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MODELING OF A PEM FUEL CELL POLARIZATION CURVE BY LOW-ORDER POLYNOMIALS FOR THE OUTPUT POWER CALCULATION ALGORITHMS

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The control and energy algorithms that govern the operation of a PEM fuel cell within an energy system must account for the nonlinear phenomena in the cell; it is often captured by the polarization curve. However, the modeling approaches used for simulation studies could be more suitable for the abovementioned algorithms due to their higher computational burden. That leads to the development of simplified approaches. Our study focuses on the algorithms used for the fuel cell output power calculation. Many of them are directed towards the maximum power point operation of the cell. We propose that the fuel cell representation for the algorithms can be used in the low-order polynomial form, thus decreasing the computational load compared to the other approaches. For the maximum power point prediction, we propose using the truncated form of polarization curve input data (ignoring the activation loss region). Our study has demonstrated that based on the same input data and MATLAB curve fitting commands, the 3rd-order polynomial provides the comparable RMSE 3.5…3.9 for the power curve approximation vs. 3.9 for the 5th-order polynomial representation. The value of the maximum PowerPoint is obtained with a 1.3 % relative error with the 2nd-order polynomial using the truncated input data compared to 1.2 % for the 5th-order polynomial representation.

1. **INTRODUCTION**

A fuel cell (FC) converts the chemical energy of the fuel into electrical energy and is dependent on the availability of the fuel itself and the oxidant. One of the most promising FC technologies is the proton exchange membrane (PEM) FC, also called hydrogen FC, with hydrogen as a fuel and oxygen as an oxidant. Due to its environmentally friendly nature (water as a by-product of power generation), it is considered a technology for sustainable development. Therefore, *an ambitious scenario* presented in Hydrogen Europe in 2019 [6] considers that hydrogen can provide up to 24 % of the total energy demands by 2050.

This technology can provide energy needs in many fields, including microgrids, uninterruptible power supplies, and portable applications [7]. PEM FC has also found its niche in the transportation sector and is rapidly developing in that direction [8,9], including cars and trucks, agricultural tractors [10] and marine transport [11], etc.

The FCs cooperate with other energy-generating and energy-storage devices in all these applications. Their cooperation is governed by different energy management systems (EMS) algorithms, which manage the energy flows between energy-generating devices, energy-storage devices, and energy consumers.

The energy flow is often based on the solution of some optimization problem. Thus, Peng *et al*. [12] developed the offline optimal EMS for an FC-based hybrid drivetrain based on Pontryagin's minimum principle and dynamic programming. The solution of Quan *et al*. [13] is based on model predictive control speed prediction of FC electric vehicles. The predicted speed influenced the EMS strategy with an objective of equivalent hydrogen consumption minimization considering the FC degradation. Unlike the previous ones, the latter approach solves the *online* optimization problem. An example of the bridge between both approaches is a hybrid strategy, like the one reported in [14], which solves the problem of optimal energy consumption and instantaneous power distribution with online adaptability. Zidane and Belaid [15] developed an EMS algorithm for a hybrid photovoltaic/battery/hydrogen system based on fuzzy logic, optimizing the overall system price and reliability. Advanced EMS strategies can also include co-optimizing energy consumption and vehicle speed planning [16].

The development of those EMS algorithms is complicated due to the FC's nonlinear nature, namely, its polarization curve [17], as explained below. For the simulation studies, a detailed representation of the FC's physical behavior is often used [18–20]. Kravos *et al*. [21] propose a thermodynamically consistent PEM FC model for control and state monitoring algorithms. Despite its applicability for various operating conditions, the other approaches prefer simpler models.

Thus, the maximum power point tracking algorithm in [22] uses the non-physics-based representation of an FC polarization curve by the ninth-order polynomial. The fifth-order polynomial was used by Pukrushpan [2] for control purposes. The dc-dc converter control algorithm used for fuel cell applications in [4] and a nonlinear voltage regulation strategy in [1] uses a much simpler representation of a polarization curve, thus requiring less computational load to implement the control algorithm.

