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THE DEPENDENCE BETWEEN THE OPTIMAL MECHANICAL ANGULAR SPEED AND THE VARIABLE WIND SPEED FROM AN ENERGETIC POINT OF VIEW

ANTONIO M. F. LUPUTI[1], MARIN SALINSCHI[2], MARIUS BABESCU[3], HORIA CIOCÂRLIE[1]

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This paper examines the fundamental elements required for operating a wind turbine (WT) at its maximum power point (MPP). It focuses on measuring the wind speed and the power output from the electric generator (EG) over time to establish the relationship between the optimal mechanical angular velocity (MAV) and the wind speed, denoted as the function $\omega_{OPTIM}(V)$. This function is crucial for controlling the power output of the electric generator, ensuring that the WT operates at its maximum power point. The study analyzes the measurements taken from the turbines in Siliștea in Dobrogea, Romania. The paper concludes by presenting an algorithm to determine the function $\omega_{OPTIM}(V)$.

1. INTRODUCTION

This paper analyzes the dependence between the optimal mechanical angular speed, $\omega_{OPTIM}(V)$, and the wind speed, V. Knowing that the function $\omega_{OPTIM}(V)$ is essential for the management of wind systems (ω_{OPTIM} being the reference parameter [1,2]) and for determining the mathematical model of a wind turbine [3–6], the efficient control of a wind system is achieved by determining the function $\omega_{OPTIM}(V)$.

Through correct modeling and efficient control of the wind system, high energy efficiency is obtained through a maximum capture of wind energy. Studies related to the topic [7–10] mainly analyze the operation of WT at constant wind speeds within time. In reality, however, in Romania, the wind speeds are significantly variable in time. In some cases, when storms last for several hours, the wind energy in such periods is considerable.

There were cases when the power generated by the wind turbines in Romania exceeded the power delivered by nuclear power plants. According to studies in wind energy capture, it becomes obvious that the wind speed variable within time ensures optimal operation of wind systems from an energy point of view [9–13]. Operating a wind turbine at its maximum power point, MPP, while maintaining the optimal mechanical angular velocity ω_{OPTIM} is a complex challenge, especially when dealing with variable wind speeds [11,12].

The conditions for operating a wind turbine (WT) at its maximum power point (MPP) at varying wind speeds over time can be derived from the kinetic moment equation [2,5,12,13] in the form

$$J\frac{d\omega}{dt} = M_{WT} - M_{EG}, \tag{1}$$

where ω represents the mechanical angular speed, ($\omega=2\pi n$), n- the frequency at the electric generator (EG), J – the total moment of inertia, M_{WT} – the moment given by the WT related to the shaft of the electric generator, and M_{EG} – the electromagnetic moment given by the EG. All the parameters are related to the shaft of the EG.

By multiplying with ω

$$J\frac{d\omega}{dt}\omega = P_{WT} - P_{EG}, \tag{2}$$

By integrating

$$J\frac{\omega_k^2-\omega_{k-1}^2}{2} = \int_{t_{k-1}}^{t_k} P_{WT}\,dt - \int_{t_{k-1}}^{t_k} P_{EG}\,dt = P_{WT\text{-}MEAN}\cdot\Delta t - E_{ELECTRIC},$$

or

$$\int_{t_{k-1}}^{t_k} P_{WT}\,dt = J\frac{\omega_k^2-\omega_{k-1}^2}{2} + \int_{t_{k-1}}^{t_k} P_{EG}\,dt, \tag{3}$$

where: ω represents MAV at time t_k, ω_{k-1} – MAV at time t_{k-1}, $\Delta t = t_k - t_{k-1}$, P_{WT} – the useful power given by the WT, $P_{WT\text{-}MEA}$ represents the average power for the interval Δt, P_{EG} – the power given by EG, and $E_{ELECTRIC}$ – the electric energy charged by the network for the interval Δt.

