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GENERALIZATION OF THE TIME INFINITE IMPULSE RESPONSE DIGITAL FILTERS

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Keywords: Frequency locked loop (FLL); Digital filter; Phase locked loop (PLL); Digital circuit; Discrete linear system.

This work describes the generalization of a new kind of infinite impulse response (IIR) digital filter to filter the pulse signal periods. This kind of digital filter was designed using the previously designed frequency-locked loops (FLL), which are based on the time measurement and processing of both the input and output periods. FLL is a linear discrete system. Starting from the general form of difference equation of the IIR FLL digital filter of the third and fourth orders, the transfer functions and Z transform of the outputs are developed for the IIR FLL digital filter of any order. To demonstrate the capabilities and utility of the general equations, they were applied to design a suitable IIR digital filter using a fourth-order FLL. The filtering abilities and the analyses in the frequency domain of the designed low pass IIR digital filter are demonstrated using the theory of IIR digital filter and the corresponding MATLAB tools. Analyses of the fourth-order IIR FLL digital filter were also performed in the time domain using computer simulation in MATLAB.

1. INTRODUCTION

In the title of this paper, the term "Time Infinite Impulse Response digital filter" with the prefix "Time" was used for the first time. The term "Time digital filter" should include IIR FLL digital filters, described in [1], as well as FIR FLL digital filters, described in [2,3]. The author has long evaluated whether using the prefix "Period" instead of "Time" is more correct, considering that pulse signal periods are processed in these systems. However, in addition to periods, time differences between the input and output periods can also be processed in these systems. Therefore, these "Time digital filters" always process time as a physical quantity. Because of that, the prefix "Time" best represents the essence of the physical process, including periods and time differences between periods. Unlike time digital filters, classical digital filters process only the signal's amplitude. Using the same principle, we can call them "Amplitude digital filters". This approach fits well with the need to make one of the essential differences between "Time digital filters" and the classic digital filters.

These time digital filters are derived from the frequency-locked loops (FLLs), which are based on measuring and processing the input and output periods of the pulse signals. In [1], it was shown how a third-order FLL can function as a time IIR digital filter. The procedure described for finding the transfer function of the third-order FLL and other necessary mathematical procedures was very long. Of course, these procedures will be very complicated for the higher-order FLLs. To reduce and simplify the process of designing the IIR FLL digital filter, it is necessary to develop all the necessary equations for the IIR FLL of the *M*-th order, that is, of any order. In addition, this article will demonstrate the application of these general mathematical solutions to the development of a fourth-order IIR FLL digital filter.

What [2] represents FIR FLL digital filters, and this article represents IIR FLL digital filters. However, unlike FIR FLL digital filters, which process only the input periods, IIR FLL digital filters process both the input and output periods. Because of that, IIR FLL digital filters are the systems with the feedback. They possess better filter characteristics than FIR FLL digital filters, which are open-loop systems. It was previously stated that the classic IIR digital filters process the amplitude, unlike IIR FLL and FIR FLL digital filters, which process the periods, *i.e.*, time instead of amplitude. Regardless, the theory of the classical IIR digital filters and its application software were used to develop IIR FLL digital filters. To achieve this, it was necessary to overcome the differences in the basic equations between classical digital filters and IIR FLL digital filters, which are theoretically and practically demonstrated in the article.

Numerous applications of FLLs are described in [4–11]. These references are also important for this article because they describe the functioning and realization of IIR FLL digital filters, their computer simulation in the time domain, and their analysis using the Z transformation and the theory of linear discrete systems. The articles and books in [12–26] provide a theoretical base for electronics implementations and development necessities.

2. GENERAL EQUATIONS OF THE IIR FLL_M

Figure 1 represents a general case of an input signal Sin and an output signal Sop of IIR FLL and shows the physical relations between the input and output variables. This article

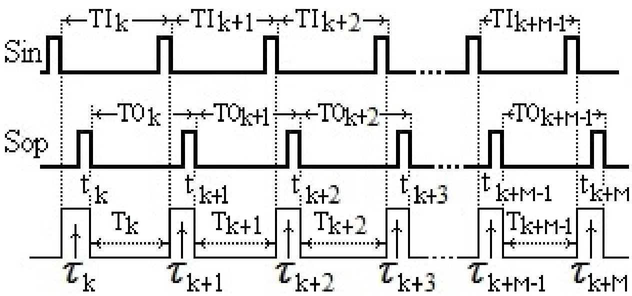

Fig. 1 – The time relations between the input and output variables of the *M*-th order IIR FLL digital filter.

