# FEED-FORWARD CONTROL DESIGN FOR ROLL/YAW ATTITUDE FLEXIBLE SPACECRAFT BASED ON THE DISTURBANCE OBSERVER

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This paper addresses the attitude control problem for the coupled roll/yaw dynamics of a flexible satellite in the presence of disturbances from flexible vibrations, unknown dynamics, and external environmental disturbances. To estimate the influence of the vibration torque generated by the flexible appendage, a disturbance observer is introduced to improve the attitude control performance and ensure the robustness as well as the stability of the proposed control. Then, the attitude controller is designed to stabilize and attenuate the estimation error of disturbances. To guarantee the stability of the closed-loop system, a stability analysis of the coupled roll/yaw dynamics of a flexible satellite in the closed-loop system is provided using the Lyapunov method, whereas the gains of the composite controller are designed based on the linear matrix inequality (LMI) approach. Finally, the simulation results of the geostationary-earth-orbit flexible satellite are presented to validate the attitude stabilization performance of the proposed approach.

## 1. INTRODUCTION

In recent three decades, the attitude control system (ACS) of flexible spacecraft has attracted the attention of many researchers. The knowledge of the interaction between the rigid hub and flexible structure dynamics is a critical task since this interaction will have a great impact on the control stability and can damage the ACS pointing accuracy requirements. Therefore, these analyses lead to considerable difficulties in designing control systems for vibrations suppression and simultaneous maneuvers.

To solve this problem, significant strategies have been developed and applied to control the attitude disturbance and suppress the flexible vibration, such as the classical PID control [1], adaptive control [2], variable structure control [3], and sliding mode control (SMC) [4]. One of the most widely used methods in ACS of flexible spacecraft is SMC because it has excellent robustness and strong anti-disturbance ability [5, 6]. However, this approach often results in a chattering phenomenon due to its discontinuous switching control. In [7, 8], a robust  $H_{\infty}$  control method is proposed in ACS of the rigid-flexible satellite. This problem is formulated in terms of linear matrix inequality which provides a good disturbance attenuation performance, but it was limited usefulness for attitude stabilization.

Other interesting approaches of robust controllers and fuzzy logic damping were performed in [9–11] to address the tracking problem of the satellite. Considering the unknown control input saturation and external disturbances using an inverse tangent-based tracking function, a backstepping attitude controller was designed for the attitude maneuver problem [12]. However, these works did not consider the vibrations caused by flexible dynamics.

To deal with the restriction faced by traditional feedforward control and improve the attitude control performance, an effective vibration suppression based on the disturbance observer strategy has been proposed [13–16], which can estimate the disturbances, and uncertainties and compensate them effectively through feedforwards, although good vibration rejection was achieved in these studies. However, the above-mentioned control theories can only be used for the stabilization of attitude control on a single axis.

Motivated by the above methods, in the presence of external or internal disturbances [15–17], a novel composite

disturbance observer and control scheme is designed for the coupled roll/yaw attitude stabilization and vibration suppression of a flexible satellite. First, an observer has been constructed to estimate the disturbances and compensate them using the controller. The equations of motion of the coupled roll/yaw of the flexible satellite are presented. The attitude controller-based observer for attitude stabilization is designed in the form of LMI, and the stability of the closedloop system is proved via the Lyapunov method. Numerical simulations are performed to show the effectiveness and performance of the composite controller.

## 2. MODEL FORMULATION

This study considers the coupled roll/yaw dynamics of a flexible spacecraft by considering the vibrations of the flexible panels. Figure 1 illustrates a typical flexible spacecraft that consists of a rigid body and several flexible appendages, such as large solar panels, and antennas.

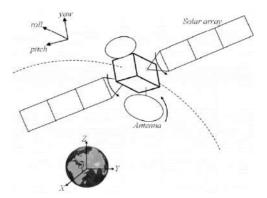


Fig. 1 – Spacecraft with flexible appendages [18].