There always needs to be a compromise between the computational complexity and the algorithm's accuracy. Maximum-power and maximum-efficiency points of an FC correspond to the non-linearity regions of the polarization curve. Therefore, they should be used as a nonlinear representation, as we are interested in the operation near one of those points. In subsection 2.2, we review several models to define the state-of-the-art on the subject. The natural question is, to what extent can we simplify the mathematical

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representation of the polarization curve to preserve the accuracy of the representation of the FC operation?

In the current study, we limit ourselves to predicting FC output power, with a special interest in the maximum output power point. We study the possibility of using low-order polynomials for an output power calculation based on the set of points (in practice, those can be obtained experimentally on the real FC). The representation can be used for EMS or control design purposes.

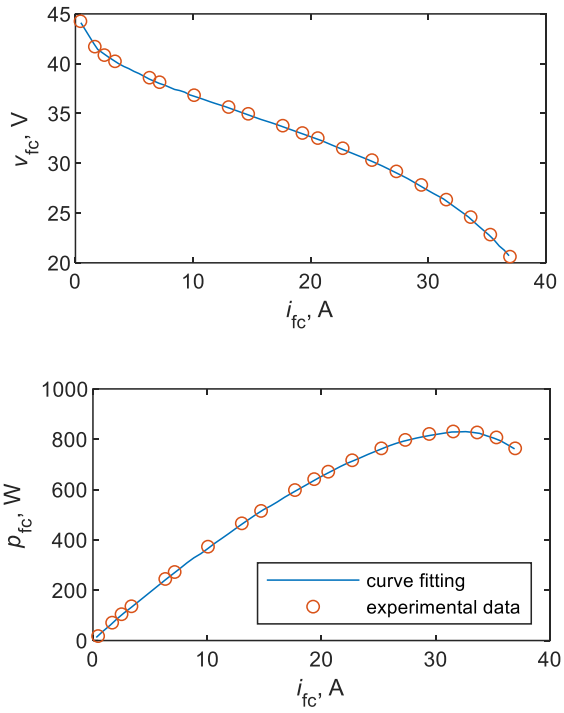


Fig. 1 – Digitalized characteristic curves of PEM FC stack SR-12 [29].

2. PEM FC POLARIZATION CURVE MODEL

2.1. PEM FC MODEL

When the FC is disconnected from the external electric circuit, its potential is obtained using the Nernst equation as E_{Nernst}, the Nernst voltage. It corresponds to the maximum energy generated in a fuel cell at the given conditions.

However, when the FC operates within a system, and the circuit is closed, the output voltage of the cell is governed by the equation

$$v_{\text{fc}} = E_{\text{Nernst}} - v_{\text{act}} - v_{\text{conc}} - v_{\Omega}, \quad (1)$$

where different voltage losses should be considered, namely, the activation voltage loss v_{act}, the difference from the equilibrium voltage required to initiate the reaction, the concentration voltage loss v_{conc} corresponding to the mass transfer phenomenon, and the ohmic voltage loss v_{Ω}. The dependence $v_{\text{fc}}(i_{\text{fc}})$ is called the polarization curve.

The ohmic voltage loss depends on the resistance of the electrolyte and other conducting fuel cell components; it is linearly dependent on the current due to Ohm's law. However, two other components of (1) are strongly non-linear [17]. Thus, nonlinearity corresponding to the activation voltage loss mainly influences the low-current region, while the concentration voltage loss influences the high-current region, as demonstrated in Fig. 1.