will rely on some results from [1]. Let us borrow the difference equation for IIR FLL_M of the *M*-th order from eq. (4) of [1]. Suppose we replace "$M=M-1$" in eq. (4) of [1], we will get eq. (1) in this article, all variables can be seen in Fig. 1. By this replacement, we reduced the number of calculation steps for one and obtained the simpler form, which is more suitable for the upcoming analysis without changing its mathematical meaning. As shown in [1], eq. (1) of this article represents an adapted form of IIR FLL_M that can function as a digital filter when its parameters are replaced with the coefficients of the corresponding

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classical digital filter. Equation (2) is the natural relation between the variables in Fig. 1. The periods' TI_k and TO_k, as well as the time difference τ_k and time interval T_k occur at discrete times t_k, t_{k+1}, $t_{k+2},\ldots,t_{k+M-1}$, t_{k+M}, defined by the falling edges of the pulses of Sop in Fig. 1. Note that the variable "k", represents the discrete time t_k when an input period is measured and taken in the calculation, according to eq. (1), there are M calculations of any output period. These calculations are performed with M system parameters b_1, b_2... b_M, M system parameters a_0, a_1, a_2... a_{M-1}, and the corresponding input and output periods TI_k and TO_k. Note that it is adopted a_0 = 1 in eq. (1). The number M represents the order of an IIR FLL_M. The calculation of M starts at discrete time $t = t_k$, just like in Fig. 1, where "k" is usually zero, but it can be any natural number. The variable τ_k will identify the phase and time relation between the input and output periods during both the locking procedure and the stable state of IIR FLL. Because of simplicity, discrete times in brackets, *e.g.*, $TO(t_{k+M})$ and $TI(t_{k+M-i})$ were changed with the corresponding index marks like TO_{k+M} and TI_{k+M-i} in eq. (1). The same changes are made in Fig. 1 and the other equations. According to Fig. 1, if we know τ_k, T_k can be calculated using eq. (3).

$$TO_{k+M}=\sum_{i=1}^{M} b_i \cdot TI_{k+M-i} + \sum_{i=1}^{M-1} a_i \cdot TO_{k+M-i}, \quad (1)$$

$$\tau_{k+1}=\tau_k + TO_k - TI_k, \quad (2)$$

$$T_k = TI_k - \tau_k, \quad (3)$$

To perform the analyses of an IIR FLL_M it is necessary to determine the Z transforms of TO_{k+M}, τ_{k+1} and T_k as well as their transfer functions. The Z transforms of eq. (1) to (3) could be derived in two ways. The first way is to develop it directly from eq. (1) to (3). The Z transform of the M-th order IIR FLL_M can also be performed from the Z transforms of multiple lower-order IIR FLLs. We will apply the second approach, which is simpler. Let us first derive the Z-transforms of TO_{k+M} and τ_{k+1}. For the IIR FLL_3 of the third order, $TO_3(z)$ and $\tau_3(z)$ is derived in [3]. They are shown in eq. (4) and (5), where TO_0 and τ_0 are the initial conditions of TO_3 and τ_3. Similarly, $TO_3(z)$ and $\tau_3(z)$ were developed in ref. [3] for the third-order IIR FLL_3, the Z transforms $TO_4(z)$ and $\tau_4(z)$ was developed for the fourth-order IIR FLL_4 and shown in eq. (6) and (7). Note that $S_{ab}=a_1+b_1+a_2+b_2$ in eq. (7). Based on eq. (4) and (6), we can derive the Z-transform $TO_M(z)$ for M-th order IIR FLL_M, given in eq. (8), where the transfer function $H_{TO_M}(z)=TO(z)/TI(z)$ is shown in eq. (9). Based on eq. (5) and (7), we can derive the Z-transform $\tau_M(z)$ for M-th order IIR FLL_M, given in eq. (10), where the transfer function $H_{\tau_M}(z)=\tau_M(z)/TI(z)$ is presented in eq. (11). According to eq. (3), $T_M(z)=TI(z)–\tau_M(z)$. $T_M(z)$ can be calculated after we have derived $\tau_M(z)$. Since $\tau_M(z)= H_{\tau_M}(z)\cdot TI(z)$, it follows that $T_M(z)=TI(z)[1–H_{\tau_M}(z)]$. Based on the last equation, the transfer function $H_{T_M}(z)=T_M(z)/TI(z)$ can be calculated using eq. (12).

