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TIME-FINITE IMPULSE RESPONSE DIGITAL FILTER BASED ON THE TIME DIFFERENCES

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Key words: Frequency locked loop; Digital filter; Phase locked loop; Digital circuit; Discrete linear system.

This work describes one model of the time-finite impulse response (FIR) digital filter whose output is based on the time differences between the input and output periods. It is intended for the filtering of the pulse signal periods. The filter is a linear, discrete system that functions as a frequency locked loop (FLL). The output correction is performed once per period. The specific properties of the FLL are described, thanks to which it is suitable to be adapted to function as a time-FIR either low-pass or high-pass digital filter. The procedure for adjusting the fourth-order FLL into the Time-FIR digital filter is presented. Mathematical analyses were performed using the Z-transform. The system's operation was simulated. For analysis in the frequency domain, the theory and the corresponding MATLAB software packages, intended for the development of the classical FIR digital filters, were used. The properties of the fourth-order FLL, as well as the filtering abilities of the developed Time-FIR digital filter, are demonstrated in the time and frequency domains.

1. INTRODUCTION

Time-infinite impulse response (IIR) digital filters are described in refs. [1,2], while Finite Impulse Response (Time-FIR) digital filters are described in [3,4]. The expression "Time-digital filter" was used for the first time in [1]. Time-digital filters may be type FIR or type IIR, depending on if only the input periods or both the input and output periods are processed. This approach is adopted, modeled on classic digital filters, which process the current values of a signal, rather than the periods of an impulse signal. The Time-digital filters, either type FIR or IIR are intended for the filtering of impulse signal periods. Unlike the classic digital filters, which possess only one output, Time-digital filters possess three outputs [4]. These are the output period TO_k, time difference τ_k between the output and input period and time interval $T_k=TI_k-\tau_k$. All of them depend on the input signal period TI_k, which means that they contain the information of TI_k. For the described FLL in [4], the frequency responses of TO_k, τ_k, and T_k are different. They indicate that all the outputs TO_k, τ_k and T_k possess some filtering characteristics. However, only the output TO_k in [4] functions precisely as the classic digital filter.

This article is a continuation of the development of Time-FIR digital filters based on the processing of periods [3,4]. In [1,4], various types of low-pass time-digital filters are described, which are characterized by the fact that the sum of all filter parameters is equal to unity. This allowed digital filter functions to be built into the output period TO_k. However, the sum of the coefficients is not equal to one in all types of digital filters. In these cases, a filtering function cannot be incorporated into the output period TO_k. Based on this knowledge, the questions are if we can overcome this problem finding an algorithm of FLL whose output τ_k, instead of the output TO_k, performs the filtering of the input signal periods. The associated questions are how to implement it using the theory of the classic digital filters and what are the advantages in comparison to the Time-digital filters described in [1-4], in which only the output TO_k functions precisely as the classical digital filter. This article answered each of these questions, using a new model of FLL of the fourth order to demonstrate the principle. The same principle can be applied to FLL of any order.

Numerous applications of FLLs are described in [5–10]. These references are also important for this article, because they describe, at the same time, the way of functioning and realization of Time-digital filters, the way of their computer simulation in the time domain, as well as the way of their design and analysis using the Z-transform and the theory of linear discrete systems. The articles and books in [11–26] serve as a theoretical basis for electronics implementations and development necessities.

2. DESCRIPTION OF THE FOURTH-ORDER FLL_4

Figure 1 represents a general case of an input signal Sin and an output signal Sop of the fourth-order FLL_4 and shows the physical relations between the input and output variables, when FLL_4 is in the stable state. The periods TI_k and TO_k, as well as the time difference τ_k and time interval T_k occur at discrete times t_k, t_{k+1}, t_{k+2}, t_{k+3} and t_{k+4}. Unlike [1–10], where discrete time t_k is defined by the falling edge of TO_k, in this article the discrete time t_k is defined by the falling edge of Sin in Fig. 1. Note that the variable "k", represents the discrete time t_k when an input period is measured and taken in calculation. To adapt the output τ_k of FLL_4 to function as the Time-FIR digital filter, let us define the basic difference equation τ_k, shown in eq. (1), in which b_1, b_2, b_3 and b_4 are the system parameters. Note that, unlike [1–4] where the filter algorithm is embedded in the output period TO_k; in this case, the filter algorithm is included within the time difference τ_k. According to eq. (1), there are four multiplications with four system parameters in calculation of any time difference τ_k.