The attitude dynamic system of the flexible appendages coupled with the rigid body can be described by the following differential equations [19,20]

$$\mathbf{I}\dot{\boldsymbol{\omega}} + \mathbf{F}^T \ddot{\boldsymbol{\eta}} = \mathbf{u} + \mathbf{d},\tag{1}$$

$$\ddot{\eta} + 2\xi\dot{\eta} + \Omega^2\eta + F\dot{\omega} = 0, \qquad (2)$$

where I is the inertia matrix,  $\omega$  is the spacecraft attitude angles vector including yaw and roll angles, I is the coupling coefficient matrix,  $\eta$  is the model coordinate,  $\xi$  and  $\Omega$  is

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the model ratio and the model frequency matrix respectively,  $\mathbf{u}$  is the control torque and  $\mathbf{d}$  represents the external bounded disturbances torques. Combining (1) with (2), yields:

$$(\mathbf{I} - \mathbf{F}\mathbf{F}^T)\dot{\boldsymbol{\omega}}(t) = \mathbf{F}(2\boldsymbol{\xi}\Omega\dot{\boldsymbol{\eta}} + \Omega^2\boldsymbol{\eta}) + \mathbf{u}(t) + \mathbf{d}(t), \qquad (3)$$

According to [21], the state space transformation can be defined as

$$\boldsymbol{\omega} = [\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{\psi}}]^T, \ \boldsymbol{x}_I = \boldsymbol{\varphi}, \ \boldsymbol{x}_2 = \boldsymbol{\psi} \ \boldsymbol{x}_3 = \dot{\boldsymbol{\varphi}}, \ \boldsymbol{x}_4 = \dot{\boldsymbol{\psi}},$$
$$\mathbf{X} = (x_I, x_2, x_3, x_4)^T, \ \mathbf{u} = (u_x, u_y)^T, \text{ and } \ \mathbf{d} = (d_x, d_y)^T.$$

Hence eq. (3) can be transformed into the following form:

$$\mathbf{X}(t) = \mathbf{A}\mathbf{X}(t) + \mathbf{B}(\mathbf{u} + \mathbf{d} + \mathbf{d}_{flex}), \qquad (4)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ (\mathbf{I} - \mathbf{F}\mathbf{F}^T)^{-1} \end{bmatrix},$$
(5)

The internal disturbances caused by the flexible vibration maybe described as:

$$\mathbf{d}_{flex} = \mathbf{F} \Big( 2\xi \mathbf{\Omega} \dot{\mathbf{\eta}} + \mathbf{\Omega}^2 \mathbf{\eta} \Big), \tag{6}$$

To handle the considerable impact on the stability of the system, two control methods are needed to enhance the performances.

## 3. COMPOSITE CONTROL DESIGN

In this section, the system composite control scheme is performed and used to estimate feedforward compensation for the disturbance observer with flexible vibration design.

It can be obtained from (1) and (2)

$$\ddot{\boldsymbol{\eta}}(t) = -\boldsymbol{\Delta}^{-1} \Big[ \mathbf{F}(2\boldsymbol{\xi}\boldsymbol{\Omega}\dot{\boldsymbol{\eta}}(t) + \boldsymbol{\Omega}^{2}\boldsymbol{\eta}(t)) + \mathbf{F}^{T}\mathbf{I}^{-1}(\mathbf{u}(t) + \mathbf{d}(t)) \Big], \quad (7)$$

where

$$\boldsymbol{\Delta} = \mathbf{I} - \mathbf{F}^T \mathbf{I}^{-1} \mathbf{F},\tag{8}$$