$$TO_3(z) = TI(z)\frac{z^2 b_1 + z b_2 + b_3}{z^3 - z^2 a_1 - z a_2} + \frac{z^3 TO_0}{z^3 - z^2 a_1 - z a_2}, \quad (4)$$

$$\tau_3(z) = TI(z)\frac{-z^2 - z(1-a_1-b_1) - b_3}{z^3 - z^2 a_1 - z a_2} + \frac{z^3 TO_0}{(z-1)(z^3 - z^2 a_1 - z a_2)} + \frac{z\tau_0}{z-1}, \quad (5)$$

$$TO_4(z) = TI(z)\frac{z^3 b_1 + z^2 b_2 + z b_3 + b_4}{z^4 - z^3 a_1 - z^2 a_2 - z a_3} + \frac{z^4 TO_0}{z^4 - z^3 a_1 - z^2 a_2 - z a_3}, \quad (6)$$

$$\tau_4(z) = TI(z)\frac{-z^3 - z^2(1-b_1-a_1) - z(1-S_{ab}) - b_4}{z^4 - z^3 a_1 - z^2 a_2 - z a_3} + \frac{z^4 TO_0}{(z-1)(z^4 - z^3 a_1 - z^2 a_2 - z a_3)} + \frac{z\tau_0}{z-1}, \quad (7)$$

$$TO_M(z) = TI(z)\cdot H_{TO_M}(z) + \frac{z^M \cdot TO_0}{z^M - \sum_{i=1}^{M-1} z^{M-i}\cdot a_i}, \quad (8)$$

$$H_{TO_M}(z)=\frac{\sum_{i=1}^{M} z^{M-i}\cdot b_i}{z^M - \sum_{i=1}^{M-1} z^{M-i}\cdot a_i}, \quad (9)$$

$$\tau_M(z) = TI(z)H_{\tau_M}(z) + \frac{z^M \cdot TO_0}{(z-1)(z^M - \sum_{i=1}^{M-1} z^{M-i}\cdot a_i)} + \frac{z\tau_0}{z-1}, \quad (10)$$

$$H_{\tau_M}(z) = \frac{-z^{M-1} - \sum_{i=1}^{M-2} z^{M-1-i}[1 - \sum_{j=1}^{i}(b_j + a_j)]}{z^M - \sum_{i=1}^{M-1} z^{M-i}\cdot a_i} - \frac{b_M}{z^M - \sum_{i=1}^{M-1} z^{M-i}\cdot a_i}, \quad (11)$$

$$H_{T_M}(z) = \frac{T_M(z)}{TI(z)} = 1 - H_{\tau_M}(z), \quad (12)$$

All general eq. (8) to (12) are very useful because, using them, we can easily derive Z transforms of the outputs and transfer functions of any order IIR FLLM, escaping long mathematical operations and significantly reducing the possibility of making errors. Let's check the correctness of the previous equations if we adopt M=3 from eq. (8) and (10), we will get eq. (4) and (5). Suppose we adopt M=4, from eq. (8) and (10), we will get eq. (6) and (7), proving the correctness of the generalized eq. (8), (9), (10) and (11).

Except for the general eq. (1), (2), (3), and (8 to 12), it is necessary to derive some additional general equations for the IIR FLL_M of the M-th order. Using eq. (8) and (10) we can find the final values $TO_{M\infty}$ and $\tau_{M\infty}$ in the stable state of IIR FLL_M, *i.e.*, for the case when $k\rightarrow\infty$. Suppose the step input is $TI(k)=TI$=const. By substituting the Z-transform of $TI(k)$, *i.e.*, $TI(z)=TI\cdot z/(z–1)$ into eq. (8), and using the final value theorem, it is possible to find the final value of the output period as $TO_{M\infty}$=lim [$(z–1)TO_M(z)$] when $z\rightarrow 1$. The result is shown in eq. (13). It comes out from eq. (13), that $TO_{M\infty}=TI$ if eq. (14) is satisfied. Equation (14) represents the general condition that the parameters of IIR FLL_M must satisfy. It is substituting now $TI(z)$ into eq. (10), and using the final value theorem, it is possible to find the final value of the time difference $\tau_{M\infty}$=lim $\tau_3(k)$ if $k\to\infty$, using $\tau_M(z)$. This is shown in eq. (15).