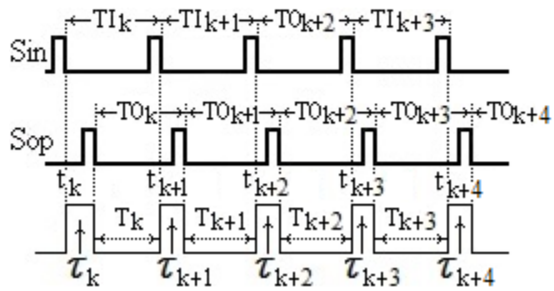

Fig. 1 – The time relations between the input and output variables of the fourth-order FLL_4.

$$\tau_{k+4}=b_1TI_{k+3}+b_2TI_{k+2}+b_3TI_{k+1}+b_4TI_k, \qquad (1)$$

One of the first checks is whether the algorithm, given by eq. (1) is feasible. For example, τ_{k+4} can be calculated at discrete time t_{k+4}, since the last input period TI_{k+3} in eq. (1) has expired at time t_{k+4}. At the same discrete time t_{k+4}, it is

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necessary to start the realization of τ_{k+4}, after it is calculated according to eq. (1). In mathematical sense, it means that τ_{k+4} is added to T_{k+3} to form TO_{k+3} in Fig. 1. The described way of functioning will significantly facilitate the realization of FLL_4, because we should not take care about the sign of τ_k. The realization of FLL_4 is feasible only if τ_k remains always positive and does not change its sign. This is ensured if TO_k always lags behind TI_k, as shown in Fig. 1. However, only through system analysis can it be determined how the system behaves, whether it will remain stable, and whether it can be adapted to function as a digital filter. For the complete analysis, it is necessary to define an expression for the output period TO_k. According to the previous description, TO_k should be calculated according to eq. (2). Note that TO_k and τ_{k+1} elapse at the same time in Fig. 1. However, τ_{k+1} is first calculated according to eq. (1). On the falling edge of τ_{k+1} an impulse of TO_k will be generated to finish the period TO_k. According to eq. (2), the output period TO_k consists of two parts. The first part, during the duration of T_k, is formed passively without any system action, while the second part τ_{k+1} is calculated according to eq. (1) and added to the time interval T_k at the discrete time t_{k+1}. Note that τ_{k+1} is added to T_k after TI_k is finished. In this way, we ensured that τ_{k+1} is always positive, *i.e.*, that TO_k always lags behind TI_k. However, for further analysis, we need to find out how TO_k depends on TI_k. We can see in Fig. 1 that $T_k = TI_k - \tau_k$. Entering this expression into eq. (2), we will get eq. (3). Note that the eqs. (2) and (3) are valid for FLL of any order.

$$\mathrm{TO}_k = T_k + \tau_{k+1}, \tag{2}$$

$$\mathrm{TO}_k = TI_k + \tau_{k+1} - \tau_k. \tag{3}$$

In the following sections, we will analyze whether the described FLL_4 can possess the expected properties. We will find the conditions for the system stability, define the transfer functions of the FLL_4 and the corresponding "b" vectors which are necessary to adapt FLL_4 to function as a time-digital filter. We will also perform the different analyses in the time and frequency domains of FLL_4.

3. STEP ANALYSIS OF FLL_4

To perform the step analysis of FLL_4, let us first find the Z-transform of eqs. (1) and (3). The Z-transform of eq. (1) is shown in eq. (4), where τ_0 is the initial condition of τ_k. Based on eq. (1), $\tau_1 = b_1 TI_0$, $\tau_2 = b_1 TI_1 + b_2 TI_0$ and $\tau_3 = b_1 TI_2 + b_2 TI_1 + b_3 TI_0$. If we enter the previous expressions into eq. (4), we can find $\tau(z)$, eq. (5). The Z-transform of eq. (3) is shown in eq. (6). Entering $\tau(z)$ from eq. (5) into eq. (6), we can calculate $TO(z)$, eq. (7). Based on eqs. (5) and (7), the transfer functions $H\tau_4(z) = \tau(z)/\mathrm{TI}(z)$ and $H_{TO4}(z) = TO(z)/TI(z)$ are shown in eqs. (8) and (9).