According to [22], the disturbance  $\mathbf{d}_{flex}$  can be formulated by the following system:

$$\begin{cases} \mathbf{d}_{flex}(t) = \mathbf{V}\mathbf{w}(t), \\ \dot{\mathbf{w}}(t) = \mathbf{W}\mathbf{w}(t) + \mathbf{H}\mathbf{d}(t), \end{cases}$$
(9)

where

$$\mathbf{w}(t) = \begin{bmatrix} \mathbf{\eta}(t), \dot{\mathbf{\eta}}(t) \end{bmatrix}^T, \mathbf{V} = \mathbf{F} \begin{bmatrix} \mathbf{\Omega}^2, 2\xi \mathbf{\Omega} \end{bmatrix}$$
$$W = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Delta}^{-1} \mathbf{\Omega}^2 & -\mathbf{\Delta}^{-1} 2\xi \mathbf{\Omega} \end{bmatrix}, \text{ and } \mathbf{H} = \begin{bmatrix} \mathbf{0} \\ \mathbf{\Delta}^{-1} \mathbf{F} \mathbf{I}^{-1} \end{bmatrix},$$

Thereby the disturbance observer is constructed as

$$\begin{cases} \hat{\mathbf{d}}_{flex}(t) = \mathbf{V}\hat{\mathbf{w}}(t), \\ \hat{\mathbf{w}}(t) = \mathbf{v}(t) - \mathbf{L}\mathbf{X}(t), \\ \dot{\mathbf{v}}(t) = (\mathbf{W} + \mathbf{L}\mathbf{B}\mathbf{V})(\mathbf{v}(t) - \mathbf{L}\mathbf{X}(t)) + \mathbf{L}\mathbf{A}\mathbf{X}(t) + \mathbf{L}\mathbf{B}\mathbf{u}(t), \end{cases}$$
(10)

where  $\hat{\mathbf{w}}(t)$  is the estimation of  $\mathbf{w}(t)$ ,  $\mathbf{v}(t)$  is the auxiliary vector as the state of the observer,  $\hat{\mathbf{d}}_{flex}(t)$  is the estimation

of the disturbance  $\mathbf{d}_{flex}(t)$ , **L** is the desired observer gain. The estimation error is denoted as

$$\mathbf{e}(t) = \mathbf{w}(t) - \hat{\mathbf{w}}(t), \tag{11}$$

Based on (4), (9), (10), and (11) it is shown that the error dynamics satisfy:

$$\dot{\mathbf{e}}(t) = (\mathbf{W} + \mathbf{LBV})\mathbf{e}(t) + (\mathbf{H} + \mathbf{LB})\mathbf{d}(t), \quad (12)$$

The aim of disturbance rejection can be reached by designing the observer gain such that (12) satisfies the desired stability and robustness performance; therefore, the structure of the controller is formulated as:

$$\mathbf{v}(t) = -\hat{\mathbf{d}}_{flex}(t) + \mathbf{K}\mathbf{X}(t), \qquad (13)$$

Combining (4) with (13), the closed-loop system is described as

$$\dot{\mathbf{X}}(t) = (\mathbf{A} + \mathbf{B}\mathbf{K})\mathbf{X}(t) + \mathbf{B}\mathbf{V}\mathbf{e}(t) + \mathbf{B}\mathbf{d}(t).$$
(14)

Thus, the composite system (11) and (14) yields

$$\begin{bmatrix} \dot{\mathbf{X}}(t) \\ \dot{\mathbf{e}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} + \mathbf{B}\mathbf{K} & \mathbf{B}\mathbf{V} \\ 0 & \mathbf{W} + \mathbf{L}\mathbf{B}\mathbf{V} \end{bmatrix} \begin{bmatrix} \mathbf{X}(t) \\ \mathbf{e}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{L}\mathbf{B} + \mathbf{H} \end{bmatrix} (t), \quad (15)$$

The output of the whole system is

$$\mathbf{z}(t) = \mathbf{C}_1 \mathbf{X}(t) + \mathbf{C}_2 \mathbf{e}(t) + \mathbf{D}_1(t).$$
(16)