Let us now demonstrate the application of the general eq. (13), (14), and (15) to develop the corresponding equations for IIR FLL_4. If we enter M=4 into eq. (13), we get eq. (16). If we enter M=4 into eq. (14), we get the general condition that the parameters of IIR FLL_4 must satisfy, eq. (17). At last, If we enter M=4 into eq. (15), the final value of the time difference $\tau_{4\infty}$, which IIR FLL_4 reaches in the stable state, is derived and shown in eq. (18).

$$TO_{M\infty} = \lim\,[(z-1)TO_M(z)]_{z\to 1} = \mathrm{TI}\,\frac{\sum_{i=1}^{M} b_i}{1-\sum_{i=1}^{M-1} a_i}, \tag{13}$$

$$\sum_{i=1}^{M} b_i + \sum_{i=1}^{M-1} a_i = 1, \tag{14}$$

$$\tau_{M\infty} = TI\,\frac{-1-\sum_{i=1}^{M-2}\left[1-\sum_{j=1}^{i}(b_j+a_j)\right]-b_M}{1-\sum_{i=1}^{M-1} a_i} + \frac{TO_0}{1-\sum_{i=1}^{M-1} a_i} + \tau_0, \tag{15}$$

$$TO_{4\infty} = TI\,\frac{b_1+b_2+b_3+b_4}{1-a_1-a_2-a_3}, \tag{16}$$

$$b_1+b_2+b_3+b_4+a_1+a_2+a_3 = 1, \tag{17}$$

$$\tau_{4\infty} = TI\,\frac{-3+2(b_1+a_1)+(b_2+a_2)-b_4}{1-a_1-a_2-a_3} + \frac{TO_0}{1-a_1-a_2-a_3} + \tau_0. \tag{18}$$

3. DEVELOPMENT OF THE TIME IIR FLL4 DIGITAL FILTER USING GENERAL EQUATIONS

With the developed general equations, we can approach the analysis or development of a wide range of IIR FLL_M applications due to three types of outputs TO_k, τ_k and T_k.

In the following text, we will emphasize the design and analysis of the filter characteristics of IIR FLL_4 using output TO_4 and compare it with the corresponding digital filter. Let's demonstrate the entire process of developing the fourth-order IIR FLL4 digital filter using the general equations. If we enter M=4 in eq. (9), we will get the Z transform of the transfer function H_{TO_4}(z) for IIR FLL4, shown in eq. (19). The next step is to define vectors b_{TO_4} and a_{TO_4} according to the Mat-lab rules for definitions of vectors "**b**" and "**a**". The vectors b_{TO_4} and a_{TO_4} are determined based on the transfer function H_{TO_4}(z) and shown in eq. (20).

$$H_{TO4}(\mathrm{z}) = \frac{TO_4(z)}{TI(z)} = \frac{z^3 b_1 + z^2 b_2 + z b_3 + b_4}{z^4 - z^3 a_1 - z^2 a_2 - z a_3}, \tag{19}$$

$$b_{TO_4} = [0 \quad b_1 \quad b_2 \quad b_3 \quad b_4] \qquad a_{TO_4} = [1 \quad -a_1 \quad -a_2 \quad -a_3]. \tag{20}$$