$$\begin{gathered} z^4\tau(z) - z^4\tau_0 - z^3\tau_1 - z^2\tau_2 - z\tau_3 = z^3 b_1 TI(z) \\ -z^3 b_1 TI_0 - z^2 b_1 TI_1 - z b_1 TI_2 + z^2 b_2 TI(z) \\ -z^2 b_2 TI_0 - z b_2 TI_1 + z b_3 TI(z) - z b_3 TI_0 + b_4 TI(z), \end{gathered} \tag{4}$$

$$\tau(z) = TI(z)\frac{z^3 b_1 + z^2 b_2 + z b_3 + b_4}{z^4} + \tau_0, \tag{5}$$

$$TO(z) = TI(z) + z\tau(z) - z\tau_0 - \tau(z), \tag{6}$$

$$\begin{gathered} TO(z) = TI(z)[z^4(b_1+1) + z^3(b_2-b_1) \\ +z^2(b_3-b_2) + z(b_4-b_3) - b_4]/z^4 - \tau_0, \end{gathered} \tag{7}$$

$$H\tau_4(z) = \frac{z^3 b_1 + z^2 b_2 + z b_3 + b_4}{z^4}, \tag{8}$$

$$\begin{gathered} H_{TO4}(z) = [z^4(b_1+1) + z^3(b_2-b_1) \\ +z^2(b_3-b_2) + z(b_4-b_3) - b_4]/z^4, \end{gathered} \tag{9}$$

Let us suppose that the step input is $TI(k) = TI = \text{const}$. Substituting the Z-transform of $TI(k)$, *i.e.*, $TI(z) = TI \cdot z/(z-1)$ into eq. (5) and using the final value theorem, it is possible to find the final value of the time difference as $\tau_{4\infty} = \lim\,[(z-1)\cdot\tau_4(z)]$, when $z \to 1$. The result is shown in eq. (10). It comes from eq. (10), that $\tau_{4\infty} = TI$ if eq. (11) is satisfied. In the same way, substituting the Z-transform of $TI(k)$, *i.e.*, $TI(z) = TI \cdot z/(z-1)$ into eq. (7) and using the final value theorem, it is possible to find the final value of the output period as $TO_{4\infty} = \lim\,[(z-1)\cdot TO_4(z)]$, when $z \to 1$. The result is shown in eq. (12). This is for the first time, relating Time FLLs and PLLs presented in [1–10], that the output period in the stable state of a FLL equals the input period TI without any condition. This fact enables vast possibilities in the usage of this model of FLL in time digital filtering applications, as well as in other FLL applications. It can be seen in [1–10] that we are allowed to use the system parameters only in the regain, which provides a stable system choice of the system parameters. The usage of this model is not limited, and it can be devoted to improving the performance of other systems,

$$\tau_{4\infty} = TI(b_1 + b_2 + b_3 + b_4), \tag{10}$$

$$b_1 + b_2 + b_3 + b_4 = 1, \tag{11}$$

$$TO_{4\infty} = TI. \tag{12}$$

Note that eq. (11) is not the condition for system stability. It is only the condition that enables $\tau_{4\infty}$ to equal TI in the stable state of FLL_4. We will see later that $\tau_{4\infty}$ can reach any value, but the system will still be functional, because TO_4 will always equal TI. To proof that, let us now simulate the functioning of FLL_4 in the time domain to show the practical meaning and benefits of the mentioned property. At the same time, the simulation will verify the accuracy of the mathematical results. All discrete values in simulations were merged to form continuous curves. All variables in the following diagram were presented in time units. The time unit can be, μsec, msec or any other, but assuming the same time units for all time variables TI, TO and τ, it was more suitable to use just "time unit" or abbreviated "t.u." in the text. It was more convenient to omit the indication „t.u.", in the diagrams.

