NONLINEAR ADAPTIVE BACKSTEPPING CONTROL OF PERMANENT MAGNET SYNCHRONOUS MOTOR

NIHAD ALI[1,2], WAQAR ALAM[2], MAHMOOD PERVAIZ[2], JAMSHED IQBAL[3]

This paper addresses the speed tracking problem of a permanent magnet synchronous motor (PMSM) under the influence of parametric uncertainties and external load torque disturbances. The nonlinear dynamics associated with both PMSM and load is considered time variant and uncertain. Two robust controllers, namely, backstepping and adaptive backstepping are designed to drive the speed of a PMSM to a predefined trajectory. The backstepping controller is used to stabilize and control the speed of motor while the uncertain parameters and disturbances are estimated by adaptive laws. These adaptation laws and the use of performance improvement term in the backstepping control reduce the gain requirements. The stability analysis of both the controllers via Lyapunov method ensures the asymptotic convergence of the overall close loop system. Theoretical analysis is presented to summarize characteristics of both the controllers. Numerical simulations are provided to verify effectiveness of the proposed controller.

1. INTRODUCTION

A permanent magnet synchronous motor (PMSM) is widely used as an actuator in several industrial applications because of its high efficiency, superior power density and large torques to inertia ratio. However, the PMSM is always subject to a highly complex and a nonlinear dynamic model. In order to get efficient performance, consequently complex controllers must be designed.

Traditional linear control schemes such as proportional derivative (PD), proportional integral (PI), and proportional integral derivative (PID) are extensively used in many industrial electric drive applications because of their simplicity [1]. However, owing to their sensitivity to parametric uncertainties and load torque perturbations, they failed to demonstrate the desired performance in high electric drive industrial applications such as speed tracking [2, 3]. To overcome this problem, a variety of nonlinear control [4] schemes have been investigated. These nonlinear control schemes are categorized in two classes: The first one is based on accurate model of PMSM such as exact feedback linearization control [5] and decoupling control [6]. The performance of these control methods degrades in case of deviation of the parameters from their nominal values. The second class constitutes of the control techniques which robustly deal with parametric variations and external disturbances. It mainly includes variable structure sliding mode control [7, 8], fuzzy control [9], H-infinity control [10] and artificial intelligence (AI) based PI–PID tuning methods [11, 12]. However, these control schemes suffer from certain drawbacks. In sliding mode variable structure control, uncertain parameters must satisfy matching condition and the main drawback is the chattering phenomena, which restricts its applications [13]. The sliding mode control scheme using smooth control law is presented to substantially alleviate the chattering phenomenon [14, 15]. Fuzzy control strategy depends upon fuzzification of Takagi-Sugeno. In H-infinity control, the operating states are ignored under special conditions [16]. In AI-based PI-PID tuning methods, the controller design is based on system model and thus exhibits excellent robustness to variable parametric uncertainties and disturbances. Nonetheless, due to complex algorithm and computational efforts, real time implementation of such methods is difficult and cumbersome.

Backstepping control is a recently developed nonlinear control scheme, which addresses the uncertainties in parameters of a nonlinear system especially those not satisfying matching conditions [17]. The key concept behind backstepping control is the decomposition of high-dimensional complex nonlinear systems into numerous low-dimensional simple subsystems [18]. Virtual control input for each subsystem is then designed. The final control law can be formulated systematically through appropriate Lyapunov functions.

Amongst various reported adaptive control methods, backstepping design offers excellent performance for highly nonlinear systems in terms of adaptation capability to parametric uncertainties and disturbance rejection capacity [17]. However, despite its benefits, backstepping control is confronted with several limitations solving complex regression matrix [19], over-parameterization caused by adaptive parameters larger than uncertain plant parameters, repetitions of differentiations process of virtual control inputs [20] and the requirement of partial or complete system model in the design of controller.

An adaptive robust controller based on backstepping control approach is derived and proposed in [21] for the speed control of PMSM. The controller exhibits robustness to system's uncertain parameters such as viscous friction, stator resistance and unknown load torque perturbations. Nonetheless, this methodology uses feedback linearization, which might cause to cancel out some useful nonlinearities. In [22], another adaptive backstepping control law for PMSM is presented. The flux linkage, stator winding resistance and load torque are considered to be unknown and bounded. Variations in the inductance and damping can degrade the performance of the presented control scheme.

To deal with these limitations, the present paper aims to design a nonlinear and robust backstepping controller based on adaptive laws to track the desired speed trajectory accurately. Unlike [22], all the parameters of the motor are

---
[1] Department of Automation, Shanghai Jiao Tong University, Shanghai, China
[2] Department of Electrical and Computer Engineering, COMSATS University Islamabad, Pakistan
[3] Department of Computer Science and Technology, Faculty of Science and Engineering, University of Hull, HU6 7RX, UK
j.iqbal@hull.ac.uk

considered to be unknown and time varying. The gain requirement is small and the transient improvement term in the control law offers fast convergence by tracking the speed error to zero asymptotically.

