Smith diagram represents complex impedances in a 4-D format that refers to two-port network parameters (which define the input-output voltages and currents relationship). Moreover, the 2D representation of impedances refers to the visualization of impedances in the complex plane (x-axis represents the real part (resistance) and y-axis represents the imaginary part (reactance). Unit radius circle in the complex plane (Fig.1) represents the definition domain of the reflection coefficient for a lossless transmission line. Amplitude higher than one for the reflection coefficient in a lossy transmission line, because of the characteristic impedance characteristic, which needs expansion of the Smith chart, [1–19].

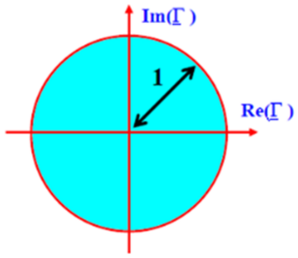

Fig. 1 – Reflection coefficient definition range for a lossless line.

The results of telegraphists' equations show the current and voltages analyzed at a specific y distance measured from the main load; therefore, we can substitute the ensemble made by the load $\underline{Z}_S$ impedance through the section line with an impedance, named the input impedance (Fig. 2), [–12].

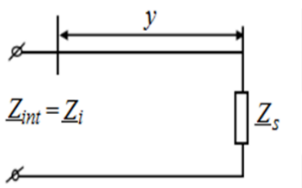

Fig. 2 – Complex input impedance definition.

$$\underline{Z}_{int} = \underline{Z}_i = \frac{\underline{U}}{\underline{I}} = \frac{\underline{U}_s ch(\underline{\gamma_1} y) + \underline{Z_0 I_s}\, sh(\underline{\gamma_1} y)}{\frac{\underline{U}_s}{\underline{Z}_0} sh(\underline{\gamma_1} y) + \underline{I}_s\, ch(\underline{\gamma_1} y)}. \quad (1)$$

where $\underline{\gamma_1} y$ is the line propagation constant.

Dividing the relation (1) by $\underline{I}_s ch(\underline{\gamma_1} y)$, we obtain:

$$\underline{Z}_i = \frac{\underline{U}}{\underline{I}} = \underline{Z}_0 \frac{\underline{Z}_s + \underline{Z}_0 th(\underline{\gamma_1} y)}{\underline{Z}_0 + \underline{Z}_s th(\underline{\gamma_1} y)}. \quad (2)$$

The complex coefficient of reflection is utilized in

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DOI: 10.59277/RRST-EE.2025.70.3.13

electrical engineering and physics while considering wave propagation in a medium containing discontinuity. The reflection coefficient is determined by:

$$\underline{\Gamma}=\frac{\underline{U}_i}{\underline{U}_d}=\frac{\underline{U}_{ref}}{\underline{U}_{inc}}=\frac{\underline{I}_i}{\underline{I}_d}=\frac{\underline{I}_{ref}}{\underline{I}_{inc}} \quad (3)$$

where: $\underline{U}_d$ is the direct voltage of complex value, equal to that incident $\underline{U}_{inc}$; $\underline{U}_i$ is the reverse voltage of complex value, equal to that reflected $\underline{U}_{ref}$; and, $\underline{I}_d$ is direct current complex value, equivalent to incident $\underline{I}_{inc}$ and $\underline{I}_i$ is reverse current of complex value, equal to that reflected $\underline{I}_{ref}$.

Representing current and voltage as an equation of reverse (reflected) and direct units will result in:

$$\underline{Z}_i=\frac{\underline{U}}{\underline{I}}=\frac{\underline{U}_d+\underline{U}_i}{\underline{I}_d-\underline{I}_i}=\frac{\underline{U}_d(1+\underline{\Gamma})}{\underline{I}_d(1-\underline{\Gamma})}=\underline{Z}_0\frac{(1+\underline{\Gamma})}{(1-\underline{\Gamma})}. \quad (4)$$

To simplify calculations, complex impedances are normalized, *i.e.*, divided by Z_0 ($\underline{z}_S=\underline{Z}_S/Z_0$). Obviously, the characteristic impedance becomes $\underline{Z}_0/\underline{Z}_0=1$ (as a rule, it is denoted by $Z_0 = 50\ \Omega$).

Defining VSWR as [11–19]:

$$\text{VSWR}=\sigma=\frac{U_{max}}{U_{min}}=\frac{1+|\underline{\Gamma}|}{1-|\underline{\Gamma}|}. \quad (5)$$

In the following, we will use normalized impedances (relative to Z_0) and denote them by lowercase. Denormalization (returning from normalized to non-normalized quantities) is achieved by multiplying by the normalization impedance.

