



NEW SOLUTION OF HIGH FORCE LINEAR ACTUATOR WITH PERMANENT MAGNETS

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This paper presents a new design, numerical modeling, and experimental validation of a high-force linear actuator with permanent magnets. The proposed linear actuator solution is designed similarly to a classical electromagnet but is equipped with NdFeB permanent magnets and an auxiliary magnetic circuit. These auxiliary components produce a high static holding force when the actuator functions in one of its main operating positions. Like most linear actuators, the new solution has two operating positions: maximum air gap (open position) and minimum air gap (close position). This article aims to present the novel design, the working principle, and the implementation and experimental results of the actuator. The numerical solution of the actuator was obtained using the FEMM package, in which the electromagnetic parameters were computed. The force produced by the actuator was determined using the LUA script incorporated in FEMM. Numerical integration obtained other mechanical parameters (acceleration, work, and speed) with MATLAB.

1. INTRODUCTION

Linear permanent magnet actuators are electromagnetic devices with similar components to classic electromagnets. They have a principal ferromagnetic circuit, an excitation coil, an auxiliary ferromagnetic circuit, and permanent magnets. The ferromagnetic circuit has two main parts: the fixed and mobile magnetic circuits.

Similar to a classical electromagnet, the actuator produces a force that moves the mobile magnetic core along its path. In a classical electromagnet, when the air gap is minimal, and the mobile core is closed, the excitation coil is energized to produce the necessary magnetic field to produce the holding force.

Contrary to classical electromagnets, which consume energy to produce a force for every position of the mobile core along its path, linear permanent magnet actuators were developed as energy-efficient solutions. The proposed actuator is equipped with permanent magnets, which produce a high-intensity magnetic field necessary to develop the holding force in a close position.

The force produced by the proposed linear actuator can be controlled for every position of the mobile magnetic core along its path. Regarding this aspect, by changing the voltage applied to the coil, the electric current flowing through it will also change proportionally. Also, the magnetic field produced by the coil is proportional to the current flowing through it. The force produced by the actuator is also proportional to the magnetic field in its air gap. Considering the above, by changing the voltage applied on the terminals of the coil, the force produced by the actuator will be modified.

Linear permanent magnet actuators have a wide range of uses. They can be utilized in electric apparatus, like high-voltage circuit breakers and other industrial applications, and in other domains, such as medical, military, aerospace, and vacuum technologies.

A problem frequently encountered in electrical systems where linear actuators are integrated is the possibility of power outages. In electrical systems in which electromagnets are integrated, the possibility of power outages leads to the interruption of energizing the excitation coil. The interruption of energy supply to the coil leads to the absence of a magnetic field. The lack of a magnetic

field in the magnetic circuit leads to the absence of a force developed by the electromagnet [1,7–9,11].

To solve this problem, some linear actuators are equipped with permanent magnets that produce a magnetic field at every moment during the device's functioning [2,3]. Therefore, when the air gap is minimal, and the actuator is in a close position, the magnetic field produced by the permanent magnets has a maximum value, and the holding force produced by the actuator is sufficient to fulfill its role.

The article's novelty is that it presents the analysis and numerical modeling process of a high-force linear actuator with a planar structure and two active air gaps. A similar solution is presented in [7] but in an axisymmetric construction. This solution presents the following disadvantage: in the manufacturing process, it was found that it is not easy to mount the permanent magnets, sliding bearings, and ferromagnetic flanges concentrically with the plunger. Due to this positioning inaccuracy, frictional forces decrease the actuator's performance. The solution presented in this article is a much airier construction and does not require fine adjustments.

The article is organized in the following manner: section 2 presents the proposed actuator design and working principle regarding the four states of operation in terms of the mechanical functioning of the device; section 3 presents the numerical computation of the proposed linear actuator, regarding the four states of operation in terms of electromagnetic functioning of the device; the experimental model of the linear actuator is presented in section 5 and finally, a brief conclusion and future work in section 6.