2. SYSTEM MODELLING

Nomenclature of various symbols for system modelling is listed in Table 1. To facilitate the controller design, the mathematical model of a PMSM has been derived in the well-known (d–q) frame using the Park's transformation as [13, 22]

$$\dot{\omega} = \frac{3p\lambda_f}{2J}i_q - b\omega - \tau,\qquad(1)$$

$$\dot{i}_q = -\frac{R}{L}i_q - i_d p\omega - \frac{\lambda_f p}{L}\omega + \frac{v_q}{L},\qquad(2)$$

$$\dot{i}_d = -\frac{R}{L}i_d + i_q p\omega + \frac{v_d}{L}.\qquad(3)$$

Table 1

Nomenclature

Symbol	Description
ω	Angular speed of rotor
i_q	q-axis current
i_d	d-axis current
R	Stator resistance
L	Stator inductance
b	Damping constant to inertia ratio
J	Rotor inertia
λ_f	Rotor to stator flux linkage
p	Number of pole pairs
τ	Load torque to inertia ratio

The derived model specifying three state variables are given by (1)–(3). The values of the parameters given in Table 1 are usually obtained with experiments. However, in practical applications, the parameters accuracy cannot be guaranteed and the dynamics of PMSM always possess uncertainties. Therefore, the following assumptions are made.

Assumption 1: The parameters resistance, damping factor, load torque, inertia of rotor and inductance are uncertain, unknown and bounded given by

$$R = R_N + \Delta_R,\qquad B = B_N + \Delta_B,\qquad T_L = T_{LN} + \Delta_T,$$

$$J = J_N + \Delta_J,\qquad L = L_N + \Delta_L,$$

where the parameters with subscript *n* are nominal values and $\Delta_{\{R,B,T_L,J,L\}}$ are disturbances in the parameters.

Assumption 2: The time derivatives of parametric perturbations in assumption 1 are bounded and satisfying the condition *i.e.*, $\lim_{t\to\infty}\dot{\Delta}_{\{R,B,T_L,J,L\}} = 0$.

3. BACKSTEPPING CONTROL

Let ω_d be the desired speed trajectory, the tracking error can be defined as $z_1 = \omega - \omega_d$. Taking the time derivative of z_1 and using (1) followed by Assumption 1, we can obtain,

$$\dot{z}_1 = \frac{3p\lambda_f}{2J_N}i_q - b_N\omega - \tau_N - \dot{\omega}_d + f_1(\Delta),\qquad(4)$$

where $f_1(\Delta)$ represents the lumped uncertain term. The control objective is to regulate the tracking error z_1 and $\dot{z}_1$ to zero by designing the control scheme. To achieve sufficient condition for the stability, a virtual controller acting as reference q-axis current can be designed as,

$$\alpha_1 = \frac{2J_N}{3p\lambda_f}\left(b_N\omega + \tau_N + \dot{\omega}_d - k_1 z_1\right).\qquad(5)$$

Defining the q-axis current error as $z_2 = i_q - \alpha_1$. Using this relationship and substituting (5) in (4) yields

$$\dot{z}_1 = -k_1 z_1 + \frac{2J_N}{3p\lambda_f}z_2 + f_1(\Delta),\qquad(6)$$

where k_1 is a positive constant. The error variable for d-axis current is defined as $z_3 = i_d$. The derivative of z_2 and z_3 can be calculated as follows,

$$\dot{z}_2 = -\frac{R_N}{L_N}i_q - i_d p\omega - \frac{\lambda_f p}{L_N}\omega + \frac{v_q}{L_N} + f_2(\Delta)$$
$$-\frac{2J_N}{3p\lambda_f}(b_N - k_1)\left(\frac{3p\lambda_f}{2J_N}i_q - b_N\omega - \tau_N + f_1(\Delta)\right),\qquad(7)$$
$$-\frac{2J_N}{3p\lambda_f}\left(\ddot{\omega}_d + k_1\dot{\omega}_d\right)$$

$$\dot{z}_3 = -\frac{R_n}{L_N}i_d + i_q p\omega + \frac{v_d}{L_N} + f_3(\Delta).\qquad(8)$$