As described in [1], we will use the theory of the IIR digital filter and its corresponding MATLAB application software to develop a time IIR FLL_4 digital filter. The procedure consists of simply replacing the system parameters of IIR FLL_4 with the digital filter coefficients; according to [1], the order of the classic digital filter, whose coefficients are to be used instead of the parameters of the IIR FLL_4, must be for one order lower than the order of the IIR FLL_4. That is the digital filter of the third order IIR DF_3, whose transfer function is shown in eq. (21). The corresponding vector subscript base and a_{DF_3} are shown in eq. (22). Note that it is adopted a_{0d} = 1. Let us now design the Butterworth low pass digital filter of the third order IIR DF_3, defined by the cutoff frequency f_g = 2 000 Hz and sampling frequency fs=10 000 Hz. Using MATLAB command [b_{DF_3}, a_{DF_3}] = butter (N, f_n), where N=3 is the order of the filter and $f_n=f_g/(f_s/2)$, we can get vectors b_{DF_3}= [0.0985 0.2956 0.2956 0.0985] and a_{DF_3}= [1 –0.5772 0.4218 –0.0563]. Comparing eq. (19) and (21) to replace the parameters with the coefficients, we can see that b_1= b_{0d} = 0.0985, b_2 = b_{1d} = 0.2956, b_3 = b_{2d} = 0.2956, b_4= b_{3d}= 0.0985, a_1= –a_{1d} = 0.5772, a_2 = –a_{2d} = –0.4218 and a_3= –a_{3d} = 0.0563. The obtained values of the coefficients satisfy eq. (17). It follows that, according to eq. (16), $TO_{4\infty}=TI$ for these values of coefficients. Replacing the parameters with the coefficients in eq. (19), the transfer function H_{TO4}(z) will turn into eq. (23). According to eq. (21) and (23), the relation between the transfer functions $H_{TO_4}(z)$ and H_{DF_3} is shown in eq. (24). Replacing the parameters with the coefficients in eq. (20), b_{TO4} and a_{TO4} will turn into eq. (25). Based on the results obtained, we can define the relation between any order transfer function of time IIR FLL_M and the transfer function of t classic digital filter, whose coefficients are used as parameters of IIR FLL_M. If the digital filter is of (M–1) order, IIR FLL must be of M-th order. The relation of their transfer functions is presented in eq. (26). The second important conclusion relates to the vectors of the transfer functions respectively H_{TO_M} of M-th order and $H_{DF_{M-1}}$ of (M–1)-th order. Their vectors are used in commands devoted to the design of digital filters. Their relations are shown in eq. (27).

$$H_{DF_3}(\mathrm{z}) = \frac{z^3 b_{0d} + z^2 b_{1d} + z b_{2d} + b_{3d}}{z^3 + z^2 a_{1d} + z a_{2d} + a_{3d}}, \tag{21}$$

$$b_{DF_3} = [\,b_{0d} \quad b_{1d} \quad b_{2d} \quad b_{3d}] \qquad a_{DF_3} = [1 \quad a_{1d} \quad a_{2d} \quad a_{3d}], \tag{22}$$

$$H_{TO_4}(\mathrm{z}) = \frac{z^3 b_{0d} + z^2 b_{1d} + z b_{2d} + b_{3d}}{z^3 + z^2 a_{1d} + z a_{2d} + a_{3d}}\, z^{-1}, \tag{23}$$

$$H_{TO_4}(z) = H_{DF_3}(z)\cdot z^{-1}, \tag{24}$$

$$b_{TO_4} = [0 \quad b_{0d} \quad b_{1d} \quad b_{2d} \quad b_{3d}] = [0 \quad b_{DF_3}] \qquad a_{TO_4} = [1 \quad a_{1d} \quad a_{2d} \quad a_{3d}] = a_{DF_3}, \tag{25}$$