## 4. STABILITY ANALYSIS

## Theorem

For a given parameter  $\gamma > 0$ , if there exist positive definite matrices yield  $\mathbf{X} > 0$ ,  $\mathbf{P}_2 > 0$ , and matrices  $\mathbf{R}_1, \mathbf{R}_2$ , satisfying (17) then:

$$\begin{bmatrix} \mathbf{M}_{1} & \mathbf{BV} & \mathbf{B} & \mathbf{XC}_{1}^{T} \\ * & \mathbf{M}_{2} & \mathbf{R}_{2}\mathbf{B} + \mathbf{P}_{2}\mathbf{H} & \mathbf{C}_{2}^{T} \\ * & * & -\gamma^{2}\mathbf{I} & \mathbf{D}_{1}^{T} \\ * & * & * & -\mathbf{I} \end{bmatrix} < 0, \quad (17)$$

where \* denotes the symmetric item in a symmetric matrix and

$$\mathbf{M}_{1} = (\mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{R}_{1}) + (\mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{R}_{1})^{2},$$
  
$$\mathbf{M}_{2} = (\mathbf{P}_{2}\mathbf{W} + \mathbf{R}_{2}\mathbf{B}\mathbf{V}) + (\mathbf{P}_{2}\mathbf{W} + \mathbf{R}_{2}\mathbf{B}\mathbf{V})^{T}.$$
(18)

Then the composite system (15) under the proposed control law (13) with gain  $\mathbf{K} = \mathbf{R}_1 \mathbf{X}^{-1}$  associated to the observer (10) with gain  $\mathbf{L} = \mathbf{P}_2^{-1} \mathbf{R}_2$  is asymptotically stable and satisfies  $\|\mathbf{z}\|_2 < \gamma \|\mathbf{d}(t)\|_2$ .

# Proof.

Considering the following system

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \bar{\mathbf{A}}\mathbf{x}(t) + \bar{\mathbf{B}}\mathbf{d}(t), \\ \mathbf{y}(t) &= \bar{\mathbf{C}}\mathbf{x}(t) + \bar{\mathbf{D}}\mathbf{d}(t). \end{aligned}$$
(19)

For given parameter  $\gamma > 0$ , if there exist **P** > 0 satisfying

$$\begin{bmatrix} \bar{\mathbf{A}}^T \mathbf{P} + \mathbf{P}\bar{\mathbf{A}} & \mathbf{P}\bar{\mathbf{B}} & \bar{\mathbf{C}} \\ * & -\gamma^2 \mathbf{I} & \bar{\mathbf{D}}_1^T \\ * & * & -\mathbf{I} \end{bmatrix} < 0,$$
(20)

where

$$\mathbf{P} = \begin{bmatrix} \mathbf{p}_1 & 0\\ 0 & \mathbf{p}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_1^{-1} & 0\\ 0 & \mathbf{P}_2 \end{bmatrix} > 0.$$
(21)

Then system (17) is asymptotically stable. Indeed, considering the Lyapunov function as [23]

$$V(t) = \mathbf{x}^{T}(t)\mathbf{P}\mathbf{x}(t) + \int_{0}^{t} \left(\mathbf{y}^{T}(r)\mathbf{y}(r) - \gamma^{2}\mathbf{d}^{T}(r)\mathbf{d}(r)\right) dr > 0, (22)$$

where  $\mathbf{P} = \mathbf{P}^T > \mathbf{0}$ ,  $\gamma > 0$ , evaluating the derivative of V(t) with respect to t along a system trajectory can yields

$$\dot{V}(t) = \dot{\mathbf{x}}^{T}(t)\mathbf{P}\mathbf{x}(t) + \mathbf{x}^{T}(t)\mathbf{P}\dot{\mathbf{x}}(t) + + \mathbf{y}^{T}(t)\mathbf{y}(t) - \gamma^{2}\mathbf{d}^{T}(t)\mathbf{d}(t) < 0.$$
(23)