The control laws to ensure the stability of the currents i_q and i_d can be selected as,

$$v_q = R_N i_q + L_N i_d p\omega + \lambda_f p\omega - \frac{3Lp\lambda_f}{2J_N}z_1$$
$$+\frac{2J_N L_N}{3p\lambda_f}(b_N - k_1)\left(\frac{3p\lambda_f}{2J_N}i_q - b_N\omega - \tau_N\right),\qquad(9)$$
$$+\frac{2L_N J_N}{3p\lambda_f}\left(\ddot{\omega}_d + k_1\dot{\omega}_d\right) - L_N k_2 z_2$$

$$v_d = R_n i_d - L_N i_q p\omega - L_N k_3 z_3,\qquad(10)$$

where k_2 and k_3 are non-zero positive constants. The convergence of z_1, z_2 and z_3 can be proven by defining $V_1 = 1/2z_1^2 + 1/2z_2^2 + 1/2z_3^2$ as a Lyapunov function. Taking its time derivative and substituting (6)–(8) and the control inputs (9) and (10) yields,

$$\dot{V}_1 = -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 + z_1 f_1(\Delta) + z_2 f_2(\Delta)$$
$$+z_3 f_3(\Delta) - z_2\frac{2J_N}{3p\lambda_f}(b_N - k_1)f_1(\Delta)\qquad(11)$$

All the error variables in (11) robustly converge to zero if the control gains satisfy the following condition

$$k_1 > |f_1(\Delta)|, \quad k_2 > \left| f_2(\Delta) - \frac{2J_N}{3p\lambda_f}(b_N - k_1) \right|. \tag{12}$$
$$k_3 > |f_3(\Delta)|$$

4. ADAPTIVE BACKSTEPPING

Adaptive backstepping control technique is the combination of backstepping control and adaptive laws and is more robust for the systems having matched as well as mismatched uncertainties. To ensure the stability of overall control system, a Lyapunov function is introduced, which guarantees that the angular speed of PMSM tracks the desired signal efficiently thus neutralizing the effects of uncertainty in the parameters. The control objective is to regulate the tracking error z_1 and the error dynamics $\dot{z}_1$ to zero by designing the control scheme. This can be achieved by taking time derivative of z_1 and using (1)

$$\dot{z}_1 = \frac{3p\lambda_f}{2J} i_q - b\omega - \tau - \dot{\omega}_d. \tag{13}$$

Defining Lyapunov candidate function $V_1 = 1/2z_1^2$ and taking its time derivative and using (13) one can obtain

$$\dot{V}_1 = z_1 \left(\frac{3p\lambda_f}{2J} i_q - b\omega - \tau - \dot{\omega}_d \right). \tag{14}$$

Let i_q be the state corresponding to virtual control that stabilizes the speed error, so the stabilizing function α_1 can be designed as

$$\alpha_1 = \frac{2\hat{J}}{3p\lambda_f} \left(\hat{b}\omega + \hat{\tau} + \dot{\omega}_d - k_1 z_1 \right), \tag{15}$$

where $\hat{b}$ and $\hat{\tau}$ are estimation of damping and load toque to inertia ratio respectively. Using (15), (14) becomes

$$\dot{V}_1 = -k_1 z_1^2 + \frac{3p\lambda_f}{2J} z_1 z_2 + z_1 \left\{ \tilde{b}\omega + \tilde{\tau} + \frac{\tilde{J}}{J} \left(\hat{b}\omega + \hat{\tau} \right. \right.$$
$$\left. \left. + \dot{\omega}_d - k_1 z_1 \right) \right\}, \tag{16}$$

where $\tilde{b}, \tilde{\tau}$ and $\tilde{J}$ are damping, load torque to ineria ratio and inertia estimation errors repectively. From (16), it can be noticed that if the estimation errors become zero and the q-axis current error is stabilized then, the actual speed will track the desired speed asymptotically. Since i_q is a state variable, which is not equal to α_1, the corresponding error dynamics can be obtained as $\dot{z}_2 = \dot{i}_q - \dot{\alpha}_1$, where $\dot{\alpha}_1$ can be calculated by taking time derivative of virtual control input (15) and can be written as

$$\dot{\alpha}_1 = \frac{2\hat{J}}{3p\lambda_f} \left(\dot{\hat{b}}\omega + \dot{\hat{\tau}} + \ddot{\omega}_d + k_1 \dot{\omega}_d \right)$$
$$+ \frac{2\dot{\hat{J}}}{3p\lambda_f} \left(\hat{b}\omega + \hat{\tau} + \dot{\omega}_d - k_1 z_1 \right) + \frac{2\hat{J}}{3p\lambda_f} \left(\hat{b} - k_1 \right) \dot{\omega}. \tag{17}$$

For the sake of brevity let,

$$A = \frac{2\hat{J}}{3p\lambda_f} \left(\dot{\hat{b}}\omega + \dot{\hat{\tau}} + \ddot{\omega}_d + k_1 \dot{\omega}_d \right)$$
$$+ \frac{2\dot{\hat{J}}}{3p\lambda_f} \left(\hat{b}\omega + \hat{\tau} + \dot{\omega}_d - k_1 z_1 \right), \tag{18}$$

with the above equation, (17) can be written as

$$\dot{\alpha}_1 = A + \frac{\hat{J}}{J} \left(\hat{b} - k_1 \right) i_q - \frac{2\hat{J}}{3p\lambda_f} \left(\hat{b} - k_1 \right) (b\omega + \tau). \tag{19}$$