Hence, the reflective coefficient $\underline{\Gamma}(y)$ can be expressed according to the normalized impedance $\underline{z}=\underline{z}(y)$ and conversely, based on:

$$\underline{\Gamma}(y)=\frac{\underline{z}(y)-1}{\underline{z}(y)+1} \text{ and } \underline{z}(y)=\frac{1+\underline{\Gamma}(y)}{1-\underline{\Gamma}(y)}. \quad (6)$$

Equations above (6) represent a two-way correspondence between the coefficients $\underline{\Gamma}$ and $\underline{z}$. Therefore, every unique spot from complex plane $\underline{\Gamma}$ matches to a normalized impedance. For instance, we would like to determine and represent the normalized impedance $\underline{z}$ values corresponding to various spots from complex plane $\underline{\Gamma}$. Cases of two-way correspondence between complex plane $\underline{\Gamma}$ and complex impedances are presented in Table 1, [1–19].

Table 1

Two-way correspondence between the complex plane $\underline{\Gamma}$ and the complex impedances.

Case	$\underline{Z}$	$\underline{z}$	$\underline{\Gamma}$
1	∞	∞	1
2	0	0	-1
3	$\underline{Z}_c$	1	0
4	$j\underline{Z}_c$	j	j
5	$-j\underline{Z}_c$	-j	-j

Therefore, the normalized complex impedances correspond biunivocal in the complex plane $\underline{\Gamma}$ to five different spots (Fig. 3), thus, the five distinct spots from the complex plane $\underline{\Gamma}$ match biunivocal to five distinct values of normalized complex impedances (Fig. 3), [11 – 19].

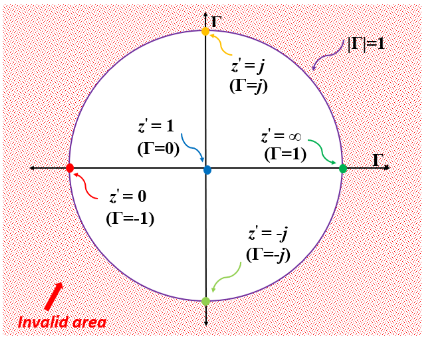


Fig. 3 – The correspondence of values of normalized complex impedances and points in the complex plane $\underline{\Gamma}$.

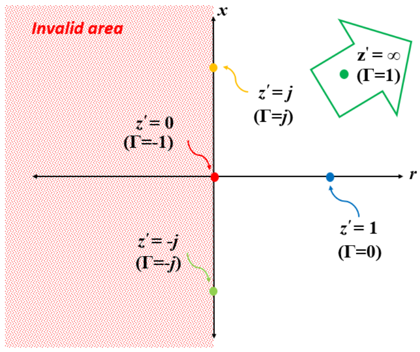


Fig. 4 – The matching values of normalized complex impedance and complex plane $\underline{\Gamma}$ points.

Smith diagram gives a Γ representation which determines quantities like VSWR or output impedance of DUT (device under test). Utilizes a Moebius transformation, representing in the complex plane $\underline{\Gamma}$, the complex impedance plane:

$$\underline{\Gamma}(y)=\frac{\underline{z}(y)-1}{\underline{z}(y)+1} \quad \text{and} \quad \underline{z}(y)=\frac{1+\underline{\Gamma}(y)}{1-\underline{\Gamma}(y)}. \quad (7)$$

As shown in Fig. 5, the half of the plane having the impedance $\underline{Z}$ real part as positive, is transposed inside the unitary circle of plane $\underline{\Gamma}$.

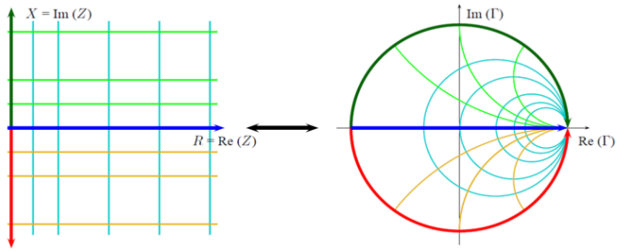


Fig. 5 – Representation of Moebius transformation from complex impedance plane to plane $\underline{\Gamma}$ frequently noted as Smith diagram.

