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A NEW HYBRID OPTIMIZATION METHOD USED IN PREDICTIVE CONTROL OF A NONLINEAR FRACTIONAL MODEL BASED ON FRACTIONAL HAMMERSTEIN STRUCTURE

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Keywords: Fractional order system; Hammerstein model; Optimization problem; Predictive control.

Fractional Hammerstein models represent various nonlinear processes, such as thermal and mechanical. Their major drawback is the non-convex optimization problem in a nonlinear model predictive control scheme due to its static nonlinearity. Indeed, an efficient optimization algorithm is needed. This work proposes a hybrid optimization algorithm combining the Nelder Mead optimization method and the Honey Badger one to synthesize a predictive control algorithm based on fractional Hammerstein models. As illustrated through simulation results, the proposed method offers clear improvements in convergence and tracking performances.

1. INTRODUCTION

In recent decades, nonlinear model predictive control (NMPC) has been considered one of the most successful control schemes. It has been used in several areas, such as energetic [1,2,3], electric [4], mechanical [5,6]. The applicability of the NMPC is extended to fractional nonlinear models. However, its application is still limited because the models are inaccurate. The complexity of controlling nonlinear dynamic fractional processes is still an open research topic, and the evolution of fractional calculus has become more attractive. Fractional models turn into an effective tool for representing linear and nonlinear systems. According to the research, the nonlinear fractional models are classified into several basic categories, such as Volterra series models [7], neural network models [8], and block-structured models. Among the structured models, we cite the Hammerstein model, which refers to a first model with a static nonlinearity followed by a second dynamic linear fractional one [9]. The Wiener structure represents an inverted form by a linear model followed by a static nonlinearity [10,11]. Combining the two previous structures forms the Hammerstein-Wiener models [12]. The Fractional Hammerstein Models (FHM) are most often manipulated in the literature considering their generality of representation. They described many processes, such as thermal systems [13] and mechanical systems [14]. Besides, FHM constitutes a powerful nonlinear modeling tool. Indeed, it allows the linear dynamic model theory to be exploited, where the nonlinearity is static.

In the literature, research works have dealt with the control of integer order Hammerstein structures, such as PID control [15], sliding mode control [16], and predictive control [17]. However, we haven't found any work dealing with the control of FHM, especially fractional predictive control. Indeed, these types of models require advanced mathematical tools and algorithms. Moreover, for fractional systems, the calculation of the optimal control uses all previous output and control sequences, which will enlarge the computing time. Therefore, to search for the control law that minimizes the optimization criterion, choosing the approach that consumes the least time to deal with fast dynamic systems and extends the exploitation of the microcontroller in real-time applications will be better. In addition, the NMPC algorithm may encounter a non-convex optimization problem due to the static nonlinearity of the model.

Furthermore, the nonlinear model predictive control can't use an analytical adaptation law for the optimal control as in the case of linear models. Thus, a numerical optimization algorithm must be run at every iteration. The optimization methods used in treating such problems in literature are deterministic, geometric, or stochastic. The first type concerns all gradient-based techniques, such as the steepest descent algorithm [18], the sequential quadratic programming [19], and the interior point algorithm [20]. The Nelder Mead (NM) method [21,22] belongs to the second type. The third type encompasses the genetic algorithms [23], particle swarm optimization [24,25], honey badger algorithm (HB) [26–28], which is a recently developed metaheuristic algorithm inspired by the intelligent foraging behavior of HB, hybrid methods HBNM [29] which apply solution obtained from HB algorithm in the NM algorithm. We demonstrated in [30] the efficiency of the NM approach in solving convex and non-convex optimization problems. It finds the best solution, minimizing the criterion in a reduced time. However, when controlling fractional systems, the control vector dimension depends on the prediction horizon. Indeed, if the prediction horizon is high, the NM approach may fail to reach the best solution. Indeed, the constraint restriction applied to the optimization variable can cause divergence of the NM optimization algorithm. However, the HB method does not block if the dimension of the control vector is high or when the constraint is restricted. Nevertheless, searching for the best solution consumes more time, which can be amplified if the predictive control is applied to fractional systems. Indeed, controlling them requires using a vast number of previous input and output measurements, significantly enlarging the calculation time.