Using (2) and (19), the q-axis current error dynamics can be obtained as

$$\dot{z}_2 = -\frac{R}{L} i_q - i_d p\omega - \frac{\lambda_f p}{L} \omega + \frac{v_q}{L} - A - \frac{\hat{J}}{J} \left(\hat{b} - k_1 \right) i_q$$
$$+ \frac{2\hat{J}}{3p\lambda_f} \left(\hat{b} - k_1 \right) (b\omega + \tau). \tag{20}$$

In order to develop the control input that stabilizes q-axis current error trajectory, defining the augmented Lyapunov function $V_2 = 1/2z_2^2$ and its time derivative as

$$\dot{V}_2 = z_2 \left\{ -\frac{R}{L} i_q - i_d p\omega - \frac{\lambda_f p}{L} \omega + \frac{v_q}{L} - A \right.$$
$$\left. - \frac{\hat{J}}{J} \left(\hat{b} - k_1 \right) i_q + \frac{2\hat{J}}{3p\lambda_f} \left(\hat{b} - k_1 \right) (b\omega + \tau) \right\}. \tag{21}$$

The control input voltage that regulates current error z_2 can be designed as

$$v_q = -k_2 \hat{L} z_2 + \hat{R} i_q + i_d p\omega \hat{L} + p\lambda_f \omega + \hat{L} A$$
$$+ \hat{L} \left(\hat{b} - k_1 \right) i_q - \frac{2\hat{L}\hat{J}}{3p\lambda_f} \left(\hat{b} - k_1 \right) \left(\hat{b}\omega + \hat{\tau} \right) - k_p z_2 \xi^2, \tag{22}$$

where $\xi = \frac{3p\lambda f \hat{L}}{2J_N} i_q - \omega - \dot{\omega}_d$ is performance improvement term with $K_p = 30$, $\hat{R}$ and $\hat{L}$ are estimated forms of the resistance and inductance, respectively. Substituting q-axis control voltage in (21) yields

$$\dot{V}_2 = -k_2 z_2^2 - k_p z_2^2 \xi^2 + z_2 \frac{\tilde{L}}{L} A_2 + \frac{\tilde{R}}{L} z_2 i_q$$
$$- \frac{\tilde{J}}{J} \left(\hat{b} - k_1 \right) z_2 i_q - \tilde{b} \frac{2\hat{J}}{3p\lambda_f} \left(\hat{b} - k_1 \right) \omega z_2 \tag{23}$$
$$- \tilde{\tau} \frac{2\hat{J}}{3p\lambda_f} \left(\hat{b} - k_1 \right) z_2,$$

$$A_2 = -k_2 z_2 + i_q p\omega + A - \left(\hat{b} - k_1 \right) i_q - \frac{2\hat{J}}{3p\lambda_f} \left(\hat{b} - k_1 \right) \left(\hat{b}\omega + \hat{\tau} \right) i_q$$

Defining d-axis current error as $z_3 = i_d$. Taking its time derivative and using (3), we obtain

$$\dot{z}_3 = -\frac{R}{L} i_d + i_q p\omega + \frac{v_d}{L}. \tag{24}$$

Define Lyapunov candidate function as $V_3 = 1/2z_3^2$. By taking time derivative and using (24) one can obtain

$$\dot{V}_3 = z_3 \left\{ -\frac{R}{L} i_d + i_q p\omega + \frac{v_d}{L} \right\}. \tag{25}$$

The control input can be written as

$$v_d = \hat{R} i_d - p\omega \hat{L} i_q - k_3 \hat{L} z_3. \tag{26}$$

Substituting (26) in (25) gives,

$$\dot{V}_3 = -k_3 z_3^2 + z_3 \left\{ \frac{\tilde{R}}{L} i_d - \frac{\tilde{L}}{L} \left(i_q p\omega + k_3 z_3 \right) \right\}. \tag{27}$$

Finally, to achieve desire stability and to develop adaptive laws that can estimate uncertain parameters of the proposed system, we have

$$V_4 = V_1 + V_2 + V_3 + \frac{\tilde{R}^2}{2Lr_1} + \frac{\tilde{b}^2}{2r_2} + \frac{\tilde{\tau}^2}{2r_3} + \frac{\tilde{J}^2}{2Jr_4} + \frac{\tilde{L}^2}{2Lr_5}, \tag{28}$$