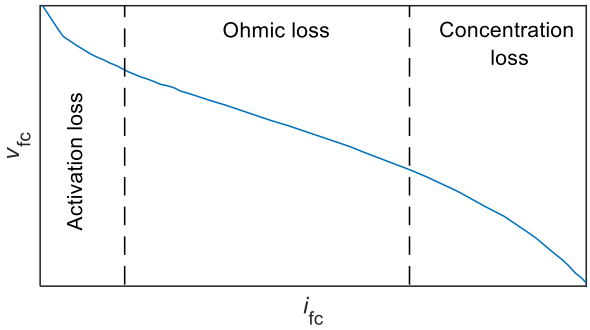


Fig. 2 – Typical polarization curve (the dependence $v_{\text{fc}}(i_{\text{fc}})$) of a PEM FC with regions of prevailing voltage losses.

2.2. EMPIRICAL AND SEMI-EMPIRICAL MODELS OF A PEM FC POLARIZATION CURVE

Due to the non-linear nature of the polarization curve, several studies were directed toward obtaining some empirical and semi-empirical models of the polarization curve. Thus, it seems that chronologically, the first was the model [3]

$$v_{\text{fc}}(i_{\text{fc}}) = E_o - b \ln i_{\text{fc}} - R i_{\text{fc}} - m \exp(n i_{\text{fc}}), \quad (2)$$

with E_o, the coefficient without the theoretical meaning (it is different from $E^0$the reversible voltage at the standard state, as the authors of [23] stress); $b \ln i_{\text{fc}}$ representing the activation loss with b standing for the parameter for oxygen reduction from the Tafel equation; R representing the resistance; $m \exp(n i_{\text{fc}})$ representing the concentration loss with m and n used to fit the experimental curve, respectively.

A few years later, its slightly modified version [5] was formulated as:

$$v_{\text{fc}}(i_{\text{fc}}) = E_o - b \ln i_{\text{fc}} - R i_{\text{fc}} + \alpha i_{\text{fc}}^k \exp(1 - \beta i_{\text{fc}}), \quad (3)$$

where the concentration loss component of (1) is fitted by parameters without physical meaning, and β representing the inverse of the limiting current density.

Below, we ignore the parameters' particular physical meaning, interpreting them only as the curve-fitting parameters independent of their physical meaning for simplification.

As the reviews of different polarization curve models conclude (*e.g.*, [24]), the most precise model is the one proposed in [25]; however, it is also the most complicated one.

In [2], the fifth-order polynomial representation is proposed

$$v_{\text{fc}}(i_{\text{fc}}) = c_0 + c_1 i_{\text{fc}} + c_2 i_{\text{fc}}^2 + c_3 i_{\text{fc}}^3 + c_4 i_{\text{fc}}^4 + c_5 i_{\text{fc}}^5, \quad (4)$$

where coefficients $c_0 \ldots c_5$ have no physical meaning.

The models (2)-(4) are suitable for representing the PEM FC in a full range of currents. However, several proposed models ignore the concentration loss region. These models can be formulated as follows [4]:

$$v_{\text{fc}}(i_{\text{fc}}) = \frac{V_o}{1 + \left(\frac{i_{\text{fc}}}{I_{\text{h}}}\right)^{\delta}}, \quad (5)$$

or as follows [1]

$$v_{\text{fc}}(i_{\text{fc}}) = V_o - a i_{\text{fc}}^b, \quad (6)$$

where V_o is the FC open circuit voltage of the fuel cell stack, δ, and I_{h} in (5) has no physical meaning, as do a and b in (6).

Again, we neglect the physical meaning of the parameters as they are used in the models, treating them as curve-fitting parameters.

It should be noted that the simplified models are applied for control purposes. Thus, model (5) is used in [4] for energy trajectory planning in a dc–dc converter system and in [24] for the development of adaptive backstepping control for a fuel cell/boost converter system. Model (6) is used in [1] to set the setting of the nonlinear state feedback for a fuel cell/supercapacitor system.

If models (5) and (6) ignore the concentration loss region, they are unsuitable for FC operation near the maximum power operating point.

Several works used more complicated models; thus, model (3) was implemented for an adaptive EMS for fuel cell vehicles to estimate the polarization curve online [27].

One of the trends in FC research is optimal parameter identification of FC parameters using multiple optimization algorithms based on the measured data to fit the model [28–30]. However, most of these algorithms are suitable for offline techniques.

