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NEW KIND OF INFINITE IMPULSE RESPONSE DIGITAL FILTERS INTENDED FOR PULSE PERIOD FILTERING

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Key words: Frequency locked loop; Digital filter; Phase locked loop; Digital circuit; Discrete linear system.

This paper describes a new kind of infinite impulse response (IIR) digital filter designed for pulse period filtering. The IIR digital filter is designed using a third-order IIR frequency locked loop (IIR FLL), which is based on the time measurement and processing of both, the input and output periods. A general form of the difference equation describing this type of IIR FLL of any order is developed and compared with the corresponding difference equation of classical digital filters. The mathematical analyses in time domain were performed using the Z transform approach and theory of linear discrete systems. An analysis of IIR digital filter was performed in time and frequency domain. The transfer functions and Z transform of the third-order IIR FLL outputs are developed. The main part of the article is devoted to design the appropriate IIR FLL digital filter using the corresponding IIR FLL. For this purpose, the theory of IIR digital filter and the corresponding MATLAB tools are used, but taking into account the differences of these systems. Filtering abilities of the designed IIR FLL digital filter are demonstrated. Computer simulation of the designed IIR FLL is made in the time domain to enable precise insight into its properties.

1. INTRODUCTION

As stated [1,2], by processing the periods of the pulse signal and the time differences between the input and output periods, new types of PLLs and FLLs can be created, which have new properties compared to classical PLLs and to FLLs. In [3–10] the numerous applications of such systems were demonstrated. One of the new applications, which is very interesting for many fields in which electronics are used, is the digital filtering of the period of the pulse signal, demonstrated in [1,2]. In these papers, FLLs process only the input periods. Although FLLs and digital filters are completely different systems, it has been shown that FLL systems have a lot of similarities in the mathematical sense with classic FIR (Finite Impulse Response) digital filters, in which only analog samples of the input signal are processed. FLLs which function as FIR digital filters, we rightly called FIR FLL digital filters. It was also proven that the complete theory of classical FIR digital filters, as well as the relevant MATLAB application software, can be used to develop FIR FLL digital filters. Of course, in this development, the differences between these systems must be taken into account. It was shown how to develop a FIR FLL digital filter and how to correctly interpret the physical phenomena obtained in its analyzes using the MATLAB software, intended for FIR digital filters.

In this paper, we will describe how the theory of classic IIR (Infinite Impulse Response) digital filters can be used to design IIR FLL digital filters, intended for filtering an impulse signal period. The term IIR in classic digital filters means that samples of both the input and output signals are used in the processing. Accordingly, IIR FLL systems process both the input and output periods. This paper, also describes a development methodology for IIR FLL digital filters of any order, using the theory of IIR digital filters and the appropriate MATLAB tools.

Refrences [3–10] are also of fundamental importance for this paper. In addition to the description of various applications of FLL and PLL systems, they describe the methodology of their analysis and the way of realization of these systems, what will be also used in the analysis and realization of IIR FLL digital filters. The articles and books in [11–25] are only used as theoretical base, for electronics implementations and the development necessities.

2. GENERAL DIFFERENCE EQUATION OF IIR FLL

The procedure in realizing an IIR FLL digital filter using a classic digital filter, consists in replacing the parameter of an IIR FLL digital filter with the coefficients of an already designed IIR digital filter. To do that, these systems must have the transfer functions of the same order, with the same number of identical parameters and coefficients. The FLL IIR digital filter is successfully realized only if, after this replacement, the magnitudes of the frequency responses of these two systems are identical. In order to achieve this task, let's first consider the difference equation of the Mth-order IIR digital filter, eq. (1). The sum of products of (M+1) filter coefficients $a_{d0},a_{d1},a_{d2},...,a_{dM}$ and the corresponding samples of the output signal y(k-i) is equal to the sum of the products of (M+1) filter coefficients b_{d0}, b_{d1}, $b_{d2},...,b_{dM}$ and the corresponding samples of the input signal x(k-i). The suffix "d" to the coefficients, indicates that they belong to digital filters. Note that the variable "k", represents the discrete time t_k when an amplitude of the input signal is sampled, measured and taken in calculation. Since it is accepted $a_{d0}=1$, eq. (1) can be transformed into eq. (2), which is structurally similar to the forms of the FLL difference equations, which are used [1–12]. The

$$\sum_{i=0}^{M} a_{d_i} \cdot y_{k-i} = \sum_{i=0}^{M} b_{d_i} \cdot x_{k-i}, \tag{1}$$

$$y_k = \sum_{i=0}^{M} b_{d_i} \cdot x_{k-i} - \sum_{i=1}^{M} a_{d_i} \cdot y_{k-i}, \tag{2}$$

general case of an input signal Sin and an output signal Sop of the M-th order FLL is shown in Fig. 1. It shows the physical relations between the variables. The periods TI_k and TO_k, as well as the time difference τ_k, occur at discrete times t_k, t_{k+1}, t_{k+2},…t_{k+M-1}, t_{k+M}, defined by the falling edges of the pulses of Sop in Fig. 1. Note that the variable "k", represents the discrete time t_k when an input period is measured and taken in calculation. The difference equation for M-th order FLL, corresponding to Fig. 1, is shown in eq. (3). According to eq. (3), there are "M+1" system parameters a_0, a_1, a_2,..., a_M and "M" system parameters b_1,

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b_2,... b_M. Note that the assumed a_0=1 in Eq. (3), just like a_{d0}=1 in digital filters, in Eq. (2). The start of the "M" calculation starts at a discrete time, just like in Fig. 1. The beginning of "M" calculations starts at discrete time $t = t_k$, just like in Fig. 1.