Thus, substituting system (19) into (23) gives

$$\dot{\mathcal{V}}(t) = (\overline{\mathbf{A}}\mathbf{x}(t) + \overline{\mathbf{B}}\mathbf{d}\mathbf{x}(t))^T \mathbf{P}\mathbf{x}\mathbf{x}(t) + + \mathbf{x}^T \mathbf{x}(t)(\overline{\mathbf{A}}\mathbf{x}\mathbf{x}(t) + \overline{\mathbf{B}}\mathbf{d}\mathbf{x}(t)) - \gamma^2 \mathbf{d}^T \mathbf{x}(t)\mathbf{d}\mathbf{x}(t) + (24) + (\overline{\mathbf{C}}\mathbf{x}\mathbf{x}(t) + \overline{\mathbf{D}}\mathbf{d}\mathbf{x}(t))^T (\overline{\mathbf{C}}\mathbf{x}\mathbf{x}(t) + \overline{\mathbf{D}}\mathbf{d}\mathbf{x}(t)),$$

with the notation

$$\widetilde{\mathbf{x}}(t) = \begin{bmatrix} \mathbf{x}^T(t) & \mathbf{d}^T(t) \end{bmatrix}$$
(25)

It is obtained

$$\dot{V}(t) = \tilde{\mathbf{x}}^{T}(t)\tilde{\mathbf{P}}\tilde{\mathbf{x}}(t) < \mathbf{0}, \qquad (26)$$

where

$$\widetilde{\mathbf{P}} = \begin{bmatrix} \overline{\mathbf{A}}^T \mathbf{P} + \mathbf{P}\overline{\mathbf{A}} & \mathbf{P}\mathbf{B} \\ * & -\gamma^2 \mathbf{I} \end{bmatrix} + \begin{bmatrix} \overline{\mathbf{C}}^T \overline{\mathbf{C}} & \overline{\mathbf{C}}^T \overline{\mathbf{D}} \\ * & \overline{\mathbf{D}}^T \overline{\mathbf{D}} \end{bmatrix} < 0, \quad (27)$$

since

$$\begin{bmatrix} \overline{\mathbf{C}}^T \overline{\mathbf{C}} & \overline{\mathbf{C}}^T \overline{\mathbf{D}} \\ * & \overline{\mathbf{D}}^T \overline{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{C}}^T \\ \overline{\mathbf{D}}^T \end{bmatrix} \begin{bmatrix} \overline{\mathbf{C}} & \overline{\mathbf{D}} \end{bmatrix} \ge 0, \quad (28)$$

with Schur complement property yields

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \overline{\mathbf{C}}^T \\ * & \mathbf{0} & \overline{\mathbf{D}}^T \\ * & * & -\mathbf{I} \end{bmatrix} \ge \mathbf{0}.$$
 (28)

Using (28) the LMI condition (27) can be written compactly as (20).

Let  

$$\overline{\mathbf{A}} = \begin{bmatrix} \mathbf{A} + \mathbf{B}\mathbf{K} & \mathbf{B}\mathbf{V} \\ \mathbf{0} & \mathbf{W} + \mathbf{L}\mathbf{B}\mathbf{V} \end{bmatrix}, \ \overline{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{L}\mathbf{B} + \mathbf{H} \end{bmatrix}, \ \overline{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_1^T \\ \mathbf{C}_2^T \end{bmatrix},$$
  
then

$$\begin{bmatrix} \mathbf{\Pi}_{1} & \mathbf{P}_{1}\mathbf{B}\mathbf{V} & \mathbf{P}_{1}\mathbf{B} & \mathbf{C}_{1}^{T} \\ * & \mathbf{\Pi}_{2} & \mathbf{P}_{2}(\mathbf{L}\mathbf{B} + \mathbf{H}) & \mathbf{C}_{2}^{T} \\ * & * & -\gamma^{2}\mathbf{I} & \mathbf{D}_{1}^{T} \\ * & * & * & -\mathbf{I} \end{bmatrix} < 0,$$
(29)