This work developed a new hybrid optimization algorithm, NMHB, which combines the NM method with the HB one. This is to exploit the advantages of both methods and avoid their drawbacks.

Main contributions:

- A new hybrid method, NMHB, was developed that combines NM and HB algorithms to find the best control solution.
- A NMPC scheme based on FHM is proposed.
- Based on the curves of static nonlinear characteristics and the criterion, the non-convex optimization problem is converted into a convex one.
- The rest of the paper is divided into four parts. The first

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section details the fractional predictive control algorithm. The second section explains the new optimization algorithm NMHB used to research the control minimizing the quadratic criterion. Then, we will test the proposed algorithms by treating convex and non-convex optimization problems for a single input, single output (SISO) system, and we will conclude with a conclusion.

2. NONLINEAR FRACTIONAL PREDICTIVE CONTROL

The Hammerstein structure will model the nonlinear system as represented in the block diagram of Fig. 1.

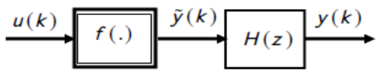

Fig. 1 – The block diagram of a Hammerstein model.

The control signal $u(k)$ is firstly executed by the static nonlinear function $f(.)$ and then passed through a linear dynamic filter described by a fractional order transfer function $H(s)$. The internal signal$\tilde{y}(k)$, which is usually assumed to be not measurable, is the result of the transformation of the input signal $u(k)$ caused by the static nonlinear function $f(.)$. The polynomial form is chosen to represent the nonlinear function. It is expressed as follows:

$$f(u) = \tilde{y}(k) = c_1 u(k) + c_2 u^2(k) + \cdots + c_{n_c} u^{n_c}(k), \quad (1)$$

where n_c is the degree of the polynomial function.

In this paper, the linear part is represented by a commensurate continuous fractional order transfer function $H(s)$, defined by

$$H(s) = \frac{b_m s^{m\alpha} + b_{m-1} s^{(m-1)\alpha} + \cdots + b_0}{a_l s^{l\alpha} + a_{l-1} s^{(l-1)\alpha} + \cdots + a_0}, \quad (2)$$

where a_0,..., a_l, b_0,..., b_m are constant coefficients and α fractional power $0 < \alpha < 1$ .a_0,..., a_l, b_0,..., b_m, α, c_1,...,c_{n_c} is the fractional Hammerstein parameters, where their identification procedure is detailed in [31].

The predictive optimal control is calculated using the model describing the system's future behavior over a finite prediction horizon. The minimization of a quadratic criterion is run over time during each sampled measurement. It allows for synthesizing an optimal control sequence. The first element in the sequence is then applied to the system. We start the calculation again, considering updated values of control and output. The matrix form of the predicted output over a prediction horizon N_p is given by:

$$\hat{\mathbf{Y}} = \mathbf{A}\mathbf{Y}_p + \mathbf{B}_p\tilde{\mathbf{Y}}_p + \mathbf{B}_f\tilde{\mathbf{Y}}_f, \quad (3)$$

where $\hat{\mathbf{Y}}$ is the predicted output vector as given by

$$\hat{\mathbf{Y}} = [\hat{\mathbf{y}}(k+1), \hat{\mathbf{y}}(k+2), \dots, \hat{\mathbf{y}}(k+N_p)]^{\mathrm{T}} \quad (4)$$

where the old output values are saved in the vector $\boldsymbol{Y_p}$:

$$\mathbf{Y}_p = [y(k-1), y(k-2), \dots, y(k-N_k)]^{\mathrm{T}}. \quad (5)$$

The old calculated values of the internal signal $\tilde{y}(k)$ are stored in the vector $\tilde{\mathbf{Y}}_p$

$$\tilde{\mathbf{Y}}_p = [\tilde{y}(k-1), \tilde{y}(k-2), \dots, \tilde{y}(k-N_k)]^{\mathrm{T}}, \quad (6)$$

where N_k are the last samples taken in the fractional calculation. The future values of the internal signal were saved in the vector $\tilde{\mathbf{Y}}_f$ as given by

$$\tilde{\mathbf{Y}}_f = \begin{bmatrix} \tilde{y}(k) \\ \tilde{y}(k+1) \\ \vdots \\ \tilde{y}(k+(N_p-1)) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n_c} c_i u^i(k) \\ \sum_{i=1}^{n_c} c_i u^i(k+1) \\ \vdots \\ \sum_{i=1}^{n_c} c_i u^i(k+(N_p-1)) \end{bmatrix}. \quad (7)$$