Therefore, the captured wind energy

$$E_{CAPTURE} = \int_{t_{k-1}}^{t_k} P_{WT}\,dt, \tag{4}$$

is found in the cinematic energy

$$E_{KINETIC} = J\frac{\omega_k^2-\omega_{k-1}^2}{2}, \tag{5}$$

by summing with the electric energy

$$E_{ELECTRIC} = \int_{t_{k-1}}^{t_k} P_{EG}\,dt. \tag{6}$$

By measuring MAV, within time, at a sampling step t, and measuring the electrical power or the power supplied to the network, the value of the useful power to the WT is obtained on the interval of time Δt:

$$P_{WT\text{-}MEAN} = J\frac{\omega_k^2-\omega_{k-1}^2}{2\cdot\Delta t} + \frac{E_{ELECTRIC}}{\Delta t}, \tag{7}$$

or

$$P_{WT\text{-}MEAN} = J\frac{\omega_k^2-\omega_{k-1}^2}{2\cdot\Delta t} + P_{EG}. \tag{8}$$

The energy balance equation determines the wind turbine's (WT) useful power. As a result, it is possible to calculate how the WT's power changes within time, represented by the function $P_{WT}(t)$.

The maximum points of the function $P_{WT}(t)$ occurring at time t^* correspond to operating in MPP at ω_{OPTIM} wind speed at the same moment, having the value V^*.

Operating in MPP is performed at optimum MAV, ω_{OPTIM}, corresponding to the maximum power $P_{WT\text{-}MAX}$. In some studies [2,3,10,11], it is shown that the value of optimum MAV, ω_{OPTIM}, is linear and dependent on the wind speed V in the form

$$\omega_{OPTIM} = k_0\cdot V. \tag{9}$$

But, in reality, the function $\omega_{OPTIM}(V)$ is not linear [5,6,12]. The proportionality factor k_o given in the technical books of the turbine is very important for the performance of the WT in MPP [9,10] and must be periodically recalculated with the changes in the weather conditions (temperature,

[1] Politehnica University of Timișoara, Department of Computer and Information Technology, Timișoara, Romania
[2] Politehnica University of Timișoara, Power System Department, Timișoara, Romania
[3] Politehnica University of Timișoara, Department of Electrical Engineering, Timișoara, Romania
E-mails: antonio.luputi@student.upt.ro, marin.salinschi@upt.r,o horia.ciocarlie@cs.upt.ro, marius.babescu@upt.ro

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humidity). To achieve maximum wind energy capture, the WT needs to operate at the optimal MAV, which is imposed by the value of the wind speed and, therefore,

$$\omega=\omega_{OPTIM}. \quad (9)$$

The function $\omega_{OPTIM}(V)$ requires experimental validation due to several factors: the adjustable positions of the turbine blades, which alter its accuracy; the changing atmospheric conditions, which affect the air density; and varying wind speeds as the function is based on wind speed ranges.

In this paper, the experimental determination of the relationship between the optimal mechanical angular velocity $\omega_{OPTIM}(V)$ and the wind speed (V) is conducted through idle operating and load operating.

2. OPTIMAL MECHANICAL ANGULAR SPEED FOR A WIND TURBINE OPERATING IN IDLE STATE

The experimental data of the wind speed, V, from Silistea 2, 141, Dobrogea, during the time interval 1:20–3:30 h, on January 7th, 2020, are given in Table 1.

Table 1

Experimental data

Hour	V[m/s]	t[s]
1:20	6.39	1·600
1:30	5.58	2·600
1:40	5.46	3·600
1:50	5.25	4·600
2:00	5.23	5·600
2:10	5.56	6·600
2:20	5.55	7·600
2:30	5.46	8·600
2:40	6.14	9·600
2:50	6.34	10·600
3:00	6.01	11·600
3:10	5.75	12·600
3:20	5.45	13·600
3:30	5.46	14·600

During this interval, the wind speed, V, varies between the minimum value of 5.23 m/s and the maximum value of 6.39 m/s. The variation of the wind speed, V, during a 2-hour interval, is shown in Fig. 1.

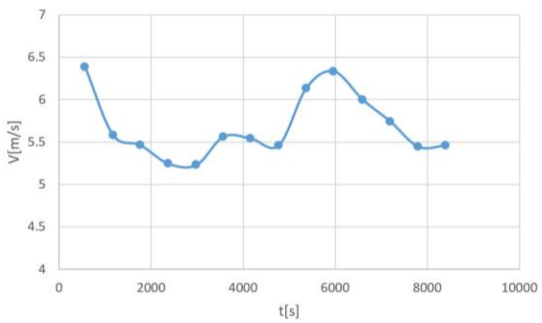

Fig. 1 – Variation of the wind speed during a time interval.

The nominal data of the WT and EG are given in Table 2.