The simulations of $TO(k)$ and $\tau(k)$ for the step input $TI_k = 6$ t.u, are shown in Fig. 2. All values for three cases of different parameters b_1, b_2, b_3 and b_4, are shown in Fig. 2. The initial conditions TI_0, TO_0 and τ_0 are equal for all of three cases. The system parameters $b_1 = b_2 = b_3 = b_4 = 0.25$ t.u. in Fig. 2a, satisfy eq. (11), *i.e.*, $b_1 + b_2 + b_3 + b_4 = 1$. In this case, as it was expected, the output period TO_∞ reached the input periods $TI = 6$ t.u. when FLL_4 is in the stable state. At the same time, according to eq. (10), the output $\tau_\infty = TI$, proving the correctness of eqs. (10), (11) and (12). The system parameters $b_1 = b_2 = b_3 = b_4 = 0.2$ t.u. in Fig. 2b, do not satisfy eq. (11), since $b_1 + b_2 + b_3 + b_4 = 0.8$ t.u. Despite that, the output period TO reached the input periods $TI = 6$ t.u. when FLL_4 is in the stable state. At the same time, according to eq. (11), $\tau_\infty = TI(b_1+b_2+b_3+b_4) = 6\cdot(0.2+0.2+0.2+0.2) = 4.8$ t.u. This result agrees with the simulated τ_∞, shown in Fig. 2b. The system parameters $b_1=b_2=b_3=b_4=0.3$ t.u. in Fig. 2c, also do not satisfy eq. (11), since $b_1+b_2+b_3+b_4=1.2$ t.u. In spite of that, the output period TO reached the input periods TI=6 t.u. when FLL_4 comes to the stable state. According to eq. (11), $\tau_\infty = TI(b_1+b_2+b_3+b_4) = 6\cdot(0.3+0.3+0.3+0.3) = 7.2$t.u. This result also agrees with the simulated τ_∞, shown in Fig. 2c. These simulation results prove the correctness of the mathematical description and step analysis of FLL_4. FLL_4 takes four steps to reach the stable state. FLL_4 takes only one step to reach the stable state, looking from the discrete time when all parameters b_1, b_2, b_3, and b_4 are taken in the calculation.

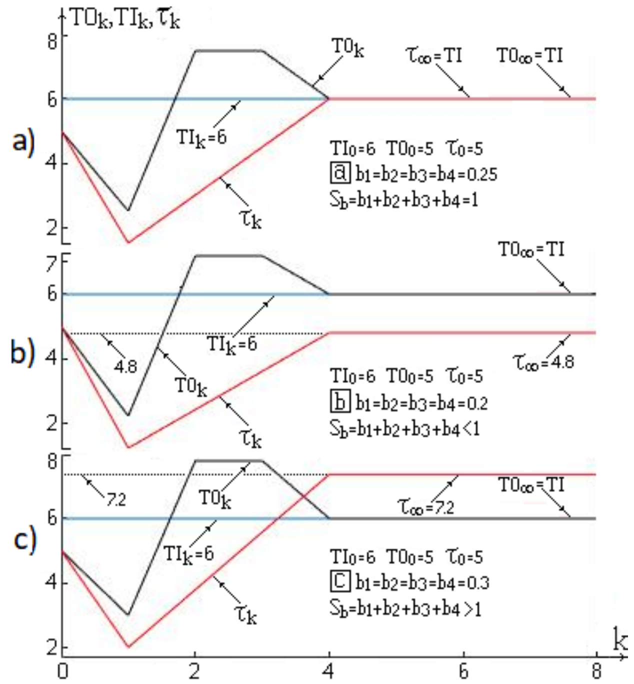

Fig. 2 – The output TO_∞ reaches TI in four steps. The system is stable regardless of the values of FLL_4 parameters: **a**. τ_∞=TI (Sum of parameters b, S_b=1), **b**. τ_∞<TI (S_b<1) and **c**. τ_∞>TI (S_b>1).

4. DEVELOPMENT OF THE TIME FIR DIGITAL FILTER BASED ON FLL_4

Since we have mathematically proven and demonstrated through simulation that the FLL_4 model is an unconditionally stable system, we can modify its parameters without altering its functionality by adjusting the coefficients of a digital filter. Unlike [1–4], where we changed the system parameters in the algorithm for the output period TO_k, in this example, we will do the same, but in the algorithm for the time difference τ_k. So the procedure is similar, only we will replace the role of the output period TO_k with the time difference τ_k. Accordingly, in this case, the filtered TI_k will appear inside the time differences τ_k instead of TO_k.

Using its derived transfer functions for FIR FLL_4, let's now demonstrate the entire process of developing the fourth-order FIR FLL_4 digital filter based on the time differences between the input and output periods. The next step is to define vectors b_4, according to the MATLAB rules for definitions of vector "**b**". Based on the transfer function $H_{\tau4}(z)$, shown in eq. (8), the vector $b_{\tau4}$ is determined and shown in eq. (13). In the same way, based on the transfer function $H_{TO4}(z)$, shown in eq. (9), the vector $\mathbf{b}_{TO4}$ is determined and shown in eq. (14).

$$b_{\tau4} = [0 \quad b_1 \quad b_2 \quad b_3 \quad b_4], \tag{13}$$

$$b_{TO4} = [(b_1+1) \quad (b_2-b_1) \quad (b_3-b_2) \quad (b_4-b_3) \quad (-b_4)], \tag{14}$$