2.3. REMARK ON MAXIMUM POWER AND MAXIMUM EFFICIENCY CALCULATION IN PEM FC CONTROL ALGORITHMS

In our work, we focus on the mathematical model of the PEM FC for the FC power calculation. It should be noted that FC control is performed using the maximum power point tracking (MPPT) and maximum efficiency point tracking (MEPT) algorithms, which assure operation with maximum output power and maximum output efficiency, respectively.

However, several control algorithms related to output power maximization in FC applications explicitly deal with the power of the fuel and the oxidant flow in FC. Thus, the MPPT algorithm developed in [31] optimizes the reactant flow. In [32], several control problems related to FC are reviewed, such as fuel and oxidant flow problems and water and thermal management.

On the other hand, due to the lower dynamic response of the FC system, the fuel and oxidant flows are uncontrolled, and the developed control techniques deal with the control from the electric side only. Several recent examples are the MPPT technique based on a hybrid artificial bee colony algorithm with fuzzy control [33] or fuzzy MPPT [34]. More advanced techniques use the characteristic curve approximation with the coefficients dependent on the power of the fuel or oxidant flow [35].

3. **INPUT DATA**

3.1. REMARK REGARDING THE INPUT DATA AND DATA PROCESSING

As we deal with the development of algorithms for control-oriented studies, we should consider the effect of errors and the limitations dictated by the application (for which the technique is developed).

Thus, we assume a real-world system with hybrid energy storage based on PEM FC operating under a control algorithm requiring power data (*e.g.*, the MPPT algorithm or an intelligent energy management algorithm). For such an application, the parameters of the FC stack are obtained from the experimental data and then utilized to adjust the control algorithm. We admit that measurement and data processing cause inevitable errors.

Therefore, we assume the only data source for all fitting curve representations. Afterward, we compare the input data and the results obtained by curve fitting. We assume that the low-order polynomials can provide a satisfactory representation of the power curve with a special interest in the maximum power point.

The operating conditions influence the FC operation and the FC characteristic curve (temperature, pressures of oxidant, and fuel, among others). Therefore, the FC characteristic curve obtained in the laboratory environment will differ from the one in the application. We aim to confirm that the low-order polynomials provide a satisfactory output for the same input data for the control algorithm. That can be obtained by comparing the RMSE for the different fitting curves.

3.2. SOURCE OF THE INPUT DATA

The input data, FC polarization (v–i) and p–i curves were obtained by digitalizing the characteristic curves of FC stack of the type of SR-12 from [29] utilizing Graph2Digit software (Fig. 2). In [29], the experimental data were initially obtained, and later, the improved gorilla troops optimization technique has been applied to evaluate the coefficients corresponding to the simplified electrochemical PEM FC model in the form as in [25]. Please also note that *simplified* corresponding to the model [25] means simplified electrochemistry; its complexity level is as high as with the accurate parameter estimation. It will be assumed below as the accurate fuel cell model and depicted in Fig. 3–5 as the *best-fit* curves.

4. **RESULTS AND DISCUSSION**

The digitalized data have been sent to the MATLAB environment for further data processing using the functions of its Curve Fitting Toolbox.

The fitting function was a polynomial of a different order. Thus, the first test case used a 5th-order polynomial as in [2]. The results are depicted in Fig. 3. The power curve is obtained as $p_{\mathrm{fc}} = v_{\mathrm{fc}} i_{\mathrm{fc}}$ using the polarization curve data.

Two other cases used 3rd (Fig. 4a) and 2nd (Fig. 5a) order polynomials. As far as our study is limited to the search of curve fitting for power calculation purposes, the common feature of the FC is that its maximum power point is mainly influenced by the ohmic losses and mass transport losses. As far as some algorithms are designed upon the assumption of the polarization curve limited to activation and ohmic loss regions, and the utilized functions (like those (5) and (6)) ignore the concentration loss region, the next assumption is that activation polarization region can be ignored, and the polynomial function can be obtained by ignoring the input data corresponding to the activation loss region. The results are depicted in Fig. 4b and Fig. 5b for 3rd and 2nd-order polynomials correspondingly.