$$H_{TO_M}(z) = H_{DF_{M-1}}(z)\cdot z^{-1}, \tag{26}$$

$$b_{TO_M} = [0 \quad b_{DF_{M-1}}],\ a_{TO_M} = a_{DF_{M-1}}. \tag{27}$$

4. PRESENTATION OF THE FUNCTIONING OF IIR FLL4 IN THE TIME AND FREQUENCY DOMAIN

The simulations in the time domain can confirm all reached math results, realized by MATLAB tools. Using simulation, let us check the correctness and compliance between some general equations and general equations and simulation based on the described algorithm for the designed IIR FLL_4 filter. All discrete values in simulations were merged to form continuous curves. All variables in the following diagram were presented in time units. The time unit can be μsec, msec or any other, but assuming the same time units for all time variables *TI*, *TO* and τ, using just "time unit" or abbreviated "t.u." in the text was more suitable. It was more convenient to omit the indication "t.u.", in the diagrams. Entering M=4 in eq. (1) we will get TO_{k+4}, eq. (28). Using eq. (28) and (2), TO_{k+4} and τ_{k+4} are simulated for TI=6 t.u., TO_0=5 t.u., and τ_0=5 t.u. and shown in Fig. 2. The system parameters used are b_1= 0.0985, b_2= 0.2956, b_3= 0.2956, b_4= 0.0985, a_1 = 0.5772, a_2 = –0.4218 and a_3 = 0.0563. It can be seen in Fig. 2 that $TO_{4\infty}=TI$. This agrees with eq. (16), since the system parameters of IIR FLL_4 satisfy eq. (17). If we calculate $\tau_{4\infty}$ by entering the system parameters, *TI*, TO_0, and τ_0 into eq. (18), we get $\tau_{4\infty}$= –2.915 t.u. The same value is obtained from the computer listing of the simulated $\tau_{4\infty}$= –2.915 t.u., shown in Fig. 2. Since $\tau_{4\infty}$ is the last in the sequence of math derivations, matching the simulated $\tau_{4\infty}$ with the calculated $\tau_{4\infty}$, confirms that both the previous math and the simulation are correct.

$$TO_{k+4} = b_1TI_{k+3} + b_2TI_{k+2} + b_3TI_{k+1} + b_4TI_k + a_1TO_{k+3} + a_2TO_{k+2} + a_3TO_{k+1}, \tag{28}$$

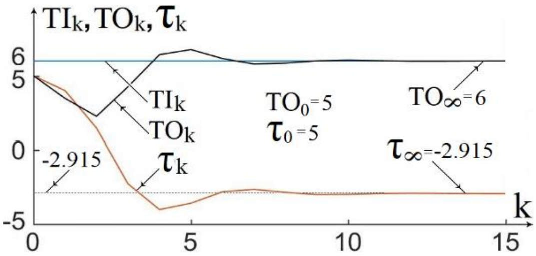

Fig. 2 –Transition and stable states of IIR FLL_4 for the designed system parameters, *TI*, and the initial conditions TO_0 and τ_0.

To determine the frequency responses of H_{TO_4} and H_{DF_3}, we need vectors b_{TO_4}, a_{TO_4}, b_{DF_3} and a_{DF_3}, which are defined in eq. (22) and (25). Based on these vectors and using MATLAB commands freqz (b_{TO_4}, a_{TO_4}, 1024, f_s) and freqz (b_{DF_3}, a_{DF_3}, 1024, *fs*), IIR FLL4 and IRR DF3 frequency responses are determined and presented in Fig. 3 for half of the sample rate. The magnitudes of the IIR DF3 and IIR FLL_4 are identical. Since IIR FLL4 and IIR DF3 are the IIR digital filters, none of their phases is linear. According to eq. (24), the ratio H_{TO_4}(z)= H_{DF_3}(z)·z–1 means that IIR FLL_4 will introduce an additional output signal delay of –2π rad compared with the phase the digital filter makes on its output signal. Note that if we consider only half of the sample rate, this delay will be –π rad. It can be seen in Fig. 3 that the phases, which two systems introduced into the output signals, differ for expected –180°, for half of the sample rate. This result proves that the adaptation of the fourth-order FLL_4 to function as a third-order IIR digital filter has been successfully realized.

To demonstrate the filtering characteristics of the Butterworth low pass IIR FLL_4 digital filter, let us suppose that the input period TI_k is defined as TI_k=6+$S_1(k)$ +$S_2(k)$ t.u., where $S_1(k)$=5·sin(2π/*fs*·f_1·*k*) and $S_2(k)$= 5·sin(2π/*fs*·f_2·*k*). The input periods are modulated by two sinusoidal signals S_1 and S_2. Suppose that the values of frequencies are f_1=500 Hz and f_2=4 000 Hz. Note that f1 is less than the cutoff frequency f_g=2 000 Hz, and f_2 is more significant than f_g. The first step in this presentation is to form a vector **TI** of 10 000 values of TI_k,