Table 2

Nominal data of WT and EG [14,15].

rated electrical power	2.5 MW
gearbox ratio	151.53
generator moment of inertia	153.3 kg·m^2
rotor and gearbox moment of inertia	8.23·10^6 kg·m^2

Experimentally, the two basic parameters-, the wind speed, V, and the current MAV, ω – are obtained by measurements. In order to know the optimal dependence of MAV on wind speed variation, in a minimum time, the EG is disconnected from the network, the WT operates at idle speed and MAV is measured at the moments of time: 0, 2, 4,...,26 s, obtaining the values given in Table 3.

For J = 511.92 kg·m^2 the time interval of Δt = 2 s, the average power is

$$P_{WT\text{-}MEAN}=J\frac{\omega_k^2-\omega_{k-1}^2}{2\cdot\Delta t}=511.92\frac{\omega_k^2-\omega_{k-1}^2}{2\cdot 2}=127.98(\omega_k^2-\omega_{k-1}^2). \quad (11)$$

Figure 2 shows the power characteristic of the WT, PWT (ω), as a function of angular velocity (ω). The operating area includes two maximum power points: MPP1 and MPP2.

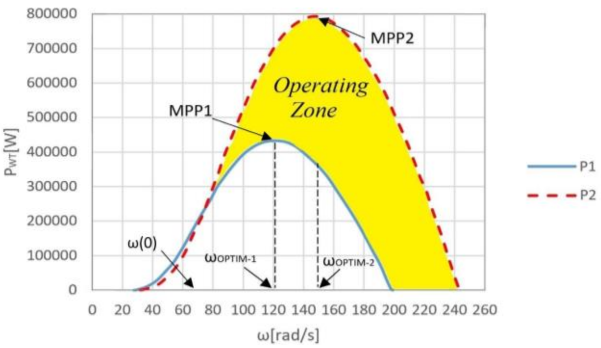

Fig. 2 – Operating zone of WT.

At the initial time, t = 0 s, MAV has the value

$$\omega(0)=66 \text{ rad/s}. \quad (12)$$

The values of MAV are obtained from measuring $\omega(2)$, $\omega(4)$, ..., $\omega(26)$ every 2 seconds. Using the values of MAV obtained from the energy balance equation, the useful power of WT can be determined during an interval of 26 s. Table 3 displays the values of the function $P_{WT}(t)$ during a time interval.

Table 3

The values of MAV and WT power

T [s]	Ω [rad/s]	T [s]	$P_{WT\text{-}MEAN}$ [W]
2	76.5	1	1.9149·10^5
4	92.1	3	3.3661·10^5
6	112.9	5	5.4571·10^5
8	135.7	7	7.254·10^5
10	156.9	9	$P_{WT\text{-}MAX}$=7.9388·10^5
12	174.7	11	7.5540·10^5
14	189	13	6.6561·10^5
16	200.4	15	5.6812·10^5
18	209.4	17	4.7202·10^5
20	216.5	19	3.8700·10^5
22	222	21	3.0866·10^5
24	226.5	23	2.5830·10^5
26	230	25	2.0448·10^5

The maximum value of WT power is

$$P_{WT\text{-}MAX}=7.9388\cdot 10^5 \text{ W}, \quad (13)$$

corresponding to MAV for t*=9

$$\omega_{OPTIM}=\frac{156.9+135.7}{2}=146.3 \text{ rad/s}. \quad (14)$$

At the time t^*=9 for which the value of WT power is maximum, the wind speed is

$$V^*=6.38 \text{ m/s}, \quad (15)$$

resulting the proportional factor k_0

$$k_0=\frac{\omega_{OPTIM}}{V^*}=\frac{146.3}{6.38}\cong 23. \quad (16)$$

The optimal MAV, ω_{OPTIM}, becomes

$$\omega_{OPTIM}=23\cdot V. \quad (17)$$

The variation of the WT power over a time interval is shown in Fig. 3.

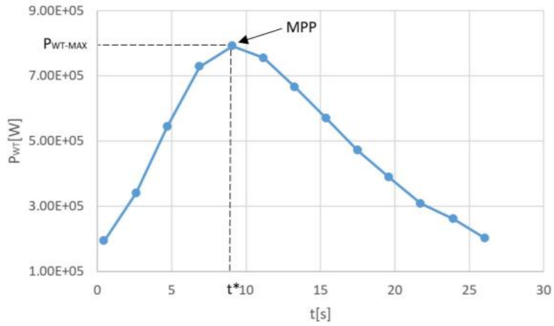

Fig. 3 – The WT power variation with time for WT operating in an idle state.