The elements of the matrix A are calculated in terms of ah_i illustrated by

$$ah_i = -\mu \sum_{l=0}^{L} \frac{a_l}{h^{l\alpha}} \sum_{i=1}^{j} (-1)^i \binom{l\alpha}{i}. \quad (8)$$

The elements of the matrix B_p and B_f are calculated in terms of ah_j and bh_j expressed by eq. (8) and (9):

$$bh_j = \mu \sum_{m=0}^{M} \frac{b_m}{h^{m\alpha}} \sum_{i=1}^{j} (-1)^i \binom{m\alpha}{i}, \quad (9)$$

where h is the sample period, $\mu = \frac{1}{\sum_{l=0}^{L} \frac{a_l}{h^{l\alpha}}}$, $\binom{l\alpha}{i}$ and $\binom{m\alpha}{i}$ are Newton's binomial generalized to non-integer orders. L and M are the number of derivations on the output and the input. More details on ah_i and bh_j calculation is developed in [32].

The dynamic criterion $J(k)$ will be minimized to determine the optimal predictive control solution at each sample iteration k. This cost function expresses, in a first term, the summed squared differences between the future set-points and the predicted outputs within a prediction horizon N_p. The second term contains the sum of squares future control increments within a control horizon N_c. The criterion is given in matrix form [33] by

$$J(k) = \frac{1}{2}[\mathbf{Y}_r(k) - \hat{\mathbf{Y}}(k)]^T \mathbf{Q}[\mathbf{Y}_r(k) - \hat{\mathbf{Y}}(k)] + \Delta\bar{\mathbf{U}}(k)^T \mathbf{R} \Delta\bar{\mathbf{U}}(k) \quad (10)$$

where $\mathbf{Y}_{\boldsymbol{r}}(k)$ is the set-point vector expressed by:

$$\mathbf{Y}_r(k) = [y_r(k+1), y_r(k+2), \dots, y_r(k+N_p)]^{\mathrm{T}}, \quad (11)$$

where $y_r(k)$ is the set-point and $\Delta\bar{\boldsymbol{U}}(k)$ is the control increment vector defined by

$$\Delta\bar{\mathbf{U}}(k) = \mathbf{U}(k) - \mathbf{U}(k-1), \quad (12)$$

$$\Delta\bar{\mathbf{U}}(k) = [\Delta\mathbf{U}(k), \Delta\mathbf{U}(k+1), \dots, \Delta\mathbf{U}(k+(N_c-1))]^T$$

$$\mathbf{U}(k) = [u(k), u(k+1), \dots, u(k+N_p-1)]^T$$

where $\mathbf{Q}$ is the weight matrix of the prediction error and $\mathbf{R}$ is the weight matrix of the control increment used to minimize the control energy? It should be noted that, at each iteration, an optimization problem with $N_p$unknown variables will be resolved, because the vector $\boldsymbol{U}(k)$ contains N_p control values. Whereas just the first component $u(k)$ of the optimal control vector $\mathbf{U}(k)$ will be applied to the system. In the following, let consider the constrained optimization problem of several variables of the criterion $J(k)$ versus the control vector $\boldsymbol{U}(k)$. The method explained below attempts to solve an optimization problem of the form

$$\min_{\boldsymbol{U}(k)} J(\mathbf{U}(k)) \text{ subject to } u_{min} \le u(k) \le u_{max},$$

where u_{min} and u_{max} are, respectively, the lower and upper bounds of the control.