2. GENERATING IMPEDANCES ON THE SMITH CHART

If it is assumed that normalized complex impedances have general form of $\underline{z}=r+jx$, and general structure of the reflection coefficient is $\underline{\Gamma}=\Gamma_r+j\Gamma_x$, therefore relations (7) are transformed into:

$$\Gamma_r+j\Gamma_x=\frac{r-1+jx}{r+1+jx} \quad \text{and} \quad r+jx=\frac{1+\Gamma_r+j\Gamma_x}{1-\Gamma_r-j\Gamma_x}. \quad (8)$$

where $\Gamma_r = R_e\{\underline{\Gamma}\}$, $\Gamma_x = I_m\{\underline{\Gamma}\}$, $r = R_e\{\underline{z}\}$,

$x = I_m\{\underline{z}\}$,

Equations (8) yield: $\Gamma_r = \frac{r^2+x^2-1}{(1+r)^2+x^2}$, $\Gamma_x = \frac{2x}{(1+r)^2+x^2}$,

$$x = \frac{2\Gamma_x}{(1-\Gamma_r)^2+{\Gamma_x}^2}, \ r = \frac{1-\Gamma_r^2-\Gamma_x^2}{(1-\Gamma_r)^2+{\Gamma_x}^2}. \qquad (9)$$

The latest relation in equations (9) is modified as follows:

$$r=\frac{1\text{-}\Gamma_r^2\text{-}\Gamma_x^2}{(1\text{-}\Gamma_r)^2+{\Gamma_x}^2}$$

$$\Rightarrow r(\Gamma_r-1)^2+(\Gamma_r^2-1)+r\Gamma_x^2+\Gamma_x^2+\frac{1}{1+r}-\frac{1}{1+r}=0$$

$$\Rightarrow\left[r(\Gamma_r-1)^2+(\Gamma_r^2-1)+\frac{1}{1+r}\right]+(1+r)\Gamma_x^2=\frac{1}{1+r}$$

$$\Rightarrow(1+r)\left[\Gamma_r^2-2\Gamma_r\frac{r}{1+r}+\frac{r^2}{(1+r)^2}\right]+(1+r)\Gamma_x^2=\frac{1}{1+r}$$

$$\Rightarrow\left[\Gamma_r-\frac{r}{1+r}\right]^2+\Gamma_x^2=\left(\frac{1}{1+r}\right)^2. \qquad (10)$$

The final relation of equation (10) describes a circle having the center coordinates $C_c = \left\{\frac{r}{1+r}, 0\right\}$ from complex plane $\underline{\Gamma}$ and the radius $\frac{1}{1+r}$. Therefore, any vertical r = ct, r≥ 0, from normalized complex impedances is transforming on complex $\underline{\Gamma}$ plane in a circle situated within $|\underline{\Gamma}|=1$ circle or equivalent normalized resistances ranging from 0 to ∞ change into a system of circles included in the range of $|\underline{\Gamma}|\le 1$ reflection coefficients, as shown in Fig. 6.

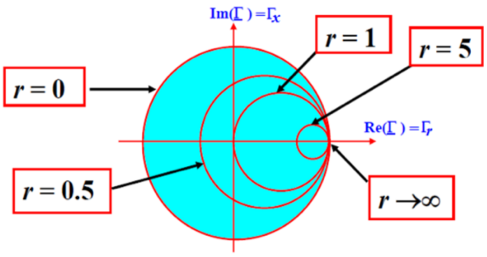


Fig. 6 – Normalized resistance represented in the complex plane.

Similarly, processing eq. (9) results the function applying the normalized reactance's plane (x∈ $(-\infty,+\infty)$) in the reflection coefficients complex plane:

$$x=\frac{2\Gamma_x}{(1-\Gamma_r)^2+\Gamma_x^2} \Leftrightarrow [x((1-\Gamma_r^2)^2+\Gamma_x^2)-2\Gamma_x \pm 1]x=0$$

$$\Leftrightarrow x^2[(1-\Gamma_r^2)^2+\Gamma_x^2]-2x\Gamma_x+1-1=0$$

$$\Leftrightarrow [(1-\Gamma_r^2)^2+\Gamma_x^2]-\frac{2}{x}\Gamma_x+\frac{1}{x^2}=\frac{1}{x^2}$$

$$\Leftrightarrow (1-\Gamma_r^2)^2+\left[\Gamma_x^2\frac{2}{x}\Gamma_x+\frac{1}{x^2}\right]=\frac{1}{x^2}$$

$$\Rightarrow (\Gamma_r-1)^2+\left(\Gamma_x-\frac{1}{x}\right)^2=\frac{1}{x^2}. \qquad (11)$$

The latest relation in equations (11) describes a circle of equal radius to $\frac{1}{|x|}$ with center coordinates $C_c = \left\{1,\frac{1}{x}\right\}$, into the complex plane $\underline{\Gamma}$ having the coordinates $\{\Gamma_r,\Gamma_x\}$. Thus, whichever vertical x = ct., $x \in (-\infty,+\infty)$ from complex plane, from complex plane normalized impedances turns into the complex plane $\underline{\Gamma}$ in an arc situated inner the $|\underline{\Gamma}|=1$ circle or equivalent normalized reactance's ranging from - ∞ to + ∞ change into circles arcs families included in the range of $|\underline{\Gamma}|\le 1$ reflection coefficients, as shown in Fig. 7, [10–19].