As it was described in [3,4], we will use the theory of FIR digital filter and the corresponding its MATLAB application software to develop FIR FLL_4 digital filter. To do that we will replace the system parameters of FIR FLL_4 with the digital filter coefficients. According to refs. [3,4], the order of the digital filter, whose coefficients are to be used instead of the parameters of the FIR FLL_4, must be one order lower than the order of the IIR FLL_4. That is the FIR digital filter of the third order DF_3, whose transfer function is shown in eq. (15). The corresponding vector b_{DF3}, is shown in eq. (16). Assigning the suffix "d" to the digital filter coefficients signifies that they belong to the digital filter DF_3. Let us now design a low-pass digital filter of the third order FIR DF_3, defined by the cutoff frequency fg=2500 Hz and sampling frequency fs=28000 Hz. If we choose triangle windowing, using the MATLAB command "fir1", we can get the vector "b_d" of the filter coefficients as b_d= fir1 (N, fn, triang (N+1)), where N=3 and the normalized cutoff frequency fn = fg/(fs/2). This command gives the next coefficients for FIR digital filters: b_{0d}=0.1152, b_{1d}=0.3848, b_{2d}=0.3848 and b_{3d}=0.1152. If we use any other kind of windowing, supported by MATLAB, the coefficients would not be the same. If we compare the transfer functions $H_{\tau4}$ and H_{DF3}, we will note that both of them consist of four parameters or coefficients. We will adopt the calculated coefficients instead of the parameters and use them in $H_{\tau4}(z)$ in a way that $b_1=b_{0d}$, $b_2=b_{1d}$, $b_3=b_{2d}$ and $b_4=b_{3d}$. If we enter the proposed parameters into eq. (8), $H_{\tau4}(z)$ changes into eq 17). The obtained values of the coefficients satisfy eq. (11), which means that for these coefficient values, $\tau_{4\infty}=TI$, according to eq. (10). Based on eqs. (15) and (17), the relation between the transfer functions $H_{\tau4}(z)$ and $H_{DF4}(z)$ is shown in eq. (18). Replacing the parameters with the coefficients in eq. (13), $b_{\tau4}$ will turn into eq. (19). Replacing the system parameters with the coefficients in eq. (14), we get new vector b_{TO4} of the transfer function H_{TO4}, shown in eq. (20).

$$H_{DF3}(z) = \frac{z^3 b_{0d} + z^2 b_{1d} + z b_{2d} + b_{3d}}{z^3}, \tag{15}$$

$$b_{DF3} = [b_{0d} \quad b_{1d} \quad b_{2d} \quad b_{3d}], \tag{16}$$

$$H_{\tau4}(z) = \frac{z^3 b_{0d} + z^2 b_{1d} + z b_{2d} + b_{3d}}{z^3} z^{-1}, \tag{17}$$

$$H_{\tau4}(z) = H_{DF3}(z)\cdot z^{-1}, \tag{18}$$

$$b_{\tau4} = [0 \quad b_{0d} \quad b_{1d} \quad b_{2d} \quad b_{3d}] = [0 \quad b_{DF3}], \tag{19}$$

$$b_{TO4} = [(b_{0d}+1) \quad (b_{1d}-b_{0d}) \quad (b_{2d}-b_{1d}) \quad (b_{3d}-b_{2d}) \quad (-b_{3d})]. \tag{20}$$

5. PRESENTATION OF THE FUNCTIONING OF FLL_4 IN THE TIME AND FREQUENCY DOMAIN

To determine the frequency responses of H_{DF3} and $H_{\tau4}$, we need vectors b_{DF3} and $b_{\tau 4}$, which are defined in eqs. (16) and (19). Based on these vectors and using Matlab commands freqz ($b_{\tau 4}$, 1024, *fs*) and freqz (b_{DF3}, 1024, *fs*), the frequency responses of FIR FLL_4 and FIR DF_3, are determined and presented in Fig. 3 for half of the sample rate. It can be seen that the magnitudes of the FIR DF_3 and FIR FLL_4 are identical. Since both FIR FLL_4 and FIR DF_3 are the FIR digital filters, their phases are linear. According to eq. (18), the ratio $H_{\tau 4}(z)=H_{DF}(z)_3 \cdot z^{-1}$ means that FIR FLL_4 will introduce an additional delay of -2π [rad] on the output signal in comparison to the phase that the digital filter makes on its output signal. Note that if we consider only half of the sample rate, this delay will be $-\pi$ [rad]. It can be seen in Fig. 3 that the phases of the two systems introduced into the output signals differ by an expected 180°, for half of the sample rate. This result demonstrates that the adaptation of the fourth-order FLL_4, designed to function as a third-order FIR digital filter, has been successfully achieved.