3. NMHB OPTIMIZATION METHOD

The nonlinearity in polynomial FHM is static and has the expression (1). The minimization of the criterion (34), which degree is equal to $2n_c$ involves generally a non-convex optimization problem. Indeed, the derivative degree becomes $2n_c - 1$ and there exist at most $2n_c - 1$ solutions in which $n_c - 1$ are bad ones corresponding to critical points of the nonlinear characteristic. These points correspond to minima or maxima in the static output-input curve. These unwanted solutions involve the failure of gradient-based methods by trapping their optimization algorithms due to the null derivatives. Whereas the other n_c solutions corresponding to minima of the static objective function J giving the optimal control solutions. The only difference is the energy the system consumes when the control law switches between several values. Thus, we will focus only on the best solution which ensures the minimum variations Δu control inputs. This can be realized by tightening the control domain using constraints with u_{min} and u_{max} around the best solution. The objectives of this work, in addition to the best tracking performances, are minimum energy consumption and short computing time to deal with real-time NMPC. The NMHB is a numerical method which minimizes a continuous function J in a space of N_p dimension. It exploits the concept of simplex, which is used in the NM method, a polygon with vertice numbers equal to several researched variables plus one $(N_p + 1)$. The objective function J evaluated at each vertex. Then, the vertices are sorted, and the worst point is changed by its symmetric versus the polygon gravity center. The initial simplex is transformed during the iterations, deformed, moved, and gradually reduced approaching to the minimum $\mathbf{U}^*$of the objective function $J(k)$. The following algorithm explains briefly the proposed NMHB optimization approach.

Initialization: Choose the maximum number of iterations N_m, sort the initial variables to verify the inequality $J(\mathbf{U}_1^0) < J(\mathbf{U}_2^0) < \ldots < J(\mathbf{U}_{N_p+1}^0)$. The initial simplex tops are formed by $\mathbf{U}_i^0$, $i \in [1, \ldots, N_p + 1]$

Loop on i, $i \in [1 \ldots N_m]$

– Compute the simplex gravity center: $\mathbf{U}_c = \frac{1}{N_p}\sum_{j=0}^{N_p} \mathbf{U}_j^i$

– Compute the reflection of $\mathbf{U}_{N_p+1}^i$ compared to $\mathbf{U}_c$: $\mathbf{U}_r = \mathbf{U}_{N_p+1}^i + 2d_i$ where $d_i = \mathbf{U}_c - \mathbf{U}_{N_p+1}^i$

- If $J(\mathbf{U}_{N_p}) > J(\mathbf{U}_r) \geq J(\mathbf{U}_1^i)$ then $\mathbf{U}^* = \mathbf{U}_r$

- If $J(\mathbf{U}_r) < J(\mathbf{U}_1^i)$ then the simplex will be expanded:

– $\mathbf{U}_e = \mathbf{U}_{N_p+1}^i + 3d_i$

- If $J(\mathbf{U}_e) > J(\mathbf{U}_r)$ then $\mathbf{U}^* = \mathbf{U}_e$ else then $\mathbf{U}^* = \mathbf{U}_r$

- If $J(\mathbf{U}_r) \geq J(\mathbf{U}_{N_p}^i)$ then, the simplex will be contracted:

- If $J(\mathbf{U}_r) \geq J\left(\mathbf{U}_{N_p+1}^i\right)$ then $\mathbf{U}^* = \mathbf{U}_{N_p+1}^i + \frac{1}{2}d_i$

- If $J(\mathbf{U}_r) < J\left(\mathbf{U}_{N_p+1}^i\right)$ then $\mathbf{U}^* = \mathbf{U}_{N_p+1}^i - \frac{3}{2}d_i$

– If the maximum number N_m has not reached $\mathbf{U}_{N_p+1}^i = \mathbf{U}^*$, $\mathbf{U}_j^{i+1} = \mathbf{U}_j^i 1j$, $j \in [1, \ldots, N_p]$, $i = i + 1$ and repeat the loop. Else if $d_i > \varepsilon$ use HB function, ε is the desired precision.

Checking constraints: $u^* = \mathbf{U}^*(1)$

– If $u^* < u_{min}$ then use the HB function

– If $u^* > u_{max}$ then use the HB function

The pseudo-code of the HB function is detailed in [23].