The adaptive laws that compensate variations in the parameters can be designed by taking time derivative of (28) along (16), (23), (27) and using Assumption 2

$$\dot{V}_4 = -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 + \frac{\tilde{R}}{L} \left(\frac{\dot{\hat{R}}}{r_1} + i_d^2 + i_q z_2 \right)$$

$$+ \tilde{b} \left\{ \frac{\dot{\hat{b}}}{r_2} + \omega z_1 - \frac{2\hat{j}}{3p\lambda_f} \left(\hat{b} - k_1 \right) \omega z_2 \right\} + \frac{3p\lambda_f}{2j} z_1 z_2$$

$$+ \tilde{\tau} \left\{ \frac{\dot{\hat{\tau}}}{r_3} + z_1 - \frac{2\hat{J}}{3p\lambda_f} \left(\hat{b} - k_1 \right) z_2 \right\} \tag{29}$$

$$+ \frac{\tilde{J}}{J} \left\{ \frac{\dot{\hat{J}}}{r_4} - \left(\hat{b} - k_1 \right) i_q z_2 + \left(\hat{b}\omega + \hat{\tau} - k_1 z_1 + \dot{\omega}_d \right) z_1 \right\}$$

$$+ \frac{\tilde{L}}{L} \left[\frac{\dot{\hat{L}}}{r_5} + A_2 z_2 - \left(k_3 z_3 + p\omega i_q \right) z_3 \right].$$

The estimator laws that estimate unknown perturbations in motor's parameter and update the control functions are designed as follows:

$$\dot{\hat{R}} = -r_1 \left(i_d^2 + i_q z_2 \right), \tag{30}$$

$$\dot{\hat{b}} = r_2 \left\{ \frac{2\hat{J}}{3p\lambda_f} \left(\hat{b} - k_1 \right) \omega z_2 - \omega z_1 \right\}, \tag{31}$$

$$\dot{\hat{\tau}} = r_3 \left\{ \frac{2\hat{J}}{3p\lambda_f} \left(\hat{b} - k_1 \right) z_2 - z_1 \right\}, \tag{32}$$

$$\dot{\hat{J}} = r_4 \left\{ \left(\hat{b} - k_1 \right) i_q z_2 - \left(\hat{b}\omega + \hat{\tau} + \dot{\omega}_d - k_1 z_1 \right) \right\}, \tag{33}$$

$$\dot{\hat{L}} = r_5 \left\{ z_3 \left(k_3 z_3 + p\omega i_q \right) - z_2 A_2 \right\}, \tag{34}$$

where r_1, r_2, r_3, r_4 and r_5 are adaptation gains. Substituting (30) to (34) in (29) gives

$$\dot{V}_4 = -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 + \frac{3p\lambda_f}{2J} z_1 z_2 - k_p z_2^2 \xi^2. \tag{35}$$

LEMMA [23]: Let Q be $n \times n$ symmetric matrix and $\dot{V}_4 \leq z^T Q z$ is in related form of polynomial of degree two, then $\dot{V}_4$ is a negative definite function if Q is Hurwitz.

Proof: The Lyapunov differential equation (35) can be expressed in the following from

$$z^T \frac{\partial \partial \left(\dot{V}_4 \right)}{\partial z \partial z^T} z \leq \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}^T \begin{bmatrix} -2k_1 & \dfrac{3p\lambda_f}{2J} & 0 \\ \dfrac{3p\lambda_f}{2J} & -2k_2 & 0 \\ 0 & 0 & -2k_3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \tag{36}$$

The Q matrix in (36) satisfies the two properties of negative definiteness i.e., the first and second minors are negative and positive, respectively. Thus, Q is negative definite if its determinant is negative

$$\det(Q) = -2k_3 \left\{ 4k_1 k_2 - \left(\frac{3p\lambda_f}{2J} \right)^2 \right\} < 0$$

$$= 4k_1 k_2 - \left(\frac{3p\lambda_f}{2J} \right)^2 > 0,$$

$$= k_1 k_2 > \left(\frac{3p\lambda_f}{4J} \right)^2. \tag{37}$$

If the feedback gains satisfy the inequality (37) and $k_3 > 0$, $\dot{V}_4$ will be negative definite. Thus, the system trajectories converge to zero asymptotically.

Remark 1: The proposed adaptive backstepping controller has three remarkable features. Firstly, by comparing the feedback gains requirement in (37) with the gains in (12) (see Section 2), we can conclude that the proposed adaptive control needs small gains to achieve robustness against the parametric uncertainties and load torque disturbances. Secondly, the adaptive laws (30-34) update the control inputs with instantaneous values of variations in the parameters, thus, providing more accurate speed tracking. Thirdly, the performance improvement term in (22) enhances the convergence rate of i_q current thus increasing the convergence of speed tracking as evidenced in (16).

5. SIMULATION RESULTS

In this section, comparative simulations are performed to verify the speed tracking performance of the two controllers designed in Section 3 and Section 4
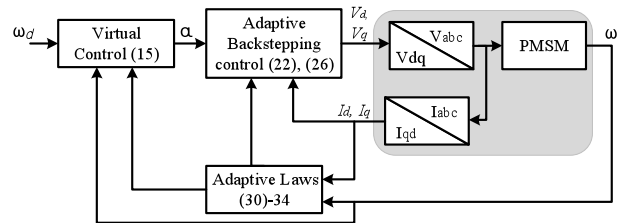

Fig. 1 – Control system block diagram.