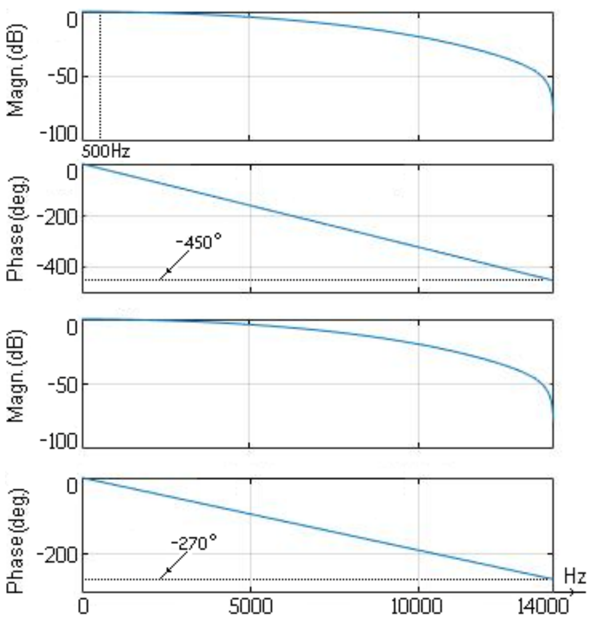

Fig. 3 – Magnitudes and phases of the frequency responses of $H_{\tau 4}(z)$ and $H_{DF3}(z)$.

Let us suppose that the input period TI_k is defined as $TI_k=6+S_1(k)+S_2(k)$[t.u.], where $S_1(k)=5\cdot\sin[2\pi/fs\cdot f_1\cdot k]$ and $S_2(k)=5\cdot\sin[2\pi/f_s\cdot f_2\cdot k]$. Suppose that the values of frequencies are f_1=500 Hz and f_2=13000 Hz. Note that the frequency f_1 is less than the cutoff frequency f_g=2500 Hz and the frequency f_2 is greater than f_g. The first step in this presentation is to form a vector TI of 28000 values of TI_k, using the above equation for TI_k. Based on the vector TI, the output vector τ = filter ($b_{\tau 4}$, 1, *TI*) is determined. This vector was also formed in simulation based on eq. (1). After that, using the "fft" command, the input and output vectors of FIR FLL_4 are formed as *X* = fft (*TI*) and *Y* = *fft* (τ). Finally, using the command "stem", stem (abs (X)) and stem (abs (Y)), the spectrums of the input TI and output τ are presented in Fig. 4. These spectrums present the absolute values of the amplitudes, covering the whole sample rate. They appear as positive values in the symmetric second half of the sample rate. It is visible in Fig. 4 that signal S_1 at 500 Hz is not attenuated, since f_1 is less than the cutoff frequency f_g=2500 Hz. This agrees with the magnitude of the FIR FLL_4 frequency response, shown in Fig. 3, since at f_1=500 Hz, the attenuation is zero. At the same time, signal S_2 at 13000 Hz is suppressed in Fig. 4, because f_2=13000 Hz is greater than the cutoff frequency f_g.

Let us now present the described processing in the time domain, in Fig. 5. All signals in Fig. 5 are generated by the simulation of the supposed input TI_k and the output τ_{k+4}, given by eq. (1). All signals are presented in 112 steps. The initial conditions in Fig. 5 are τ_0=0 t.u. and $TI_0=TI$=6 t.u.

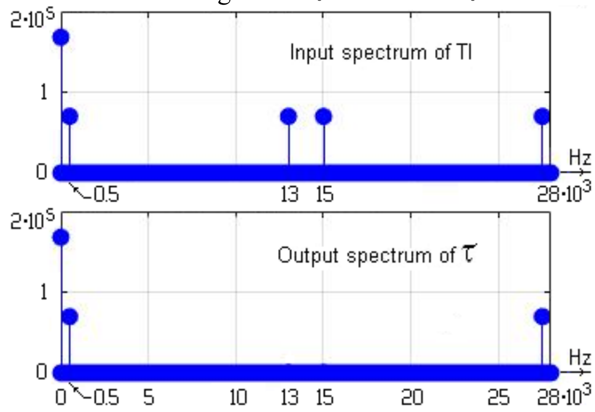

Fig. 4 – The component of S_2 signal exists in the spectrum of TI, but it is eliminated from the spectrum of τ.