Initialization: Initialize the position of nB honey badgers: $\mathbf{U}_i = u_{min} + r_1(u_{max} - u_{min})t_{max}$, where t_{max} is the maximum number of iterations.

Step 1:

- while $t \leq t_{max}$ do (loop on i)

– Compute the density factor: $\beta = 2\exp(\frac{-t}{t_{max}})$

- For $j = 1$ to nB do

– Defining intensity: $I_j = r_2 \frac{S}{4\pi d_j^2}$, $S = (\mathbf{U}_j - \mathbf{U}_{j+1})^2$, $d_j = \mathbf{U}^* - \mathbf{U}_j$

– If $r_6 < 0.5$ then

$\mathbf{U}_i = \mathbf{U}^* + 6I_j\mathbf{U}^* + r_3\beta d_j\cos(2\pi r_4)(1 - \cos(2\pi r_5))$ ||

Else $\mathbf{U}_i = \mathbf{U}^* - r_7\beta d_j$

- If $J(\mathbf{U}_i) \leq J(\mathbf{U}_j)$ then $\mathbf{U}_j = \mathbf{U}_i$ and $J(\mathbf{U}_j) = J(\mathbf{U}_i)$

- If $J(\mathbf{U}_i) \leq J(\mathbf{U}^*)$ then $\mathbf{U}^* = \mathbf{U}_i$and $J(\mathbf{U}^*) = J(\mathbf{U}_i)$.

Step 2: $u^* = \mathbf{U}^*(1)$

– If $u^* < u_{min}$ then $u^* = u_{min}$

– If $u^* > u_{max}$ then $u^* = u_{max}$

where r_1, r_2, r_3, r_4, r_5, r_6 and r_7 are random numbers between 0 and 1.

This hybrid method is proposed to exploit the advantages of the two methods. Indeed, the NM method is a fast convergence algorithm, and the HB method uses extended exploration. In addition, we take away their drawbacks: failure with multidimensional problems for NM and high oscillations for HB.

4. SIMULATION RESULTS

This part deals with the nonlinear system with the Hammerstein structure. We will consider models where the search for the best control law is a convex or non-convex optimization problem. We will compare the system tracking performances of stability, accuracy, and speed when we employ the NM and HB optimization algorithms with their hybrid approach NMHB. The NMPC is verified under the MATLAB environment.

4.1 MODEL 1

The fractional Hammerstein expression of the following model is detailed in the reference [31]:

$$\begin{cases} \tilde{y}(k) = 1.6174u^3(k), \\ H(s) = \frac{1.7397s^{0.4988}+2.1601}{0.9852s^{1.4964}+1.7439s^{0.9976}+3.2185s^{0.4988}+1}, \end{cases} \tag{13}$$

where $y(k)$ is the output, $u(k)$ is the control and $\tilde{y}(k) = f(u)$ the nonlinear signal is nonmeasurable.

Let N_s be the whole number of considered samples and h sampling period. The fractional predictive algorithm is simulated using the following parameters: $N_c = 1$; $N_s = 120$; $h = 0.1$; $\mathbf{R} = 0.1\mathbf{I}$; $\mathbf{Q} = \mathbf{I}$; where $\mathbf{I}$ is the identity matrix.

The curve plotted in Fig.2 represents the evolution of the static nonlinear characteristic.

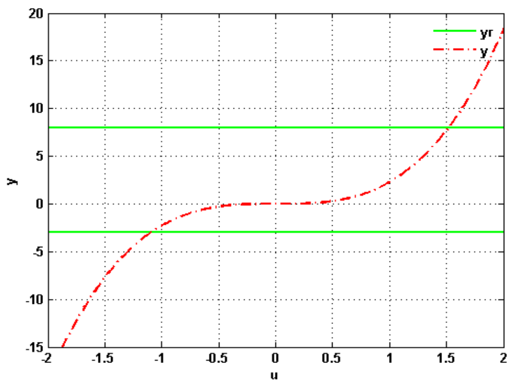

Fig. 2 – The static nonlinearity of model (13) for y_r swaying between 8 and -3.