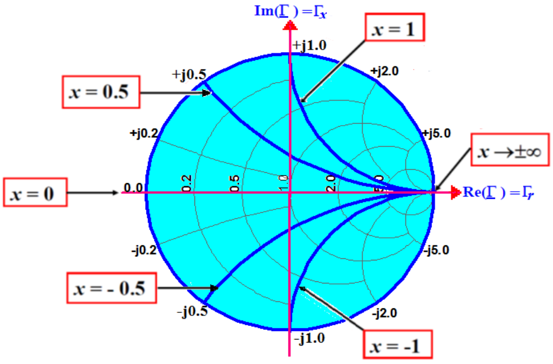


Fig. 7 – Normalized reactance represented in $\underline{\Gamma}$ complex plane.

3. GENERATING ADMITTANCES ON THE SMITH CHART

Recall of generic shape of normalized complex admittances is $\underline{y} = \frac{\underline{Y}}{Y_0} = \underline{Y}Z_0 = g + jb = \frac{1-\underline{\Gamma}}{1+\underline{\Gamma}}$, when g represents conductance, and b representing susceptance. The complex reflection coefficient has the general structure $\underline{\Gamma} = \Gamma_r + j\Gamma_x = \frac{1-\underline{y}}{1+\underline{y}}$. Considering these relationships, the following equations are obtained:

$$g+jb=\frac{1-\Gamma_r-j\Gamma_x}{1+\Gamma_r+j\Gamma_x}$$

$$\Rightarrow g=\frac{1-\Gamma_r^2-\Gamma_x^2}{(1+\Gamma_r)^2+\Gamma_x^2} \Rightarrow \left(\Gamma_r+\frac{g}{g+1}\right)^2+\Gamma_x^2=\frac{1}{(g+1)^2}$$

$$b=\frac{-2\Gamma_x}{(1+\Gamma_r)^2+\Gamma_x^2} \Rightarrow (\Gamma_r+1)^2+\left(\Gamma_x+\frac{1}{b}\right)^2=\frac{1}{b^2}. \qquad (12)$$

The relationship $\left(\Gamma_r+\frac{g}{g+1}\right)^2+\Gamma_x^2=\frac{1}{(g+1)^2}$ describes, in the complex plane, a circle having a radius of $\frac{1}{g+1}$ and center coordinates equal to $C_c = \left\{-\frac{g}{g+1}, 0\right\}$. Therefore, any vertical g = ct., g≥0, from the complex plane of normalized admittances is transforming on $\underline{\Gamma}$ complex plane in a circle which is situated within the $|\underline{\Gamma}| = 1$ circle or equivalent to normalized conductance ranging from 0 to ∞ change into circles families included in the range of $|\underline{\Gamma}|\le 1$ reflection coefficients, as shown in Fig. 8.

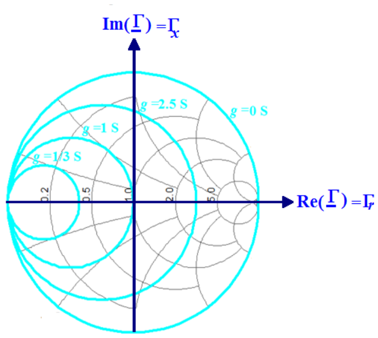


Fig. 8 – Normalized conductance represented in $\underline{\Gamma}$ the complex plane.

The equation $(\Gamma_r+1)^2+\left(\Gamma_x+\frac{1}{b}\right)^2=\frac{1}{b^2}$ describes a circle with $\frac{1}{|b|}$ radius, and $C_c = \left\{-1,-\frac{1}{b}\right\}$ center coordinates, in the complex plane $\underline{\Gamma}$. Thus, any vertical b = ct., $b \in (-\infty, \infty)$ on the complex plane, from the normalized complex admittance plane turns into an arc situated inside the $|\underline{\Gamma}| = 1$ circle, in the complex plane $\underline{\Gamma}$. Similarly, normalized susceptance ranging from $-\infty$ to $+\infty$ change into families of arcs of circles included in the range of reflection coefficients $|\underline{\Gamma}| \leq 1$, as shown in Fig. 9, [10–18].