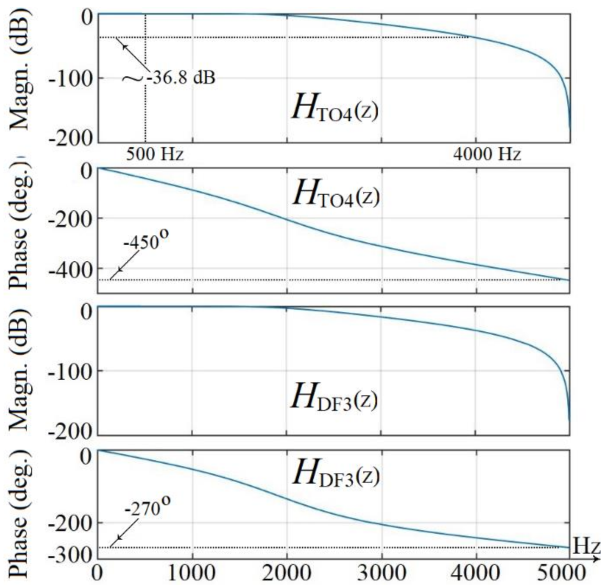

Fig. 3 – Magnitudes and phases of the frequency responses of HTO4(z) and HDF3(z).

using the above equation for TI_k. The output period vector **TO** = filter (**bTO**, **aTO**, **TI**) is determined based on the vector TI. This vector was also formed in simulation based on eq. (28). After that, using the "fft" command, the input and output vectors of IIR FLL_4 are formed as X = fft (*TI*) and Y = fft (*TO*). Finally, using the command "stem", stem (abs (X)) and stem (abs (Y)), the spectrums of the input and output periods are presented in Fig. 4. These spectrums present the absolute values of the amplitudes, covering the whole sample rate. They appear as positive values in the symmetric second half of the sample rate. It is visible in Fig. 4 that signal S_1 at 500 Hz is not attenuated since f_1 is less than the cutoff frequency f_g = 2 000 Hz. This agrees with the magnitude of the IIR FLL_4 frequency response shown in Fig. 3, since at f_1=500 Hz, the attenuation is zero. At the same time, signal S_2 at 4 000 Hz is suppressed for about –36.8 dB in Fig. 3 because f_2 = 4 000 Hz is greater than the cutoff frequency f_g. It can be seen in Fig. 4 that the zero component at the frequency close to zero is not attenuated, which is also in agreement with the magnitude of IIR FLL_4, shown in Fig. 3. A complete description regarding the zero component is presented in [2,3].

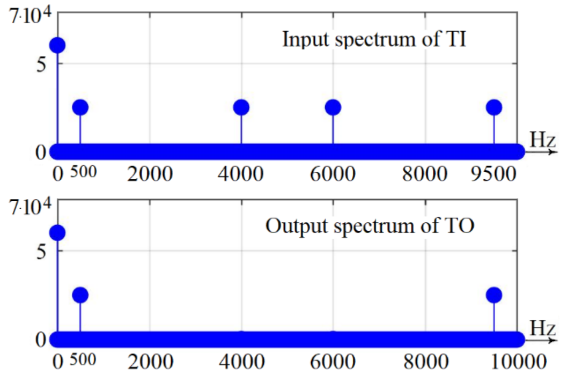

Fig. 4 – The input spectrum of TI and the output spectrum of TO.

To gain further insight into the physical process of IIR FLL_4, we will now present the inputs and outputs of IIR FLL_4 in the time domain, which are shown in Fig. 5. All signals in Fig. 5 are generated by the simulation of the supposed input TI_k and the output TO_{k+4}, given by eq. (28). All signals are presented in 60 steps. The initial conditions in Fig. 5 are TO_0=0 t.u., τ_0=0 t.u. and TI_0=TI=6 t.u. Signal S_{1k} is presented in Fig. 5a. Since the frequency of S_{1k} is f_1=500 Hz and the sampling frequency fs = 10 000 Hz, signal S_{1k} is sampled 10000/500 = 20 times per period. Figure 5a shows almost no deformation of the sinusoidal signal due to the large number of samples per period. However, if this image is enlarged, the deformation can be observed, but only in the part of the signal where its change in discrete time is minimal, *i.e.*, in its maximum and minimum values. Signal S_{2k} is presented in Fig. 5b. Since the frequency of S_{2k} is f_2 = 4 000 Hz, signal S_{2k} is sampled 10 000/4 000 = 2.5 times per period. Both S_{1k} and S_{2k} in Figs. 5a and 5b are deformed sinusoidal signals. However, the number of samples per period of S_{2k} is significantly smaller, so the S_{2k} signal is highly deformed