We notice from the curve given in Fig. 2 that each desired set-point corresponds to only one control value ($y_r = 8$ correspend to $u = 1.5$ and $y_r = -3$ correspend to $u = 1.1$), illustrated by the static criterion curve traced in figures 3 and 4 corresponding to the set-point values $y_r = 8$ and $y_r = -3$. Therefore, for this example, the research of the optimal control $u^*(k)$ minimizing the criterion $J(k)$ which is a convex optimization problem. However, the curve of the minimized criterion changes with the prediction horizon N_p. Indeed, small values of N_p, the curve presents a palliate around the inflection point $(0, 0)$ of the characteristic in Fig. 2. In fact, the palliate is larger when N_p decrease.

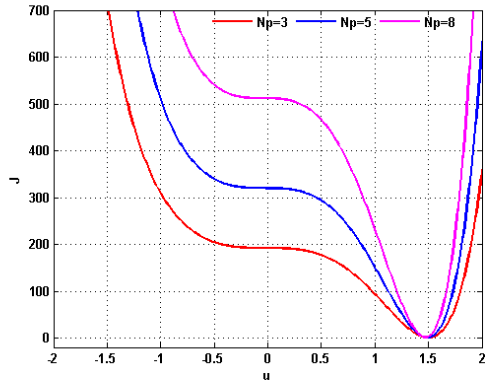

Fig. 3 – The objective function J of model (13) for $y_r = 8$.

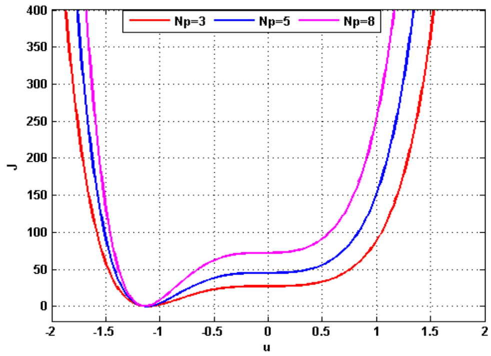

Fig. 4 –The objective function J of the model (13) for $y_r = -3$.

The prediction horizon N_p appears in eq. (10), then it will influence the shape of the static objective function curve and the temporal response. Therefore, we will start by comparing the behavior of the system using the NM and HB methods for different values of the prediction horizon N_p. The curves plotted in Fig. 5 and 6 illustrate the temporal responses and the control evolution obtained, respectively, with NM and HB methods for $N_p = 3, N_p = 5$, and $N_p = 8$.

Curves drawn in Fig. 5 and 6 indicate that the search for the optimal control minimizing the criterion $J(k)$ depends on the prediction horizon N_p. In fact, when using NM method, for the prediction horizon $N_p = 8$, and the output succeeds in tracking the set-point changes. Meanwhile for $N_p = 3$ and $N_p = 5$, the output fails when trapped in a palliate. This is explained by the fact that the simplex of the NM approach degenerates if the criterion curve presents a palliate. This is demonstrated in the curves illustrated in Fig 3 and 4 around $u = 0$. However, with the HB algorithm, the output successfully tracks the set-point changes.

Nevertheless, both output and control signals present considerable oscillations. Indeed, this method, which uses the HB strategy, evaluates the criterion for a large population to search for the best solution. It uses a flag to change the search direction to get high opportunities to explore the search space extensively. Therefore, it succeeds in reaching the best solution.

In this work, the NMHB algorithm is proposed to combine the two methods, NM and HB obtain a response without oscillations and independent of the dimension of the control vector $\boldsymbol{U}$, which is N_p.

The curve plotted in Fig. 7 represents the temporal responses evolution when using NMHB algorithm for $N_p = 5$ compared to a gradient-based method trust region (TR) and NM and HB methods.