2. PROPOSED ACTUATOR DESIGN AND WORKING PRINCIPLE

The proposed solution of a linear permanent magnet actuator has the following main components: a ferromagnetic circuit, an excitation coil, and permanent magnets. It is also equipped with an opening spring and numerous auxiliary non-magnetic components, which fix the main components and help with optimal operation.

As shown in Fig. 1, the proposed linear permanent magnet actuator has the following parts: 1 – guiding rod; 2 – upper plate; 3, 3' – flat spacers; 4, 4' – auxiliary plates; 5, 5' – permanent magnets; 6, 6' – excitation coil; 7, 7' –

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support plates; 8, 8' - side plates; 9 – middle plate; 10, 10' – lower plates; 11, 11' – guiding plates; 12 – inferior plate.

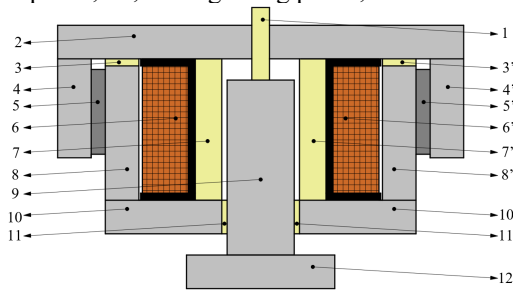


Fig. 1 – Design of the proposed linear actuator (without opening spring) in the open position.

The actuator is designed with fixed, mobile, and auxiliary ferromagnetic parts. The fixed ferromagnetic circuit is composed of upper plate (2), side plates (8, 8') and lower plates (11, 11'). The mobile ferromagnetic circuit comprises a middle plate (9) and an inferior plate (12). The auxiliary ferromagnetic circuit comprises plates (4 and 4') and the permanent magnets (5, 5').

The ferromagnetic plates are made of 1010 Steel, with high permeability, to direct the magnetic field, and the coil is made of copper wire. The guiding rod (1), flat spacers (3, 3'), support plates (7, 7'), and guiding plates (11, 11'), which are used to guide the mobile core and sustain the whole structure, are made of brass so that they do not influence the magnetic flux direction.

The excitation coil produces a magnetic field to realize the closing and opening maneuvers by moving the mobile core along the air gap, as seen in Fig. 4 and Fig. 7. Also, permanent magnets are used to produce a magnetic field, which is necessary to maintain the mobile ferromagnetic circuit in the closed position. The ferromagnetic circuit guides the magnetic field produced by the excitation coil and by permanent magnets in the actuator's air gaps, where the electromagnetic force is exerted.

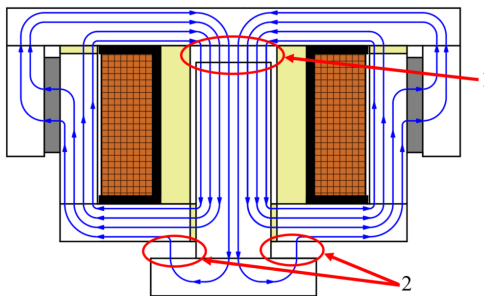


Fig. 2 – Magnetic field produced by the excitation coil during the closing maneuver and directed in the main gap (1) and additional air gaps (2).

This solution's main advantage and novelty consists of a special geometry of the mobile magnetic core, which is designed in an improved version by multiple iterations using a finite element method program (FEMM). Existing solutions have a simple geometry for the mobile ferromagnetic circuit, with only one main air gap (which is also present for the current solution, between the middle (9) and upper plate (2)). The new solution of the ferromagnetic circuit forces the magnetic flux to pass through two additional air gaps (delimited by the inferior (12) and lower plates (11)), as seen in Fig. 2 and Fig. 3.

The advantage leads to a higher force the actuator produces in the closed position and a lower mass for the

mobile ferromagnetic circuit (and a higher speed and acceleration).