where

$$\boldsymbol{\Pi}_{1} = \boldsymbol{P}_{1}(\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K}) + [\boldsymbol{P}_{1}(\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})]^{T},$$
  
$$\boldsymbol{\Pi}_{2} = \boldsymbol{P}_{2}(\boldsymbol{W} + \boldsymbol{L}\boldsymbol{B}\boldsymbol{V}) + [\boldsymbol{P}_{2}(\boldsymbol{W} + \boldsymbol{L}\boldsymbol{B}\boldsymbol{V})]^{T}.$$
(30)

Pre-multiplied and post-multiplied simultaneously the LMI (29) by diag( $\mathbf{Q}_1$ ,  $\mathbf{I}$ ,  $\mathbf{I}$ ), thus (17) can be obtained by defining  $\mathbf{X} = \mathbf{Q}_1$ ,  $\mathbf{R}_1 = \mathbf{K}\mathbf{Q}_1 = \mathbf{K}\mathbf{X}$ , and  $\mathbf{R}_2 = \mathbf{P}_2\mathbf{L}$ . Consequently, by considering the control gain as  $\mathbf{K} = \mathbf{R}_1\mathbf{Q}_1^{-1}$  and the observer gain as  $\mathbf{L} = \mathbf{P}_2^{-1}\mathbf{R}_2$  it can be concluded that the composite system (15) is asymptotically stable.

### 5. SIMULATION RESULTS

To validate the performance of the proposed controller, numerical simulations have been developed in MATLAB/SIMULINK environment and M.file, and the composite control scheme has been applied to the satellite with one flexible appendage while the roll/yaw attitude stabilization is accomplished by LMI theory.

The satellite is designed to move in the geostationary orbit with an altitude of 36 000 km, while the orbital rate is the same as the angular velocity in one real day which is  $\omega_0 = 7.2921e - 5$  rad/s.

 Table 1

 Single solar array flexibility model

 OP is out of the plane, T torsion AND IP in-plane

CANTILEVER MODE	CANTILEVER FREQUENCY	COUPLING SCALARS, $\sqrt{\text{Kg.m}^2}$		
DESCRIPTION	ω, rad / s	ROLL,	PITCH,	YAW,
		$\delta_1$	δ2	δ3
OP-1	0.885	0	0	35.372
OP-2	6.852	0	0	4.772
OP-3	16.658	0	0	2.347
OP-4	33.326	0	0	0.548
T-1	5.534	0	2.532	0
T-2	14.668	0	0.864	0
T-3	33.805	0	0.381	0
IP-1	1.112	35.865	0	0
IP-2	36.362	2.768	0	0

Table 2

Satellite simulation parameters

Saterinte Simulation Parameters				
Parameters	Value			
Inertia [kgm2]	diag(3026,3164)36			
Initial attitude[rad]	[1.5 2]			
Initial attitude rate [rad/s]	[-0.1 0.2]			
Desired attitude [rad]	[0 0]			
External Torque [N.m]	$\begin{cases} T_x = 2*10^{-5}(1-2\sin\omega_0 t) \\ T_y = -5*10^{-5}(\cos\omega_0 t) \end{cases}$			

The flexible satellite parameters are shown in Table 1 from reference [20]. The satellite simulation parameters are detailed below (see Table 2).