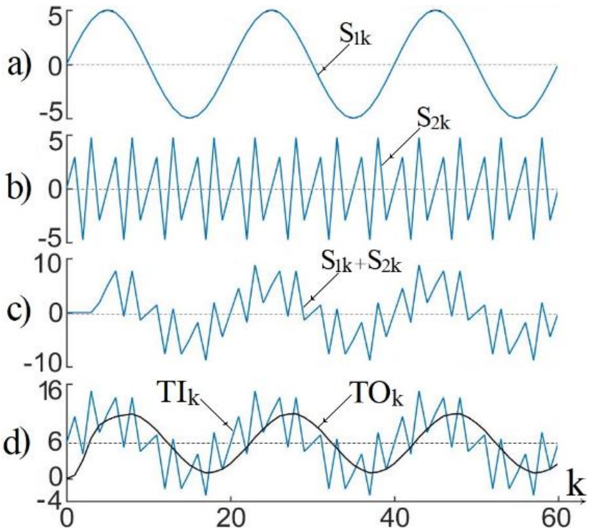

Fig. 5 – The simulation of the input and output signals of IIR FLL$_4$, using supposed TI_k and TO_{k+4} given by eq. (28).

into needle-like shapes, which create a wider range of higher-frequency components in the frequency domain. The sum of S_{1k} and S_{2k} is shown in Fig. 5c. The input TI_k, as the sum of 6 t.u, S_{1k}, and S_{2k}, is presented together with TO_k in Fig. 5d. Figure 5d shows that the IIR FLL$_4$ generates TO_k, which is almost identical with S_{1k}, while S_{2k} signal is eliminated. This agrees with Fig. 4, where we can see that, in the output spectrum of TO_{k+4}, the component of 4 000 Hz belonging to S_{2k} has completely disappeared. The identical results of the simulations in the time domain, shown in Fig. 5, with the analysis results in the frequency domain, prove that the entire Z transform mathematical analysis of IIR FLL$_4$ and analyses in the frequency domain are correct.

5. **CONCLUSIONS**

This article, which continues [1], describes the basic theory and development approach to new time IIR digital filters to filter pulse signal periods.

This is the first article in the literature describing the general development approach to time IIR FLL$_M$ digital filter of any order, using its general equations. The article describes the methodology, procedures, math, simulation support, analysis in time and frequency domains, and all necessary general equations that develop any order IIR FLL$_M$ digital filter. Due to these general equations for IIR FLL$_M$ of any order, the procedure for their development is enormously shorted and practically reduced to the development of the classical digital filters. If we did not use the general equations, developing the necessary equations for a higher-order IIR FLLM digital filter would be impossible without making an error due to the extensive mathematical operations.

This article, like [2] in the domain of time FIR FLL digital filters, opened wide possibilities for using IIR FLL digital filters in electronics, telecommunications, control, and measurements, which use different forms of periodic and non-periodic pulse signals. There is an obvious need to filter them in some applications.

Due to the complexity of the presented material, some additional efforts have been made to connect all segments of different presentations and analyses into a logical whole, such as mathematics, simulation, time presentations of signals, frequency responses of the transfer functions, and frequency presentations of signals for IIR FLL of the fourth order. This helps not only to prove the correctness of all the material presented but also to enable the understanding of the described physical process and to facilitate the revision simultaneously.

Compared with FIR FLL$_M$ digital filters, IIR FLL$_M$ digital filters of the same order require more calculations. Because of that, the realization of any IIR FLL$_M$ digital filter would not be possible without a microprocessor, which is necessary to perform numerous calculations.

The results of this article represent the basis for further possible development of time IIR FLL$_M$ digital filters. These results simultaneously enable the frequency analysis of all types of IIR FLLs and thus enable their more comprehensive application. However, the most likely and useful next step is to deepen the theory of time IIR FLL$_M$ digital filters or discover further methods to facilitate and shorten their development and analysis. All the novelties reached in the next steps will also apply to the classical digital filters and all discrete linear systems.

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