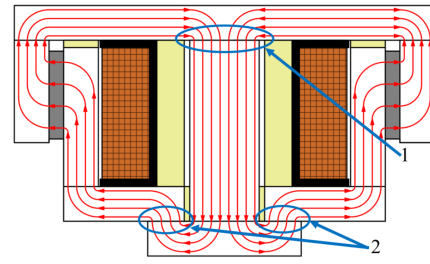


Fig. 3. Magnetic field produced by the permanent magnets in the closed position directed in the main air gap (1) and additional air gaps (2).

The linear permanent magnet actuator has been designed to meet the following technical requirements:

- Minimal closing force (at maximum air gap): 5200 N.
- Minimal holding force (at minimum air gap): 4100 N.
- Minimal force (during opening): 2000 N.
- Stroke length: 22 mm.
- Rated coil voltage: 230 Vdc.

Regarding the working principle, four states of operation of this device can be identified, which can be classified according to the position of its movable armature (as can be seen in Fig. 5 and Fig. 7), as well as the power supply to its excitation coil. From this point of view, the four states of operation are:

- The open position is when the coil is not powered, and the air gap has the maximum value.
- The closing maneuver is when the coil is open, and its coil is supplied with energy to modify the mobile armature's position; during this maneuver, the mobile armature moves to reduce the air gap and exerts the closing force needed.
- The close position is when the coil is not powered and the air gap is minimal.
- The opening maneuver occurs when the actuator is closed, and its coil is supplied with energy to modify the mobile armature's position; during this maneuver, the mobile armature moves to enlarge the air gap and exert the needed opening force.

In the open position (at maximum air gap), the spring is pre-compressed and exerts a stabilizing force that ensures the actuator's ability to remain open if an accidental force is applied to close it:

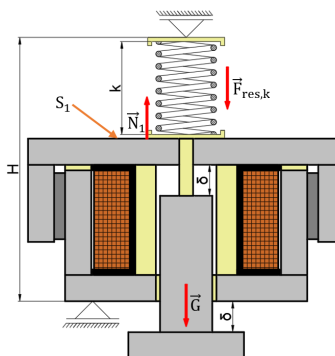


Fig. 4 – Linear permanent magnet actuator in open position.

In this operating state, the mechanical equation can be written as follows:

$$\vec{G} + \vec{F}_{res,k} + \vec{N}_1 = 0, \quad (1)$$

where \vec{G} is the weight of the moving components, \vec{N}_1 is the

normal force at surface S_1 , and $\vec{F}_{res,k}$ is the pre-compression force exerted by the opening spring, where \vec{N}_1 can be considered a stabilizing force that ensures the actuator's ability to remain in open position if an accidental force is applied to close it; also, δ is the maximum air gap of the permanent magnet actuator.

During the linear permanent magnet actuator's closing maneuver, the electromagnetic force it developed drives the mobile magnetic core towards the fixed ferromagnetic circuit, as seen in Fig. 5.

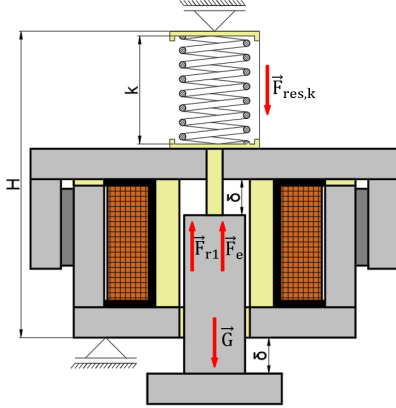


Fig. 5 – Linear permanent magnet actuator during closing maneuver.

To complete the closing maneuver and to reduce the gap between the moving and the fixed magnetic core, it is necessary for the electromagnetic actuator to produce a force stronger than antagonist forces:

$$|\vec{F}_e| > |\vec{F}_{res,k}| + |\vec{G}|, \quad (2)$$

where \vec{F}_e is the electromagnetic force exerted by the opening spring, corresponding to the gap specified above, and \vec{G} as the mobile armature's weight.