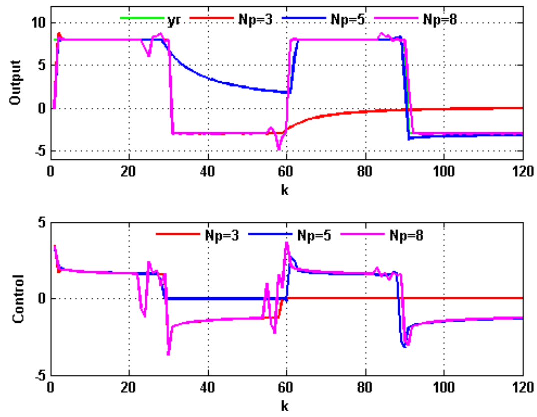

Fig. 5 – The temporal responses and the control law using the NM method.

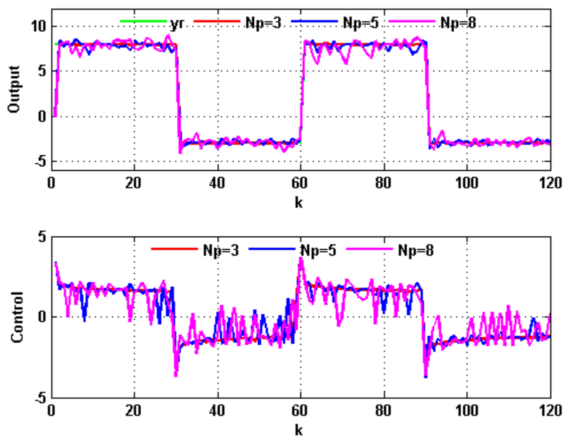

Fig. 6 –The HB method's temporal responses and control law.

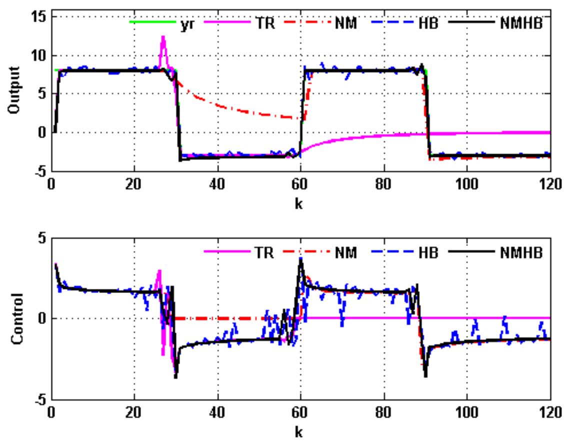

Fig. 7 – The temporal responses and the control law using TR, NM, HB, and NMHB methods for $N_P = 5$.

By comparing the responses obtained using NMHB to those obtained using TR, NM, and HB methods, the new algorithm NMHB succeeds in finding the best solution, unlike that of NM which diverges due to the simplex degeneration in the palliate around the point (0,0). The divergence of the gradient-based method TR is explained by the little value of gradient obtained into the palliate presented in Fig. 3. With the HB algorithm, the output converges to the set-point. However, it presents oscillations for the control and output signals.

Table 1 contains the control signal variance (σ_u) and the mean squared error (mse) calculated for the four methods.

Table 1

The control signal variance and the squared mean error.

Optimization methods	*TR*	*NM*	*HB*	*NMHB*
σ_u	0.6959	0.4735	0.3748	0.3600
mse	22.3663	10.9508	0.6489	0.5689

Table 1 illustrates the simulation of Fig. 7. The NMHB approach gives better tracking performances in terms of the control signal variance $\boldsymbol{\sigma_u} = 0.36$ and the mean squared error $mse = 0.5689$.

4.2 MODEL 2

The model (14) represents a SISO system with a polynomial structure expressing the nonlinear function. The following FHM, given by (14), is detailed in [34]:

$$\begin{cases} \tilde{y}(k) = 0.0548u(k) + 0.0041u^2(k) + 0.1877u^3(k) - 0.0306u^4(k), \\ H(s) = \frac{1}{2.4882s^{0.4888} + 4.4867}, \end{cases} \quad (14)$$

where $y(k)$, $u(k)$ and $\tilde{y}(k) = f(u)$ are as defined in the previous examples.

The curve plotted by Fig. 8 represents the evolution of the static nonlinear characteristic of y and the static criterion J versus the control u.