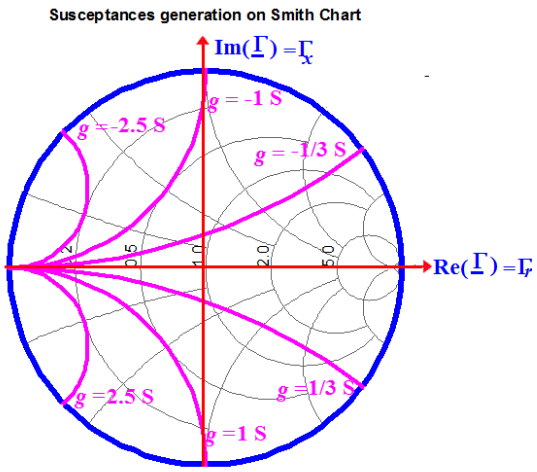


Fig. 9 – Normalized susceptance represented in $\underline{\Gamma}$ complex plane.

The basic techniques of the Smith diagram for lossless transmission lines consist of:

- It is known $\underline{Z}(y)$ and required to be determined $\underline{\Gamma}(y)$ or it is known $\underline{\Gamma}(y)$ and required $\underline{Z}(y)$;
- It is known Γ_r and $Z\_R = R$ needed to be determined $\underline{\Gamma}(y)$ and $\underline{Z}(y)$ or given $\underline{\Gamma}(y)$ and $\underline{Z}(y)$, required Γ_r and $Z\_R = R$;
- Find the distances d_{min} and d_{max} (minimum and maximum positions of stationary voltage wave character);
- Identifying the VSWR
- It is known $\underline{Z}(y)$ and required to be determined $\underline{Y}(y)$ or it is given $\underline{Y}(y)$ and required $\underline{Z}(y)$.

4. EXEMPLES

Let's find the coefficient of reflection $\underline{\Gamma}(y)$, when giving the complex impedance $\underline{Z}(y)$. The method is described below:

P1. Normalizing the complex impedance, according to the equation below:

$$\underline{z}_{(y)} = \frac{\underline{Z}_{(y)}}{Z_0} = \frac{R}{Z_0} + j\frac{X}{Z_0} = r + jx. \tag{13}$$

P2. The normalized constant resistance circle r (given), is represented in the complex plane $\underline{\Gamma}(y)$;

P3. The arch correspondent to normalized constant x reactance is generated on complex plane $\underline{\Gamma}$ (y);

P4. The curves intersection at steps P2 and P3 indicates complex coefficient of reflection $\underline{\Gamma}$ (y) sought. The Smith diagram gives immediately the mode and phase angle of $\underline{\Gamma}$ (y).

Let's find now $\underline{\Gamma}$ (y) when $\underline{Z}$ (y) = 25 + j100, Z_0=50Ω. The method for identifying the $\underline{\Gamma}(y)$ reflection coefficient, given $\underline{Z}(y)$, is shown in Fig. 10.

Let's also find $\underline{\Gamma}(y)$, respective $\underline{Z}(y)$, when $y = 0.18\lambda$ where $\underline{Z}_s = 25 + j100$ is load impedance and characteristic impedance $|\underline{Z}_0| = Z_0 = 50\Omega$ are given. The method for identifying $\underline{Z}(y)$ complex impedance and $\underline{\Gamma}(y)$ reflection coefficient is described in Fig. 11.

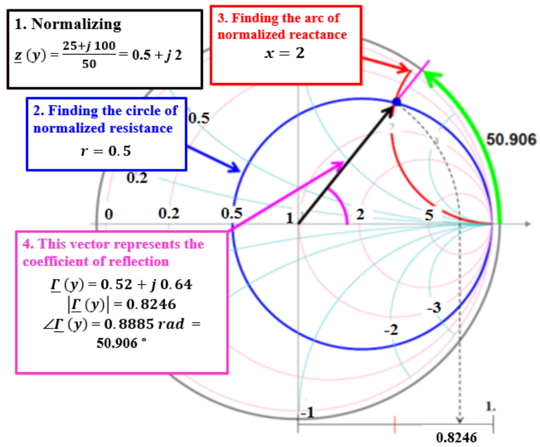


Fig. 10 – Method description of identifying $\underline{\Gamma}(y)$, having $\underline{Z}(y) = 25 + j100$ with $|\underline{Z}_c| = Z_c = 50\Omega$.