The block diagram for the simulation is shown in Fig. 1, which includes dynamical backstepping controller with the

adaptive laws. The instantaneous values of all the uncertain parameters are estimated and updated in controller by the adaptive laws. For the sake of fair comparison, same gain values are selected for both the controllers according (37). The values of these gains and the adaptation gains are mentioned in Table 2.

Table 2
Designed feedback and adaptive gains

Gain type	Symbol	Value
Feedback gains	k_1	350
	k_2	15000
	k_3	5000
Adaptive gains	r_1	0.000016
	r_2	25000
	r_3	0.0105
	r_4	47
	r_5	85000

The Initial conditions of the state variables are taken as zero. In addition, the initial values of all the designed estimators are assumed to be zero to observe the estimation performance. The desired speed trajectory is chosen as the exponential profile. The robustness property of both the controllers is analysed in detail under the influence of parametric uncertainties. All the model parameters are initially nominal which are taken from [14]. In Fig. 2, speed tracking performance of PMSM is shown where actual speed tracks the desired speed with high accuracy counteracting all the parametric perturbations. The sudden changes in parameters of PMSM for backstepping case drives the actual speed of the motor away from its reference trajectory thus causing severe steady-state error as shown in Fig. 3. To overcome this issue, higher gains are required. The adaptive control causes a undershoot of 1.2 rad/s and an overshoot of 0.39 rad/s. However, it restores nominal performance within 50 ms and 45 ms as shown in the zoomed view of Fig. 2 and is also indicated as proof in Remark 1.
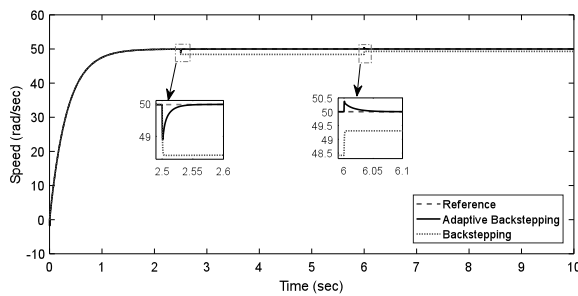

Fig. 2 – Speed tracking of PMSM for exponential reference profile.

In Fig. 3, the tracking error between actual and desired speed of synchronous motor is demonstrated.
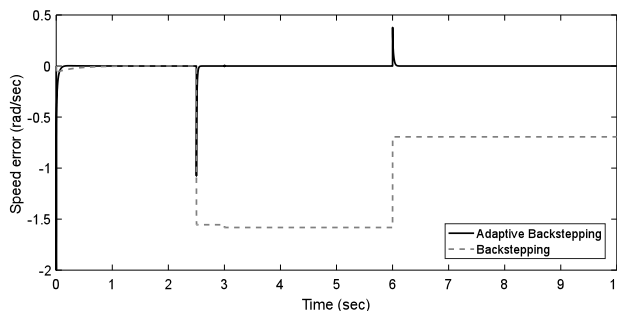

Fig. 3 – Tracking error between actual and desired speed.

It is quite clear that the proposed adaptive law accurately tracks the desired trajectory thus minimizing the tracking error in a very short duration of time as compared to the robust backstepping control.

Figure 4 depicts the effectiveness of the adaptive law in load torque estimation. A constant sudden load of 1 Nm is applied to the motor between 0 and 2.5 s, then at time $t = 2.5$ s the load torque is increased by 200 %. Finally, at time $t = 6$ s the load torque is reduced by 100 %. It is very obvious that when the synchronous motor is suddenly subjected to several changes in the load torque, the adaptive algorithm exactly estimates those changes, thus the undesired behavior of synchronous motor is compensated.
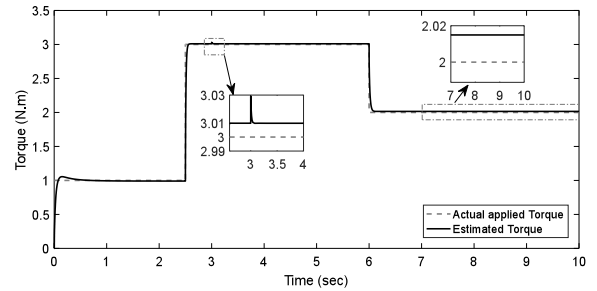

Fig. 4 – Load torque disturbances and their estimation.