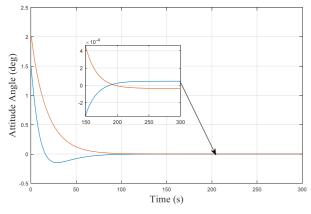
Based on LMI (17), the controller gain is

$$\mathbf{K} = \begin{bmatrix} -107.7 & -99.4 & -1234.4 & -50.3 \\ -234.5 & -420.9 & -2294.2 & -6115.4 \end{bmatrix}$$

and the observer gain is

$$\mathbf{L} = \begin{bmatrix} 0 & 0 & 0.00010 & -0.0503 \\ 0 & 0 & -0.0311 & -0.0001 \\ 0 & 0 & -0.0001 & 0.01740 \\ 0 & 0 & 0.00600 & -0.0001 \end{bmatrix}.$$

Figures 2 and 3 show the time evolutions of the coupled roll/yaw attitude angles and angular velocity respectively, the attitude control performance and attitude stabilization are evidently enhanced by the proposed approach where the pointing accuracy has been reached which can be illustrated in their partial amplifications.





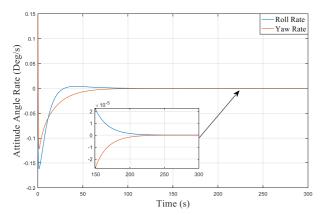


Fig. 3 - Time response of attitude rate angle.

Figures 4 and 5 show the time responses of disturbances, and estimations errors, respectively. It can be concluded that the effects of the vibrations can be reduced to the lowest, which yields better vibrations suppressions by estimating the disturbances through the observer and compensating them through the proposed controller.

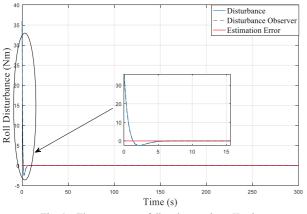


Fig. 4 - Time response of disturbance along X-axis.

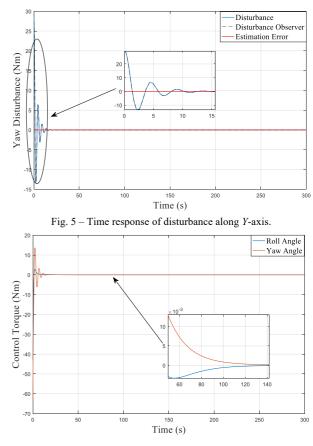


Fig. 6 - Time response of control torque.

In Figure 6 it can be clearly observed that the control torque is a little important at the beginning of the simulation and fast reduced to the lowest, which confirms the effectiveness of the proposed approach and its better dynamic response performance, moreover, no excessive control energy is required to reject the disturbances.

For more details analysis, the root mean square (RMS) errors of the attitude angles and the attitude rate obtained by the proposed method and two types of controller methods (disturbance observer-based controller (DOBC) and extended state observer (ESO)) are listed in Table 3. It is seen that the proposed method gives the best results in controlling the satellite model against the elastic motion disturbance and external disturbance effects.

 Table 3

 Comparison results of attitude control performance during two-axis maneuvers: RMS error (deg)

	RMS error of the proposed method	RMS error of PD DOBC	RMS error of PD ESO
Roll (deg)	0.007	0.008	0.050
Yaw (deg)	0.0022	0.0031	0.010
Roll rate (deg/s)	0.096 e-3	1.009 e-3	2.851 e-3
Yaw rate (deg/s)	0.121 e-3	0.369 e-3	0.651 e-3
	Magnitude of error	Magnitude of error	Magnitude of error
Angles (deg)	0.0028	0.0086	0.051
Rate (deg/s)	0.154 e-3	1.107 e-3	2.916 e-3

# 6. CONCLUSIONS

The attitude control and stabilization issue for coupled roll/yaw dynamics of a flexible spacecraft is considered in this paper. To attenuate the effects of vibrations and disturbances, a robust disturbances-observer-based controller has been developed. To minimize the residual vibration, an active suppression technique is perfectly adopted. Based on Lyapunov theory, the control laws can guarantee that composite closed-loop systems are asymptotically stable in the presence of disturbances. Even more, the observer and controller gains are well designed via LMI. Finally, the effectiveness of the proposed controller is verified through practical numerical simulation as requested in most practical missions, while the pointing accuracy and stability satisfy the requirements.

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