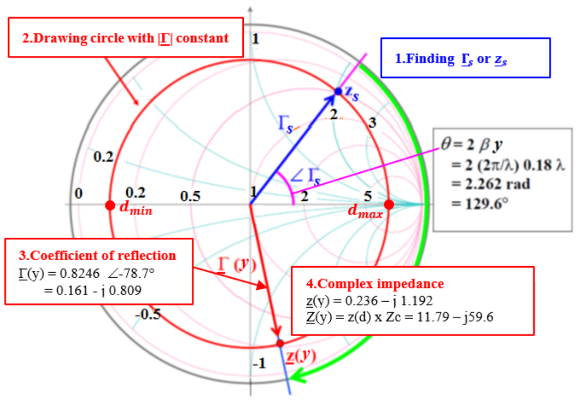


Fig. 11 – Method of identifying $\underline{Z}(y)$ and $\underline{\Gamma}(y)$, having y = 0.18λ, load impedance $\underline{Z}_s = 25 + j100$ and characteristic impedance

$|\underline{Z}_0| = Z_0$= 50Ω are given.

Among the lossless transmission line, we have a constant reflection coefficient for a specified load, because: $|\underline{\Gamma}(y)| = \left|\underline{\Gamma}_s \, exp\left(-j2\gamma_l y\right)\right| = |\underline{\Gamma}_s|$. Hence, a circle whose origin coincides with radius $|\underline{\Gamma}_s|$ define all possible reflections among the line of transmission in the complex coordinate plane $\{\Gamma_r, \Gamma_x\}$. The impedance values of the line at any distance can be determined when the constant mode circle of the reflection coefficient is known and described through the Smith chart.

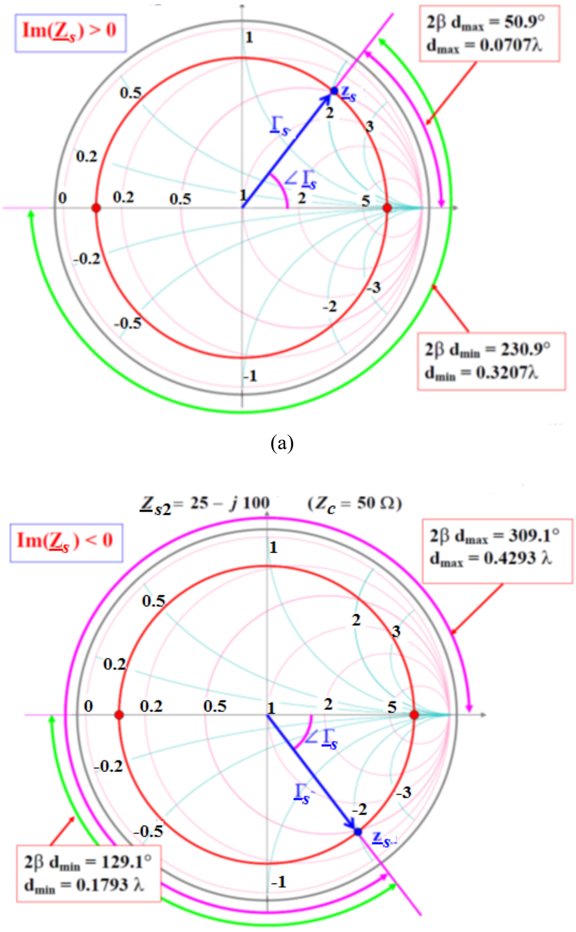
The steps of the algorithm for determining distances *dmax* and *dmin* when given $\underline{\Gamma}_s$ and $\underline{Z}_s$ are:

P1. The reflection coefficient $\underline{\Gamma}_s$ or normalized complex load impedance $\underline{z}_s$ shall be determined on the Smith chart.

P2. Represent the circle corresponding to the coefficient of reflection of constant mode $|\underline{\Gamma}(y)| = |\underline{\Gamma}_s|$ on the Smith diagram. This circle will intersect the reflection coefficients real axis at two points identifying *dmin* (when $\underline{\Gamma}(y)$ it is real negative) and *dmax* (when $\underline{\Gamma}(y)$ it is real positive).

P3. Smith diagram gives a horizontal gradation considering normalized distances in relation to λ wavelength can be directly interpreted. Angles, among real axis, respective vector $\underline{\Gamma}_s$, give also a path for calculating the distances *dmax* and *dmin*.

Finding the distances d_{max} and d_{min}, when the load impedances are known $\underline{Z}_{s1} = 25 + \mathrm{j}100$, $\underline{Z}_{s2} = 25 - \mathrm{j}100$ and the characteristic impedance $|\underline{Z}_0| = Z_0 = 50\Omega$. The procedure for determining the distances d_{max} and d_{min} is described in Fig. 12, a respectively b.