In Fig. 5, the deviant behavior of synchronous motor's inertia is displayed. Initially rotor has 0.0008 kgm^2 inertia between 0 and 2.5 s, 0.00145 kgm^2 between 2.5 and 6 s. and 0.00127 kgm^2 between 6 and 10 s. Thus, the adaptive law accurately predicts the irregular changes in inertia and achieves the nominal control performance.
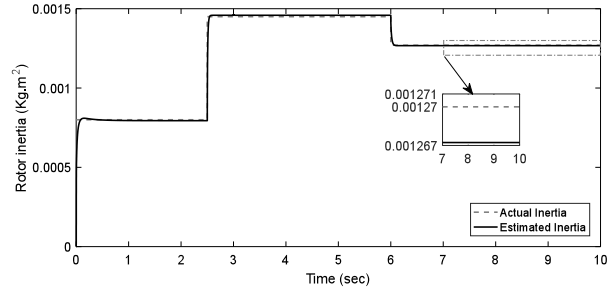

Fig. 5 – Perturbations in rotor inertia and estimation.

Figure 6 illustrates the variant nature of damping coefficient. It is evident that in continuous mode of operation for a long time, the motor's damping coefficient fluctuates around the nominal value which causes perturbation in system. So, the damping estimator precisely estimates the changes occurred and update the system parameters in control input. The zooms in Figs. 4–6 show the steady-state response of the adaptation laws, which concludes high estimation accuracy.
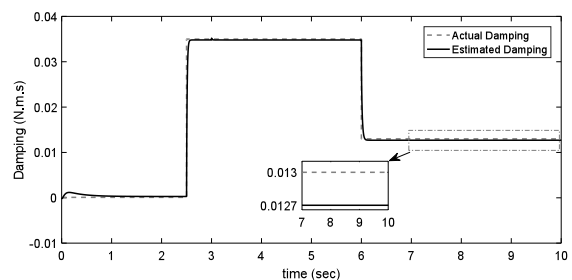

Fig. 6 – Variations in damping and estimation.

In Figs. 7 and 8, efficiency of the proposed adaptive law is demonstrated for the uncertain behavior of armature resistance and inductance. In fact, the resistance and inductance are variant parameters, whose behavior change with continuous operation. Thus, to compensate these changes and to achieve the desired performances in the presence of perturbed parameters, an accurate adaptive law is investigated, whose effectiveness can be clearly seen in estimating the updated value of variant parameters.
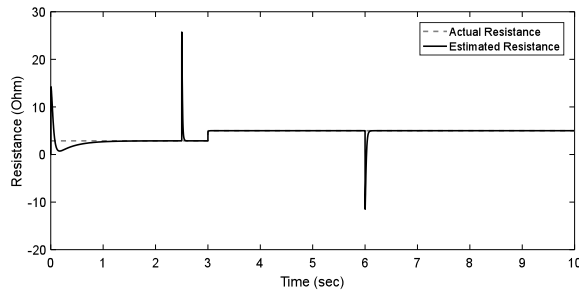

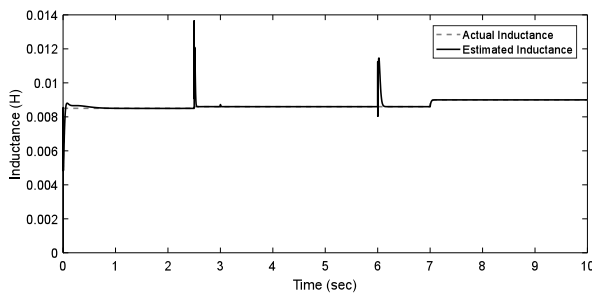Fig. 7 – Perturbations in stator resistance and estimation.

Fig. 8 – Perturbations in stator inductance and estimation.

5. CONCLUSION

In this paper, adaptive backstepping controller has been presented for a PMSM to control the angular speed under parametric perturbations. The controller and estimation laws are designed by defining Lyapunov function. Speed performance is independent of parametric uncertainties and unknown load torque dynamics. The problem of over-parametrization is eradicated since the designed adaptive parametric dynamics are equal to number of uncertain parameters. The impact of all the parametric variations and estimation is also studied thoroughly. It is noticed that load torque and inertia disturbances have very strong distorting impact on speed performance. The proposed control scheme restores nominal speed performance and compensates the effect of all the uncertainties and disturbances in PMSM dynamics.