The previous equation can be expressed as follows:

$$\vec{F}_e + \vec{F}_{res,k} + \vec{G} + \vec{F}_{r1} = 0, \quad (3)$$

where \vec{F}_{r1} is the resultant force during the closing maneuver.

When the linear actuator is in a close position (as seen in Fig.6), its force is maximum. The spring is at maximum compression, and its force is highest during the closing maneuver.

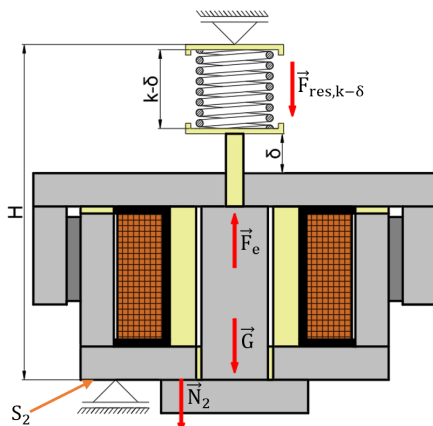


Fig. 6 – Linear permanent magnet actuator in closed position.

To maintain the mobile armature in a close position (at minimum air gap), the holding force produced by it must be stronger than the antagonist ones:

$$|\vec{F}_{hold}| > |\vec{F}_{res,k-\delta}| + |\vec{G}|, \quad (4)$$

where \vec{F}_{hold} is the holding force produced by the actuator, $\vec{F}_{res,k-\delta}$ is the force exerted by the opening spring, and \vec{G} is the weight of the mobile magnetic core.

The previous equation can be expressed as follows:

$$\vec{F}_{hold} + \vec{F}_{res,k-\delta} + \vec{G} + \vec{N}_2 = 0, \quad (5)$$

where \vec{N}_2 is the normal at the surface S_2 . \vec{N}_2 can be considered a stabilizing force that ensures the actuator's ability to remain in a close position if an accidental force is applied to open it.

During the linear actuator's opening maneuver, the coil is energized so that the device's electromagnetic force is negligible and not considered in equations (as seen in Fig 7). The spring has maximum compression at the beginning of the maneuver and the force exerted by it has its maximum value $\vec{F}_{res,k-\delta}$.

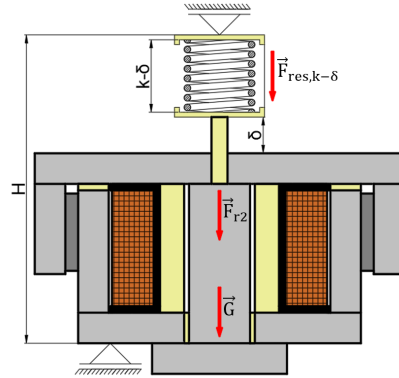


Fig. 7. Linear permanent magnet actuator during opening maneuver

The equation which describes the opening maneuver can be written as:

$$\vec{G} + \vec{F}_{res,k-\delta} + \vec{F}_{r2} = 0, \quad (6)$$

where $\vec{F}_{res,k-\delta}$ is the force exerted by the opening spring, \vec{G} the mobile core's weight and \vec{F}_{r2} is the resultant force during the opening maneuver.

3. NUMERICAL COMPUTATION OF THE PROPOSED LINEAR ACTUATOR

The new linear permanent magnet actuator solution can be analyzed using different approaches. The main parameters and geometrical dimensions were predetermined analytically based on technical requirements and the material's properties. A 2D numerical model was implemented to validate the analytical solution, as seen in Fig. 8.

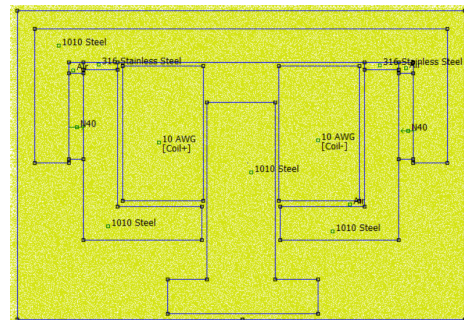


Fig. 8 – Numerical modeling of the linear permanent magnet actuator.