(a) (b)

Fig. 12 - Description of the procedure for determining distances *dmax* and *dmin*, when giving load impedances and characteristic impedance Ω:

a) For $\underline{Z}_{s1} = 25 + \mathrm{j}100$; b) for For $\underline{Z}_{s2} = 25 - \mathrm{j}100$.

$\underline{\Gamma}_s$ and $\underline{Z}_s$ is also given and asked to determine the VSWR, which is determined by equation (5).

The complex impedance normalized at a maximum distance of the stationary wave model is defined below:

$$\underline{z}(\mathrm{d_{max}}) = \frac{1+|\underline{\Gamma}(\mathrm{d_{max}})|}{1-|\underline{\Gamma}(\mathrm{d_{max}})|} = \frac{1+|\underline{\Gamma}_s|}{1-|\underline{\Gamma}_s|} = \mathrm{VSWR}. \quad (14)$$

This size is always real ≥ 1. VSWR is easily identified on the Smith diagram by interpreting the normalized impedance's real value at the d_{max} distance, considering that it is real and positive.

The algorithm for determining VSWR when known $\underline{\Gamma}_s$ and $\underline{Z}s$ has the following steps:

P1. The reflection coefficient corresponding to the load $\underline{\Gamma}_s$ and the normalized complex load impedance $\underline{z}s$ shall be determined on the Smith diagram.

P2. Draw the circle corresponding to the constant modulus of the coefficient of reflection $|\underline{\Gamma}(d)| = |\underline{\Gamma}_s|$.

P3. Look for the circle intersection with the actual axis for the reflection coefficient (corresponding to d_{max} distance of the transmission line);

P4. Normalized constant resistance circle is passing also through this point, reading and interpolating the normalized resistance value to find VSWR value.

The method to determine the value of VSWR for distances *dmax* and *dmin*, when the load impedances are known, and character impedance = 50 Ω. The method for finding the value of the ratio of stationary wave voltages when giving load impedances $\underline{Z}_{s1} = 25 + \mathrm{j}100$, $\underline{Z}_{s2} = 25 - \mathrm{j}100$ and characteristic impedance $|\underline{Z}_0| = Z_0 = 50\Omega$, is shown in Fig. 13.

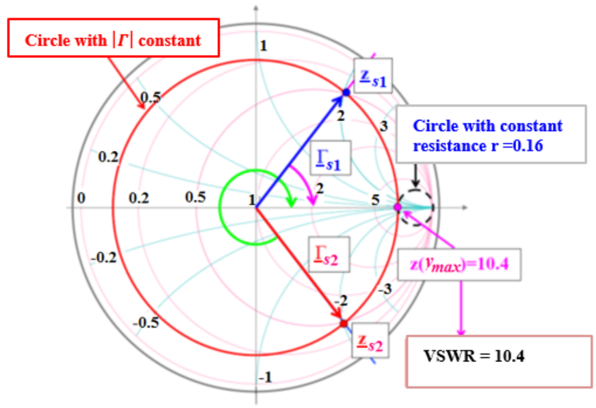


Fig. 13 – Procedure description to determine the Stationary Wave Voltage value.

Determination of complex admittance $\underline{Y}(d)$ when complex impedance $\underline{Z}(d)$ is known. Reminding that normalized complex impedance $\underline{z}(d)$ and normalized complex admittance $\underline{y}(d)$, for any distance d on the transmission line, are given by the expressions:

$$\underline{z}(d) = \frac{1+\underline{\Gamma}(d)}{1-\underline{\Gamma}(d)} \text{ and } \underline{z}(d) = \frac{1+\underline{\Gamma}(d)}{1-\underline{\Gamma}(d)}. \quad (15)$$

Because $\underline{\Gamma}\left(d + \frac{\lambda}{4}\right) = -\underline{\Gamma}(d)$

$$\Rightarrow \underline{z}\left(d + \frac{\lambda}{4}\right) = \frac{1+\underline{\Gamma}\left(d+\frac{\lambda}{4}\right)}{1-\underline{\Gamma}\left(d+\frac{\lambda}{4}\right)} = \frac{1-\underline{\Gamma}(d)}{1+\underline{\Gamma}(d)} = \underline{y}(d). \quad (16)$$

The relation (17) is true only for normalized impedances and admittances. Their values are:

$$\underline{Z}\left(d + \frac{\lambda}{4}\right) = Z_0 \cdot \underline{z}\left(d + \frac{\lambda}{4}\right); \ \underline{Y}(d) = Y_0 \cdot \underline{y}(d) = \frac{\underline{y}(d)}{Z_0}. \quad (17)$$

where $Y_0 = \frac{1}{Z_0}$ is representing the characteristic admittance of the transmission line.