Signal S_{1k} is presented in Fig. 5a. Since the frequency of S_{1k} is f_1=500 Hz and the sampling frequency *fs*=28000 Hz, it means that signal S_{1k} is sampled 28000/500=56 times per period. Signal S_{2k} is presented in Fig. 5b. Since the frequency

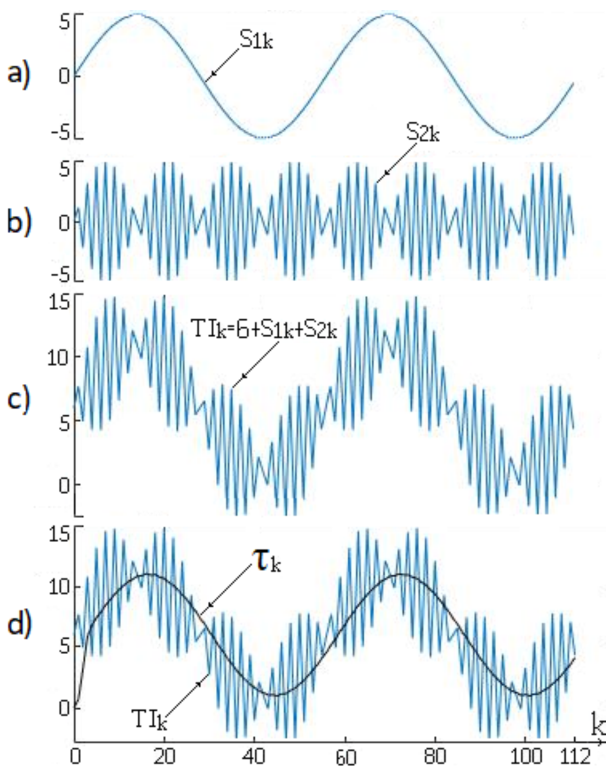

Fig. 5 – The simulation of the input and output signals of FIR FLL_4, using supposed TI_k and τ_k given by eq. (1).

of S_{2k} is f_2=13000 Hz, it means that signal S_{2k} is sampled only 28000/13000=2.15 times per period. Due to that, signal S_{2k} is highly deformed. Note that if the number of samples per period is equal to or less than 2, the sampled signal will not appear in the spectrum. The input TI_k, as the sum of 6 t.u, S_{1k}, and S_{2k}, is presented in Fig. 5c. Figure 5d shows TI_k and τ_k. Signal τ_k is almost identical to S_{1k}, while signal S_{2k} is eliminated. This is in agreement with Fig. 4, where we can see that, in the output spectrum of τ_k, the component of 13000 Hz belonging to S_{2k} has been eliminated. The identical results of the simulations in the time domain, shown in Fig. 5, with the results of the analysis in the frequency domain, shown in Figs. 3 and 4 are proof that the entire previous analysis of FIR FLL_4 is correct.

One of the essential features of the described FLL_4 algorithm is the ability to track rapid changes in the direction coefficient of the input signal. We can see in Fig. 6 how the output period TO_k tracks the input period TI_k, which is the sum of signals S_1 and S_2 raised to a level of 6 t.u. Only the first 14 steps of the enlarged TO_k and TI_k signals are shown in Fig. 6. Even though the signal S_2 is needle-shaped and rapidly changes the direction coefficient, we can see that the output period very quickly reduces the difference between the initial conditions TO_0 and TI_0 to zero error. After four steps, the output period TO_k tracks TI_k with a negligible error that is generated when the direction coefficient of TI_k changes by almost 180 degrees.

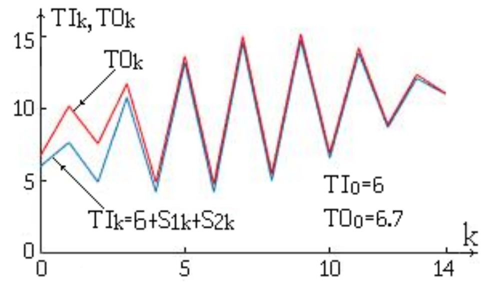

Fig. 6 – Despite the rapid changes in the direction coefficient of the input signal and the initial difference, TO_k tracks TI_k almost without error after four steps.