To analyze the mechanical functioning of the linear actuator, the force must be computed for every position of the mobile magnetic core along its path. To realize this, the

numerical solution was computed with a finite element method (FEMM) program, in which an LUA script was implemented [11]. The numerical problem implemented in the program is a planar type; the depth of the problem (and the ferromagnetic circuit, respectively) is 120 mm, and a direct current (dc) analysis is performed. The ferromagnetic circuit was modeled using non-laminated iron, and the coil is made of copper wire (AWG 10) with a diameter of 2.588 mm [13]. The permanent magnets are modeled using NdFeB 45 alloy with high energy density. The magnetomotive force produced by the coil is set at 40.000 At to obtain the value of the magnetic flux density necessary for the actuator's closing maneuver. The maximum air gap between the mobile and fixed ferromagnetic circuit is 22 mm, and the minimal value of the air gap is 0.1 mm.

When the linear permanent magnet actuator is in the open position, as seen in Fig. 9, the air gap has the maximum value (22 mm, respectively). The magnetic field is produced only by the permanent magnets, and the magnetic flux density is low in the air gap. Because of this, the force developed by the actuator is negligible (and has a maximum value of approximately 60 N, depending on the type of permanent magnets used in numerical computation).

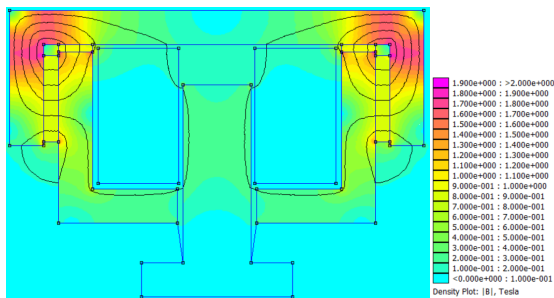


Fig. 9 – Linear permanent magnet actuator in open position.

During the closing maneuver, the actuator has a variable air gap, starting from a maximum value $\delta_{max} = 22$ mm to the minimum value, *i.e.*, the technological air gap (approximately $\delta_{min} = 0.1$ mm). To move the mobile magnetic core, the excitation coil is energized to produce a stationary magnetic field, which is directed in the main air gap by the ferromagnetic circuit, where the electromagnetic force is developed [4–6].

During the closing maneuver, the magnetic field produced by the excitation coil has the same direction and orientation in the magnetic circuit as that produced by the permanent magnets, as seen in Fig. 10.

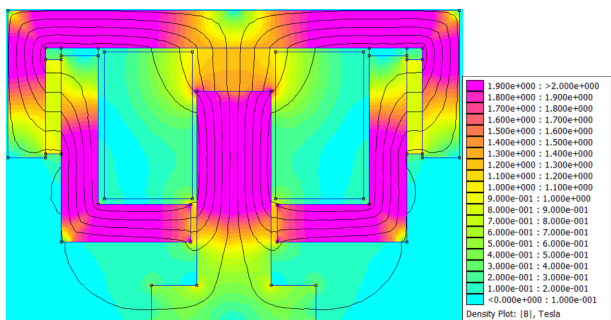


Fig. 10 – Linear permanent magnet actuator during closing maneuver.

Exerting a force between the mobile and fixed ferromagnetic circuits reduces the air gap between them and, thus, lowers the air gap's magnetic reluctance [12].

Considering that a constant electric current passes through the excitation coil, producing a constant magnetomotive force, reducing the air gap and its magnetic reluctance will lead to a higher magnetic flux, as seen in Fig. 11.