The obtained function ω_{OPTIM} (V), is valid for values of wind speed in the range

$$V\cong 6.=>7 \text{ m/s}. \quad (18)$$

At other wind speed values, a recalculation of the function ω_{OPTIM} (V) is required.

3. OPTIMAL MECHANICAL ANGULAR SPEED FOR A WIND TURBINE WORKING IN LOAD STATE

In this case, the experimental data on wind speed during the time interval from 1:20 to 3:30 remain consistent with the previously mentioned values. The power from EG remains constant over the whole interval

$$P_{EG}=5.7632\cdot 10^5 \text{ W}. \quad (19)$$

The WT power values, in this case, are calculated by summing the inertial power

$$P_{INERTIAL}=J\frac{\omega_k^2-\omega_{k-1}^2}{2\cdot\Delta t}, \quad (20)$$

with the EG power

$$P_{WT\text{-}MEAN}=P_{INERTIAL}+P_{EG}\ =J\frac{\omega_k^2-\omega_{k-1}^2}{2\cdot\Delta t}+5.7632\cdot 10^5. \quad (21)$$

At the initial time, MAV is

$$\omega(0)=111 \text{ rad/s}. \quad (22)$$

At specific moments, 0, 2, 4,...,26 s MAV values are obtained from measurements and calculating the power $P_{WT\text{-}MEAN}$. Table 4 shows the values of MAV and the power for WT operating in a load state.

Table 4

The values of MAV and power

T [s]	ω [rad/s]	T [s]	$P_{WT\text{-}MEAN}$ [W]
2	113.3	1	$6.4234\cdot 10^5$
4	116.29	3	$6.6418\cdot 10^5$
6	120.03	5	$6.8943\cdot 10^5$
8	124.54	7	$7.1748\cdot 10^5$
10	129.72	9	$7.4488\cdot 10^5$
12	135.39	11	$7.4488\cdot 10^5$
14	141.31	13	$7.687\cdot 10^5$
16	147.24	15	$7.8596\cdot 10^5$
18	152.96	17	$7.9531\cdot 10^5$
20	158.31	19	$P_{WT\text{-}MAX}=7.9608\cdot 10^5$
22	163.2	21	$7.8944\cdot 10^5$
24	167.58	23	$7.7753\cdot 10^5$
26	171.45	25	$7.6174\cdot 10^5$

At the time t^* = 9 when the WT power reaches its maximum value, the MAV value is:

$$\omega_{OPTIM}=\frac{152.96+147.24}{2}=150.1 \text{ rad/s}. \quad (23)$$

WT power reaches the maximum value of

$$P_{WT\text{-}MAX}=7.9608\cdot 10^5 \text{ W}, \quad (24)$$

at time t^* for which the wind speed is

$$V^*=6.385 \text{ m/s}, \quad (25)$$

resulting the proportional factor k_1:

$$k_1=\frac{\omega_{OPTIM}}{V^*}=\frac{150.1}{6.385}=23.508. \quad (26)$$

When operating in a load state, the optimal MAV dependence on wind speed has the form

$$\omega_{OPTIM}=23.508\cdot V. \quad (27)$$

The variation over time of the WT power in load state is shown in Fig. 4.

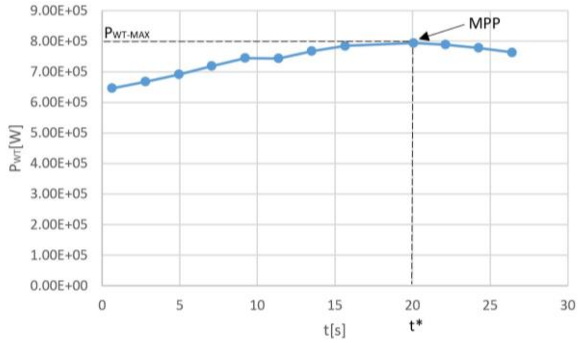

Fig. 4 – The WT power varying over time for WT operating in load state.