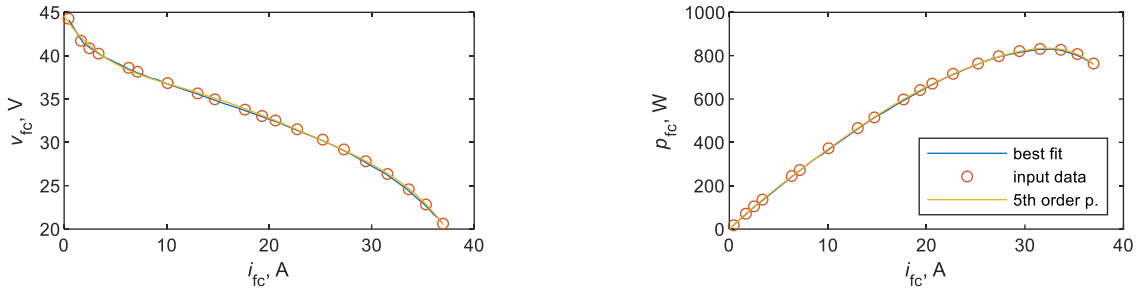
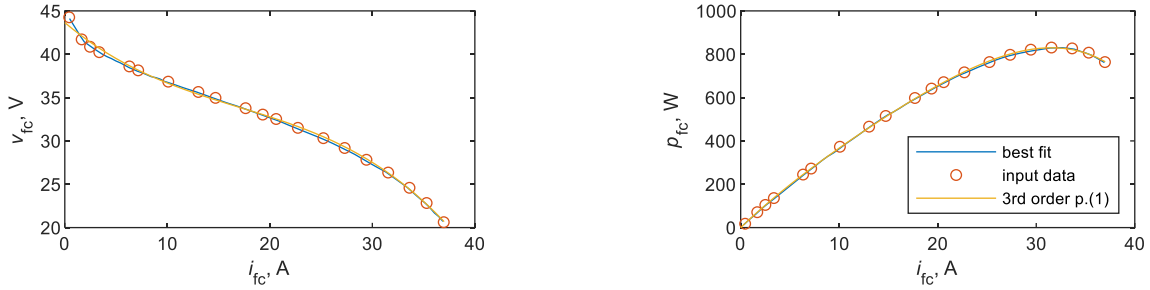
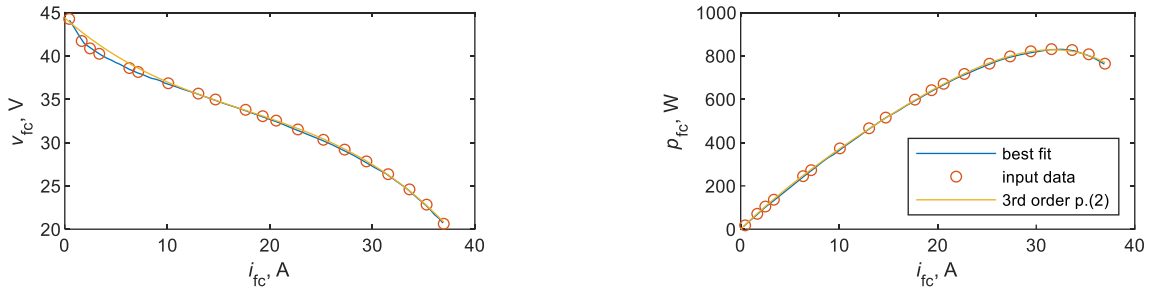


Fig. 3 – Polarization curve fitting of PEM FC stack SR-12 using 5th order polynomial and the corresponding power curve.