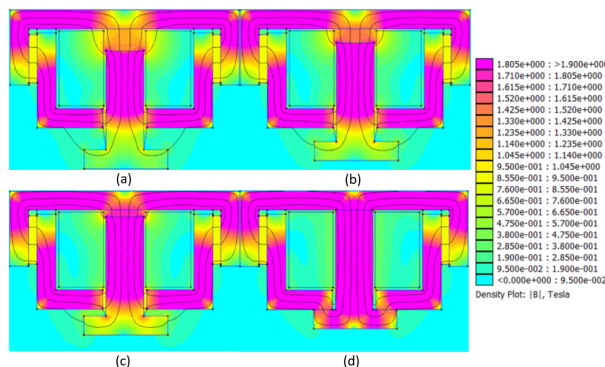


Fig. 11 – Magnetic flux in the linear actuator during closing maneuver for an air gap of 22 mm (a) – maximum value, 14 mm (b), 7 mm (c), and 0,1 mm (d) – minimum value.

Considering this, the higher the magnetic flux, the higher the actuator's force will develop during the closing maneuver, as seen in Fig. 12.

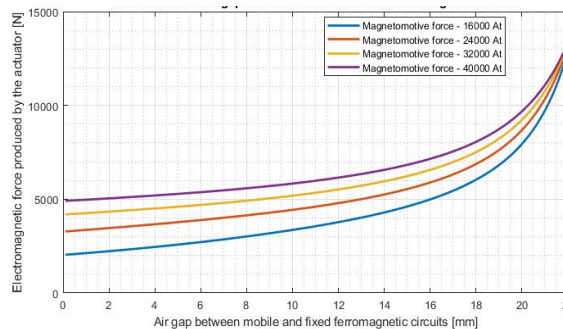


Fig. 12 – Electromagnetic force developed by the actuator as a function of distance traveled by the mobile magnetic core for different magnetomotive forces produced by the actuator.

When the linear actuator is in a close position, the air gap has a minimum value, and the coil is not energized. During this operation, the permanent magnets strictly produce the magnetic field. The magnetic flux density has a high value in the air gap, as shown in Fig. 13, which leads to a high electromagnetic force between the fixed and the mobile magnetic core.

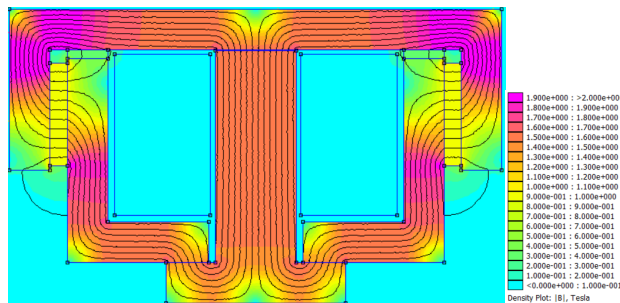


Fig. 13 – Linear permanent magnet actuator in close position.

The magnetic field produced by the permanent magnets is directed by the auxiliary ferromagnetic circuit in the air gap between the lower plates (10 and 10') and the inferior plate (12) (as shown in Fig. 2). The size of the air gaps must be correlated with the overall size of the ferromagnetic circuit so that the magnetic flux produced by the permanent

magnets is directed into this air gap (and the inferior plate - 12) only during the close position of the linear actuator.

The holding force exerted by the actuator in a closed position depends on the energy density of the permanent magnets implemented in the device. Table 1 shows values for the holding force for different magnet types.