Let us now demonstrate why the described algorithm is essential for the realization of time-domain FIR digital filters. If we want to realize, for instance, a high-pass digital filter based on FLL_{11} (FLL of eleventh order), we should use the coefficients of digital filter DF_{10} instead of the parameters of FLL_{11}. Let us first design a tenth-order FIR digital filter, DF_{10}, with a cutoff frequency of fg = 10000 Hz and a sampling frequency *of* fs = 28000 Hz. Using the MATLAB command "fir1", we can get the vector "b_d" of the filter coefficients as b_d=fir1(N, fn, 'high'), where N=10 and the normalized cutoff frequency $fn=fg/(fs/2)$.

This command gives the next coefficients for FIR DF_{10} digital filter: b_{0d}=0.0051, b_{1d}= -0.0060, b_{2d}=-0.0190, b_{3d}=0.1095, b_{4d}= -0.2349, b_{5d}=0.2956 b_{6d}=-0.2349 b_{7d}=0.1095 b_{8d}=-0.0190 b_{9d}=-0.0060 and b_{10d}=0.0051. Note that the sum of all coefficients gives 0.0053. This sum is not equal to one, as in the case of FLL_4, eq. (11). For all FIR FLLs in refs. [1 to 10], the algorithms were defined by the output TO_k. We have seen that for the systems to be functional, the sum of the parameters "b" must be equal to one. It turns out that we are unable to adapt FLL_{11} to function as a FIR high-pass digital filter if the algorithm is defined by the output period TO_k. Instead, in TO_k, the digital filter algorithm is incorporated into $_k$, as previously described for FLL_4. Due to the unconditional stability of that model, we can use the same described approach but for FLL_{11} instead of FLL_4. If we adopt the presented coefficients of the DF_{10} digital filter instead of the FLL_{11} parameters, we can, in the same way as for FLL_4, present the input and output spectra using the same input signal TI_k. These spectra are presented in Fig. 7. We can see that the zero component and the component of signal S_1 are eliminated from the output spectrum, because their frequencies are less than fg=10000 Hz. In contrast, the component of signal S_2 kept the same value as in the spectrum of the input signal, due the fact that the frequency of S_2 is higher than fg.

It can be concluded that FLL_{11}, which is implemented in the same way as FLL_4, functions as a Time-FIR high-pass digital filter, even though the sum of its parameters is not equal to unity.

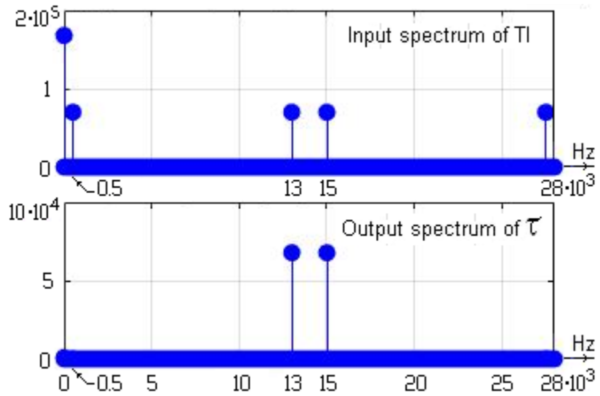

Fig. 7 – The component of S_1 is eliminated from the spectrum of τ. The component of S_2 signal kept the same value in both spectra.

6. **CONCLUSION**

As a continuation of the development of Time-FIR digital filters based on the processing of periods, refs. [3, 4], this is the first article in the literature that illustrates how the time differences between the input and output periods can be used to filter the period of the input pulse signal. In the previous articles describing this type of frequency-locked loop, it was indicated that these systems have three outputs that contain information about the input signal. It was also shown that if the basic algorithm of such systems is expressed within the output period, then the sum of all parameters of such a system must be equal to unity for the system to be stable. However, the sum of coefficients "b" in FIR classical digital filters, whose coefficients are used in this implementation, is not always equal to unity. In such cases, the described systems cannot be used as time-digital filters if the digital filter algorithm is built into the output period. However, this article demonstrates that the defect can be overcome by implementing the function of a classical digital filter in the time difference, rather than in the output period. Thanks to this approach, any classical digital filter can be used for the realization of the corresponding Time-FIR digital filter.

In addition, it is worth pointing out that this approach uses discrete time defined by the pulses of an input signal, which makes it possible to define an unconditionally stable FLL. Compared to the previously described FLLs, this FLL allows for complete freedom in choosing the system parameters, thereby facilitating the solution of a broader set of conflicting technical requirements.

At the same time, this article illustrates how great the possibilities are in the application of the systems based on the processing of the periods and time differences of the impulse signals. One of the possible aims in this area could be to develop a unique FLL system that simultaneously filters the input signal in two ways on its separate outputs, using two different types of digital filter algorithms.

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