(a)



(b)

Fig. 4 – Polarization curve fitting of PEM FC stack SR-12 using 3rd order polynomial and the corresponding power curve: a) input data are taken from the full i_{fc} range; b) input data are taken from the truncated i_{fc} range.

The goodness of fit for all test cases is evaluated using RMS values based on the v–i and p–i curves. The results are summarized in Table 1.

Table 1

RMSE comparison of polarization curve approximations

Polynomial	RMSE, v–i curve	RMSE, p–i curve
5th order	0.1657	3.8217
3rd order, using the full data set	0.2358	3.9544
3rd order, using the truncated dataset	0.3717* 0.1409**	3.5339* 3.6512**
2nd order, using the full data set	0.7141	13.7063
2nd order, using the truncated dataset	1.6324* 0.2811**	7.4873* 7.1532**

* RMSE calculated comparing the whole range of polarization curve
** RMSE calculated comparing the truncated dataset

When comparing the RMSE values corresponding to the v–i curve in Table 1, it is obvious that higher-order polynomials result in better goodness of fit. When the truncated input dataset is used, the lower-order polynomials predictably better fit the curve in the whole range but provide a better representation if the selected range is analyzed.

Another result is that when the RMSE values for the curve fitting using the truncated data are compared, the values decrease when the whole data range is analyzed. That is due to the addition of the low-current region, where the power is small.

Table 2

Comparison of the values of current for the maximum FC output power

Polynomial	i_{fc} for p_{fcmax}, A	δ_i, %
Input data	32.645	—
5th order	32.247	–1.22
3rd order, using the full data set	31.551	–3.352
3rd order, using the truncated dataset	31.665	–3.004
2nd order, using the full data set	35.18	7.764
2nd order, using the truncated dataset	32.214	–1.322

The comparison of the current values for the maximum FC output power (Table 2) demonstrated that while the 5th-order polynomial representation provides the best current value with –1.22 % relative error, the 3rd-order polynomial representation provides the results at 3 – 4 %. The 2nd order polynomial representation with the full input data provides the worst result, 7.764 %. In contrast, the truncated input data (ignoring the activation loss region) provides the current value with a –1.322 % relative error, close to the one obtained using the 5th-order polynomial.

Nevertheless, suppose the curve fitting is used in the algorithms requiring the FC operation near the maximum power point. In that case, it is well represented even by low-order approximations.