REFERENCES

1. J. Iqbal, R. U. Islam, S. Z. Abbas, A. A. Khan, S. A. Ajwad, *Automating industrial tasks through mechatronic systems – A review of robotics in industrial perspective*, Tehnički vjesnik, **23**, *3*, pp. 917–924 (2005).
2. R. Shanthi, S. Kalyani, P. M. Devie, *Design and performance analysis of adaptive neuro-fuzzy controller for speed control of permanent magnet synchronous motor drive*, Soft Computing, pp. 1–15 (2020).
3. A. Larbaoui, B. Belabbes, A. Meroufel, D. Bouguenna, *Commande par mode glissant floue de la machine synchrone (fuzzy sliding mode control for synchronous machine)*, Rev. Roum. Sci. Techn.-Electrotech. Et Energ., **62**, *2*, pp. 192–196 (2017).
4. J. Iqbal, M. Ullah, S. G. Khan, B. Khelifa, S. Ćuković, *Nonlinear control systems¬A brief overview of historical and recent advances*, Nonlinear Engineering – Modeling and Application, (2009).
5. H. C. Cho, K. S. Lee, M. S. Fadali, *Adaptive control of pmsm systems with chaotic nature using lyapunov stability based feedback linearization*, Int. J. of Innova. Comp., Info. and Cont., **5**, *2*, pp. 479–488 (2009).
6. D. Q. Wei, B. Zhang, X. S. Luo, S. Y. Zeng, D. Y. Qiu, *Effects of couplings on the collective dynamics of permanent magnet synchronous motors*, IEEE Trans. Cir. and Syst. II: Express Briefs, **60**, *10*, pp. 692–696 (2013).
7. Y. Deia, M. Kidouche, M. Becherif, *Decentralized robust sliding mode control for a class of interconnected nonlinear systems with strong interconnections*, Rev. Roum. Sci. Techn.-Electrotech. Et Energ., **62**, *2*, pp. 203–208 (2017).
8. T.-B.-T.Nguyen, T.-L.Liao, J.-J.Yan, *Adaptive sliding mode control of chaos in permanent magnet synchronous motor via fuzzy neural networks*, Math. Probl. in Eng., (2014).
9. Y. B. Jeddou, R. Abdessemed, E. Merabet, L. Bentouhami, *Fuzzy controller for self-excited dual star induction generator with online estimation of magnetizing inductance used in wind energy conversion*, Rev. Roum. Sci. Techn.-Electrotech. Et Energ., **63**, 4, pp. 417–422 (2018).
10. G. Rigatos, P. Siano, A. Melkikh, N. Zervos, *Nonlinear H-infinity Control Approach to Stabilization of Distributed Synchronous Generators*, IEEE Systems Journal, **99**, pp. 1–10 (2017).
11. M. Demirtas, *Off-line tuning of a PI speed controller for a permanent magnet brushless DC motor using DSP*, Energy Conversion and Management, **52**, *1*, pp. 264–273 (2011).
12. R.W. Jan, C.S. Tseng, R.J. Liu, *Robust PID control design for permanent magnet synchronous motor: a genetic approach*, Electric Power Systems Research, **78**, *8*, pp. 1161–1168 (2008).
13. N. Ali, A. U. Rehman, W. Alam, H. Maqsood, *Disturbance observer based robust sliding mode control of permanent magnet synchronous motor*, Journal of Electrical Engineering, **14**, *6*, pp. 2531–2538 (2019).
14. J. Iqbal, M.I. Ullah, A.A. Khan, M. Irfan, *Towards sophisticated control of robotic manipulators: Experimental study on a pseudo-industrial arm*, Strojniški vestnik-Journal of Mechanical Engineering, **61**, *7–8*, pp. 465–470 (2015).
15. N. Ali, I. Tawiah, W. Zhang, *Finite-time extended state observer based nonsingular fast terminal sliding mode control of autonomous underwater vehicles*, Ocean Engineering, **218** (2020).
16. J.-H. Hao, X.-W. Wang, and H. Zhang, *Chaotic robust control of permanent magnet synchronous motor system under uncertain factors*, Acta. Physica. Sinica., **63**, *22*, pp. 2202032-2202038 (2014).
17. M. Krstic, I. Kanellakopoulos, P.V. Kokotovic, *Nonlinear and Adaptive Control Design*, Wiley, New York, 1995.
18. O. Khan, M. Pervaiz, E. Ahmad, J. Iqbal, *On the derivation of novel model and sophisticated control of flexible joint manipulator*, Rev. Roum. Sci. Techn.-Electrotech. Et Energ., **61**, *1*, pp. 103–108 (2017).
19. C. M. Kwan, F. L. Lewis, *Robust backstepping control of induction motors using neural networks*, IEEE Tran. Neural Networ., **11**, *5*, pp. 1178-1187 (2000).
20. J.-H. Hu J.-B. Zou, *Adaptive backstepping control of permanent magnet synchronous motors with parameter uncertainties*, Control and Decision, **21**, 11, pp. 1264-1269 (2006).
21. J. Zhou, Y. Wang, *Real-time nonlinear adaptive backstepping speed control for a PM synchronous motor*, Control Engineering Practice, **13**, *10*, pp. 1259–1269 (2005).
22. B. Bossoufi, M. Karim, A. Lagrioui, M. Taoussi, *FPGA-Based Implementation Nonlinear Backstepping Control of a PMSM Drive*, International Journal of Power Electronics and Drive System, **4**, 1, pp. 12–23 (2014).
23. R. Bhatia, *Positive definite matrices*, Princeton University Press, 2009.