Table 1

Electromagnetic force developed by the linear actuator at minimum air gap (in a closed position without energizing the coil) for different types of permanent magnets used in numerical modeling

Magnet type	Remanence B_r [T]	Coercivity H_c [kA/m]	Maximum energy product BH_{max} [kJ/m ³]	Force F_{hold} [N]
NdFeB N30	1.08	796	223	7695.1
NdFeB N35	1.17	867	263	7847.8
NdFeB N40	1.24	923	302	7955.9
NdFeB N45	1.32	875	342	8034.9
NdFeB N50	1.4	796	382	8052.9

The opening maneuver of the linear actuator is realized by the force exerted by the compression springs and the excitation coil [5]. During this operation, the excitation coil is energized, and an electric current flows through it, producing a magnetic field opposite the direction the permanent magnets produced. Its magnetic field has certain values along the magnetic circuit for different currents that flow through the excitation coil. The resultant magnetic field depends on the magnetic field produced by the permanent magnets and the one produced by the excitation coil, as seen in Fig. 14:

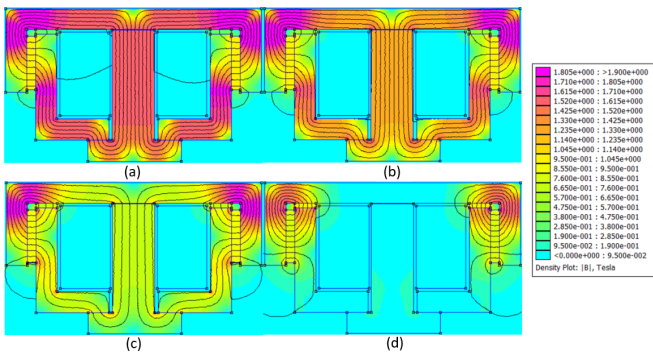


Fig. 14 – Magnetic field along the magnetic circuit for different magnetomotive forces produced by the excitation coil, (a) – 0At, (b) – 1080 At, (c) – 2160 At, (d) – 3240 At, respectively.

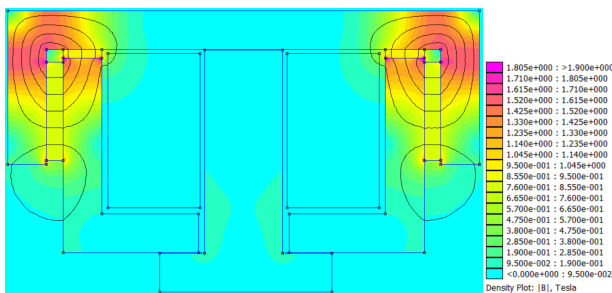


Fig. 15 – Magnetic flux during the opening maneuver for the optimal value of the electric current flowing through the excitation coil.

For a certain value of the electric current that flows through the excitation coil, the magnetic field produced by it has the same value as that produced by the permanent

magnets but has an opposite direction along the magnetic circuit. Considering this, the magnetic circuit's special geometry is designed to make the flux in the air gaps between the mobile and fixed ferromagnetic plates negligible, as presented in Fig. 15.

As seen in Fig. 15, an increase in the value of the electric current flowing through the excitation coil will increase its magnetic field. For a certain optimal value (as presented in the previous paragraph), it is equal but in the opposite direction to that produced by the permanent magnets. Considering this, the force will decrease with the increase of the electric current.

If the electric current exceeds the optimal value, the magnetic field produced by the coil will be higher than the magnetic field produced by the permanent magnets. In this case, the resultant magnetic field, which has an opposite direction along the magnetic circuit produced by the permanent magnets, will produce a force that can maintain the mobile armature in a close position.

5. EXPERIMENTAL MODEL

After the linear actuator was computed and simulated using the FEMM package, an experimental model was implemented to validate the device's working principle and the mechanical parameters. The experimental model has the same cross-sectional dimensions as the numerical model. The ferromagnetic circuit of the experimental model was made of solid steel (S235R, respectively) and was coated with a layer of zinc to protect it from rusting.

The excitation coil is made of insulated copper wire (with a diameter including insulation—2.1 mm) with an effective diameter of 2 mm; it is multi-layered and has 350 turns. The auxiliary components (support and guiding plates) were brass, and the guiding rod was stainless steel.