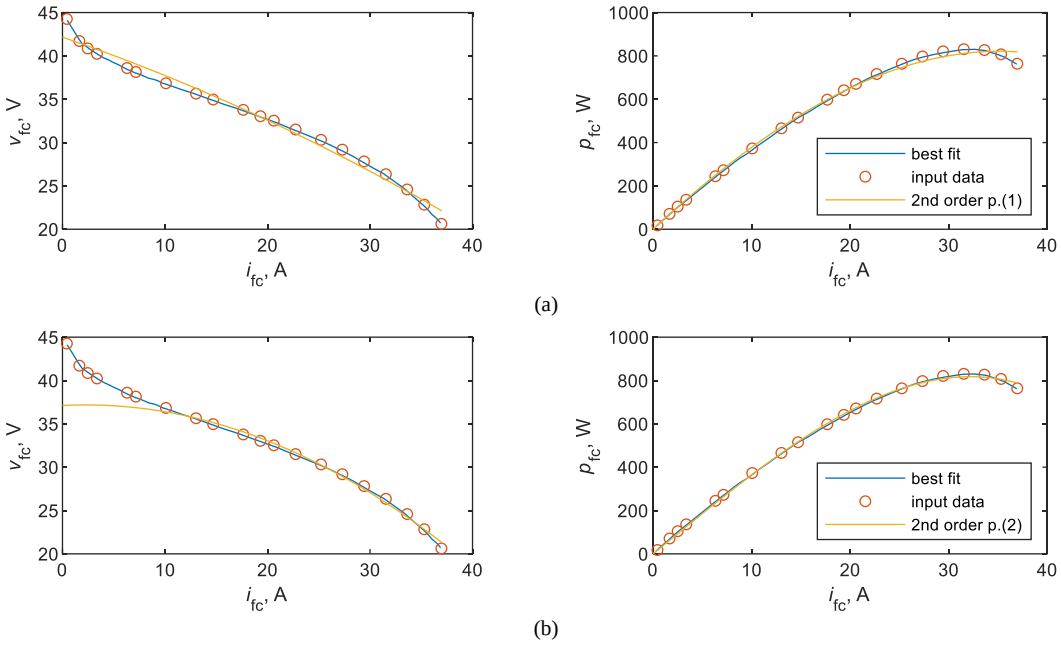


Fig. 5 – Polarization curve fitting of PEM FC stack SR-12 using the second order polynomial and the corresponding power curve: a) input data are taken from the whole i_{fc} range; b) input data are taken from the truncated i_{fc} range.

5. CONCLUSIONS

A simplified representation of the PEM fuel cell polarization curve (unlike more complex models used for simulation studies) is useful for developing control and energy management algorithms. One possible simplification is that the polarization curve can be used to predict fuel cell output power and assess the maximum output power point, as is the case for numerous algorithms reported in the literature.

Our study compares the 5th-order polynomial representation in [2] and the low-order, 3rd, and 2nd-order polynomials as possible substitutes in the algorithms. The input data for the fitting polynomials was selected similarly, and the curve fitting functions were the standard ones from the MATLAB environment. The other idea of the paper was that for the maximum power point prediction, the input data can be taken, ignoring the activation loss region.

As a result, the 3rd-order polynomial representation of the power curve is obtained using the comparable values of RMSE as compared to the 5th-order polynomial (3.5…3.9 *vs* 3.8), with the better results obtained using the truncated input dataset. When comparing the prediction of the maximum output power of the fuel cell, the 2nd-order polynomial representation with the truncated dataset provided a 1.3 % relative error as compared to the 1.22 % with the 5th-order polynomial and 3…3.5 % with the 3rd-order polynomial representation.

The other input dataset for the other fuel cell stack would result in the other numerical values. However, reducing the order of the polynomial representation of the fuel cell polarization curve for the algorithms requiring the cell's output power will reduce the computational burden of the algorithm, resulting in comparable accuracy.

Further study will be directed towards implementing the proposed solution in a hybrid energy storage system based on FC to demonstrate its feasibility and determine its possible advantages and drawbacks.

The other direction for future research is to study a similar solution of using low-order polynomials for maximum efficiency calculation algorithms.

Also, future studies will consider the change of the fitted curve parameters depending on the FC system's operating conditions, like the one performed in [35]. In that case, a family of characteristic curves can be fitted using low-order polynomials.

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NOMENCLATURE

Abbreviations

EMS – energy management system
FC – fuel cell
MEPT – maximum efficiency point tracking
MPPT – maximum power point tracking
PEM – proton exchange membrane
RMSE – root mean square error

Variables

a – curve fitting coefficient from [1]
b – curve fitting coefficient from [1]
b – parameter for oxygen reduction from the Tafel equation
$c_0 \dots c_5$ – curve fitting coefficients from [2] with no physical meaning
E_{Nernst} – Nernst voltage
E_o – parameter from [3] with no physical meaning
E^0 – reversible voltage at the standard state
i_{fc} – fuel cell output current
I_{h} – parameter from [4] with no physical meaning
k – parameter from [5] with no physical meaning
m – parameter from [3] with no physical meaning
n – parameter from [3] with no physical meaning
p_{fc} – fuel cell output power
p_{fcmax} – maximum fuel cell output power
R – fuel cell resistance
v_{act} – activation voltage loss
v_{conc} – concentration voltage loss
v_{fc} – fuel cell output voltage
v_{Ω} – ohmic voltage loss
V_o – fuel cell open circuit voltage
A – parameter from [5] with no physical meaning
B – inverse of the limiting current density
Δ – parameter from [4] with no physical meaning
δ_{i} – relative error of current