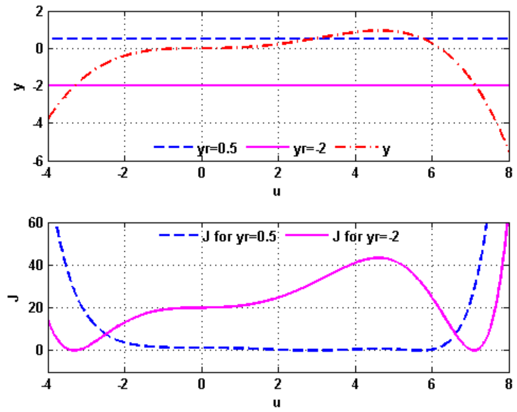

Fig. 8 – The static nonlinearity of the model (14) and the objective function J.

The nonlinear characteristic given in Fig. 8 shows that for the same set-point, there correspond two control values minimizing the criterion J for each set-point value. However, for the set-point value $y_r = 0.5$, the static criterion presents two weak minima for $u = 2.8$ and $u = 5.8$ and involve a large palliate. When considering the two solutions, the energy consumption increases for each set-point value as the control switches two distant values. It also enlarges the calculation time. For this reason, we will reduce the search domain using tightened constraints $u_{min} = -5$ and $u_{max} = 5$ around the best solution and is close to zero.

Figure 9 depicts the temporal responses of the model (14) for $N_p = 5$. Curves plotted in Fig. 9 show that when reducing the constraints around the best solution the NM algorithm is trapped in the upper limit of the constraint, $u = 5$, corresponding to the critical point of the characteristic of Fig. 8 for $y_r = 0.5$. This is due to the swaying of the algorithm to the other solution which is $u = 5.8$.

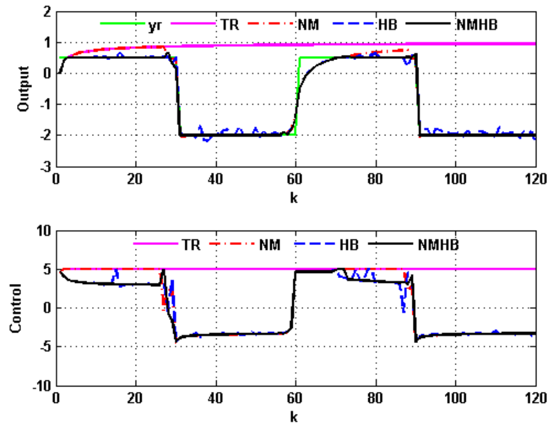

Fig. 9 – The temporal responses and the control law for $N_p = 5$.

This is a drawback of the NM method. As for the HB algorithm, it allows the output to reach the set-point ($y_r = 0.5$) but with the presence of oscillations. As for the TR algorithm, it diverges by being trapped in a critical point ($u = 5$) due to the null value of the gradient. However, when using the NMHB method, the insufficiencies cited above are remedied. Therefore, the hybridized NMHB algorithm achieves the best tracking performances in a reduced calculation time compared to that consumed by the HB algorithm.

5. **CONCLUSION**

In this work, we have developed a predictive control for nonlinear systems represented by FHM. We employed the optimization methods studied in the literature, such as the NM and HB algorithms, for fractional calculations. Then, we proposed a new algorithm, NMHB, taking advantage of both methods. We profit from the power of the NM approach to solve the optimization problem in the least computing time, and we exploit the HB method, which deals with issues of enormous dimensions and copes with local and global optimization problems. Two forms of nonlinear characteristics, as well as different orders of transfer functions and different nonlinearity degrees, are exploited to study the efficiency of the new approach. The static nonlinear characteristic and the criterion are used to tighten the control constraints around the best solution, convert a non-convex optimization problem into a convex one, and guide algorithms' convergence toward the best solution that consumes the least energy.

In future work, the optimization algorithm will be improved by minimizing calculation time to deal with fast dynamic systems such as robots. Furthermore, the proposed predictive controller will be applied to real processes.

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CREDIT AUTHORSHIP CONTRIBUTION

The authors have equally contributed to this work.

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