Finding complex load admittance $\underline{Y}d$, when the complex load impedance is $\underline{Z}_s = 25 + \mathrm{j}100$, also the characteristic impedance $|\underline{Z}_0| = Z_0 = 50\Omega$. The procedure for determining the complex load admittance $\underline{Y}(d)$ is represented in Fig. 14.

On the diagram of complex impedance ($\underline{Z} = R + \mathrm{j}X$), the valid reflection coefficient is represented all the time by the vector corresponding to the normalized complex impedance ($\underline{z} = r + \mathrm{j}x$). Diagrams specially prepared for complex admittances ($\underline{Y} = G + jB$) are modified in such a way as to give the correct coefficient of reflection in correspondence with the complex admittance generated.

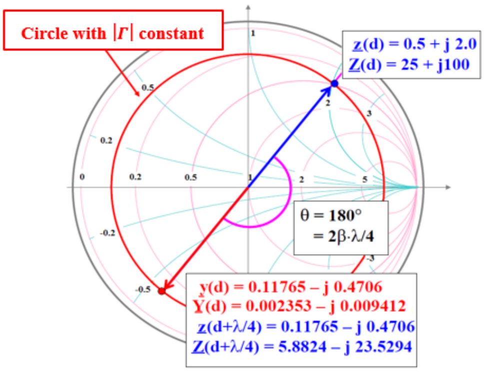


Fig. 14 - The procedure for determining the complex admittance of the load when giving the complex load impedance and the characteristic impedance.

Because the associated impedance and admittance have different positions on the Smith diagram, their imaginary elements always have different signs (see Fig. 15). Hence, a positive reactance corresponds to a negative susceptance. However, a negative reactance corresponds to a positive susceptance, as shown in Fig. 15.

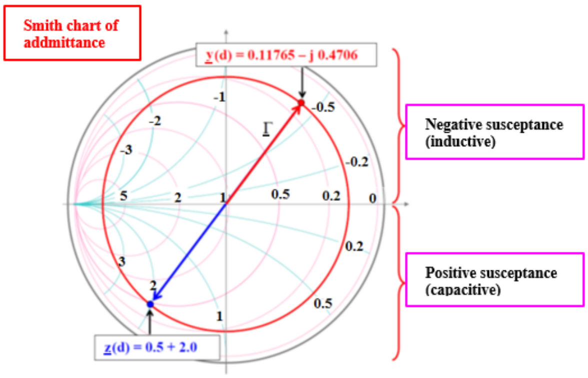


Fig. 15 - Distribution of inductive and capacitive susceptance on the Smith diagram.

5. CONCLUSIONS

The Smith diagram is helpful in various applications, from adapting the impedances of a small-signal amplifier to minimize noise, to performing a load-pull analysis for a power amplifier. In the latter case, a power amplifier is forced to work on various impedances, at which the phase is rotated 360 degrees. The output power is then displayed on the Smith diagram, allowing the optimal output impedance for that transistor/amplifier to be determined to achieve maximum power, maximum linearity, or maximum efficiency, as appropriate.

The usefulness of the Smith diagram is not limited to professionals in the field but also extends to radio amateurs who are increasingly using modern instruments for analyzing circuits or antennas. Whether it is amateur radio antenna analyzers such as SARK 110, Rig Expert AA55, KC901 or a mini-VNA (Vector Network Analyzer), the correct understanding of the Smith diagram usage allows the user to see precisely what the nature of the impedance is and implicitly which is the real (resistive) and imaginary (reactive) component. From this, conclusions can be drawn about whether the antenna is too short or too long, as well as findings on how to build the adaptation circuit.

The chart provides a robust and accurate representation of the passive impedances (those with a positive real part) from 0 to ∞. Negative real part impedances, for example the reflection amplifier or other device as active, will appear outer from the Smith diagram. It is nice to observe, especially in the field of radio frequency and microwave studies, because mapping transforms impedances or acceptances (y = 1/z) into reflective factors and conversely. The conversion among impedance, respective intake in the diagram would be mainly simple: Γ(y = 1/z) = -Γ(z); "Text".

CREDIT AUTHORSHIP CONTRIBUTION STATEMENT

MG: Conceptualization, editing, simulation
MI, MS: Methodology, supervision, validation
DN, LB, AG: Results analysis, validation.

ABBREVIATIONS

SWR - Standing Wave Ratio
VSWR – Voltage Standing Wave Ratio
DUT – Device Under Test
VNA – Vector Network Analyzer

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