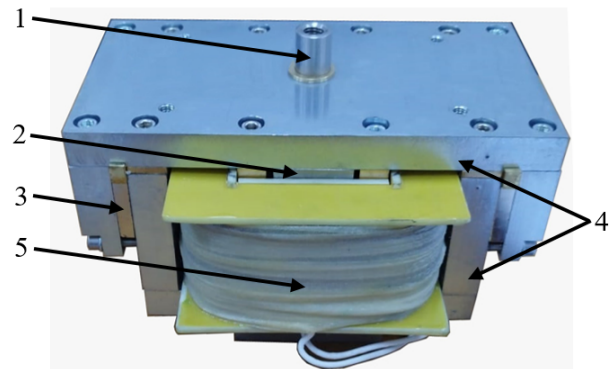


Fig. 17 – Physical model of the direct current actuator (1 – guiding rod, 2 – mobile ferromagnetic circuit, 3 – permanent magnet, 4 – fixed ferromagnetic circuit, 5 – excitation coil).

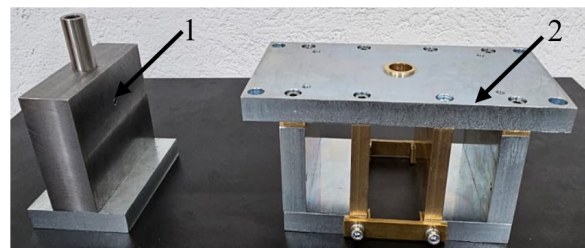


Fig. 18 – Main ferromagnetic components of the linear actuator (1 – mobile ferromagnetic circuit, 2 – fixed ferromagnetic circuit).



Fig. 19 – Measuring the holding force of the linear permanent magnet actuator (1 – variable spacing support, 2 – load cell sensor, 3 – linear actuator, 4- stainless steel support, 5 – digital force gauge).

To measure the electromagnetic force produced by the actuator during a closed position and to compare with the computed results, the experimental setup presented in Fig. 19 was used. The linear actuator (3) was placed on a non-magnographic support to keep the device fixed. The force produced by the linear actuator was measured using a digital force gauge (5) and a load cell sensor (2). The load cell sensor is mounted using a specially designed variable spacing support.

The electrical and mechanical parameters are presented in the following table:

Table 2

Proposed actuator electrical and mechanical parameters

Linear electromagnetic actuator	
Coil Voltage [Vdc]	$U_{coil} = 230 \text{ Vdc}$
Coil electrical resistance	$R_{coil} = 1.2 \Omega$
Current through coils during closing maneuver	$I_c = 115 \text{ A}$
Current through coils during closing maneuver	$I_o = 8.6 \text{ A}$
Number of turns	348 turns
Number of layers	12 layers
Number of turns/layers	29 turns / layer
Magnetomotive force produced during closing maneuver	$\Theta_c = N \cdot I = 40000 \text{ At}$
Magnetomotive force produced during closing maneuver	$\Theta_o = N \cdot I = 3000 \text{ At}$
Holding force (Measured)	$F_{holding} = 7520 \text{ N}$
Holding force (Computed)	$F_{holding} = 7955 \text{ N}$

One of the most important mechanicals of the linear electromagnetic actuator is the holding force exerted in a closed position. As seen in Table 3, the computed force is 7955 N, and the force obtained during experimental results is 7,520 N. The actuator can perform its function in the system in which it is integrated, considering that the minimum holding force needed is 5,200 N.

6. CONCLUSIONS

This paper presented a novel solution for a linear permanent magnet actuator. The article presents the working principle of the actuator, the mechanical and electromagnetic

parameters, and the physical model implemented to validate the computed model. The main novelty of the presented solution consists of a special geometry for the magnetic circuit, which can reduce the overall size of the actuator, considering that other solutions (presented in the introduction) have approximately the same mechanical and electrical parameters. If the permanent magnets and excitation coil are properly implemented, the actuator can produce a high force during the closing maneuver and at a minimal air gap (close position). In future work, the authors want to improve the existing model and implement the actuator in an industry application.

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