The difference between the two values of the proportionality factor k, obtained in idle and load states, is acceptable in practice.

$$\frac{23.508-23}{23.508}100=2.161 \text{ \%}. \quad (28)$$

4. THE ALGORITHM FOR DETERMINING THE OPTIMAL MECHANICAL ANGULAR SPEED

The designed algorithm is based on measuring the parameters at a sampling step Δt: the wind speed, V; the frequency/MAV, n/ω; and the energy from EG, $E_{ELECTRIC}$. Knowing MAV ω_k, ω_{k-1}, the value of WT power is obtained from the energy balance equation:

$$P_{WT\text{-}MEAN}=J\frac{\omega_k^2-\omega_{k-1}^2}{2\cdot\Delta t}+\frac{E_{ELECTRIC}}{\Delta t}, \quad (29)$$

over a period Δt, which includes a sufficiently large number of sampling steps to ensure that ω_{OPTIM} falls within the range. The maximum power of the WT, $P_{WT\text{-}MAX}$, is obtained at

$$\omega=\omega_{OPTIM}. \quad (30)$$

The maximum power output, $P_{WT\text{-}MAX}$, determines the coordinates of a point on the $\omega_{OPTIM}(V)$ function. The estimated duration of the function $\omega_{OPTIM}(V)$ depends on the number of sampling steps t. If the estimation time for determining the function $\omega_{OPTIM}(V)$ is not limited, then the end of the process is considered at

$$\omega = \omega_{OPTIM}. \quad (31)$$

The minimum estimation duration is obtained at a minimum MAV interval

$$\omega\in[\omega(0), \omega_{FINAL}] \quad (32)$$

where the optimal MAV, ω_{OPTIM}, is present in this interval.

Figure 5 illustrates the algorithm that identifies the moment when the WT power is maximum, when the wind speed, V, and MAV, ω are: V^* and ω_{OPTIM}.

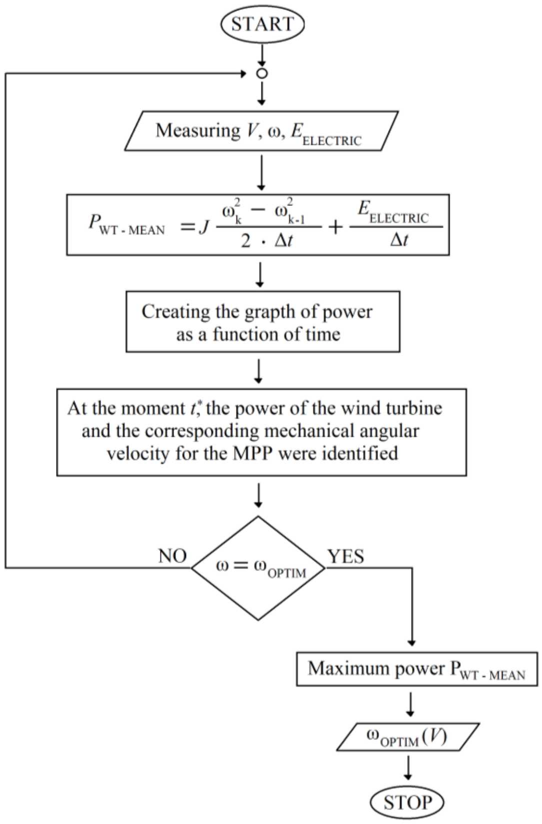

Fig. 5 – Algorithm for determining the function $\omega_{OPTIM}(V)$.

With this algorithm, the function ω_{OPTIM}(V) is determined to achieve optimal energy performance in a minimum time.

5. **CONCLUSIONS**

This paper explores the relationship between the mechanical angular velocity and the wind speed. Comprehending that this relationship is key to controlling the power output of the electric generator, it ensures that the wind turbine operates at its maximum power point. The study specifically focuses on the power of wind turbines at Siliştea in Dobrogea, Romania. The maximum power point is identified by measuring the wind speed, rotation speed (mechanical angular velocity), and generator power. Thus, the optimal mechanical angular velocity is known for a certain wind speed value. For constant wind speeds over time, the problem is relatively simple. However, for variable wind speeds over time, the problem becomes complex due to the large mechanical inertia of the wind system. A no-load operation test is the simplest method to identify the maximum power point. The optimal mechanical angular velocity dependency through a load test has also been analyzed. The last part of the paper presents an algorithm for determining the function $\omega_{OPTIM}(V)$, which is crucial for optimizing wind turbine performance. It is considered appropriate to use this algorithm periodically as weather conditions and mechanical demands change over time. This solves the problem of maximizing the capture of wind energy.

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