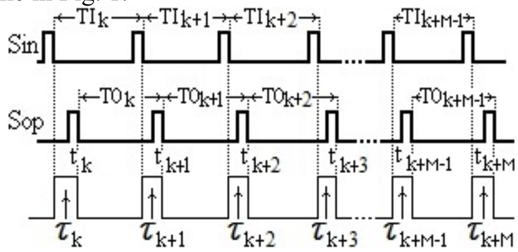

Fig. 1 – The time relations between the input and output variables of the *M*-th order FLL.

$$TO_{k+M} = \sum_{i=1}^{M} b_i \cdot TI_{k+M-i} + \sum_{i=1}^{M} a_i \cdot TO_{k+M-i}. \quad (3)$$

The number of parameters "b" of the M-th-order FLL, in eq. (3), is "M" and the number of coefficients "b_d" of the digital filter of the M-th order, in eq. (2), is equal "M+1". Obviously we have to choose the (M+1)-th order FLL to be able to adopt the digital filter coefficients instead of the FLL "b" parameters. On the other hand, the number of parameters "a_d" and "a" in eqs. (2) and (3) is identical, i.e., "M+1". When we increase the order of FLL by one, *i.e*,. on (M+1)-th order, the number of parameters "a" will be one more than the number of coefficients "a_d". Therefore, if we want the number of parameters "a_d" and "a" to be identical, we must give up the parameter "a_{M+1}". Increasing the order of FLL from M-th order to (M+1)-th order and adopting a_{M+1}=0 was done by modifying eq. (3) into eq. (4). Taking into account that the parameter a_0=1 and coefficient a_{d0}=1,

$$TO_{k+M+1} = \sum_{i=1}^{M+1} b_i \cdot TI_{k+M+1-i} + \sum_{i=1}^{M} a_i \cdot TO_{k+M+1-i}. \quad (4)$$

the number of parameters “b” and “a” in eq. (4) and the number of coefficients “b_d” and “a_d” in eq. (2), are identical. That number is "M+1". Now it is possible to use all the calculated coefficients of the digital filter, given by eq. (2), instead of the parameters of the FLL difference equation, given by eq. (4). So, eq. (4) represents the general form of the (M+1)-th order FLL when the FLL is to function as an IIR FLL digital filter. But, the order of the digital filter, whose coefficients are to be used instead of the parameters of the FLL, must be of the M-th order, *i.e.*, for one order lower than the order of the IIR FLL.

3. EXAMPLE OF AN IIR FLL FILTER DESIGN USING THEORY OF CLASSIC DIGITAL FILTER

Let us now demonstrate the entire process of developing an IIR FLL digital filter. In order to make the description clearer, we will choose the lower order of IIR FLL and digital filter. If we choose the third order IIR FLL_3 than, according to the previous conclusion, digital filter should be of the second-order. From the general eq. (2) of the digital filter, the difference equation of the second-order filter can be easily obtained, if we adopt M=2 and replace k=k+2 in eq. (2). Equation (2) will turn to eq. (5). In a similar way, for M+1=3, from Eq. (4) can be obtained the difference equation of IIR FLL_3, shown in eq. (6). Note that these two equations are structurally similar. They describe systems of the same order and they have the same number of coefficients, *i.e.*, parameters. For further analysis, we also need eq. (7). Equation (7) comes out as natural relation between the variables in Fig. 1. The variable τ_k will serve to identify the phase relation, as well as the time relation between the input and output periods, during both the locking procedure and the stable state of a FIR FLL_3. In the first step, we need to find the Z transforms of the transfer functions of the described IIR FLL_3 using eqs. (6) and (7). The Z transforms of eqs. (6) and (7) are presented in eqs. (8) and (9). In eqs. (8) and (9), TO_0, TI_0 and τ_0 are the initial conditions of the variables TO_k, TI_k and τ_k. Based on eq. (6), for k=-2, $TO_1=b_1TI_0+a_1TO_0$ and for k=-1, $TO_2= b_1TI_1+b_2TI_0+a_1TO_1+a_2TO_0$. Using the given expressions and eq. (8), TO(z) is calculated and shown in eq. (10), where $R(z)=z^3TO_0/(z^3-z^2a_1-za_2)$. It is now of interest to investigate under which conditions this IIR FLL_3 is the stable system. To do that, let us suppose that the step input is TI(k) = TI = constant. Substituting the Z transform of TI(k), *i.e.*, TI(z) = TI·z/(z-1) into eq. (10) and using the final value theorem, it is possible to find the final value of the output period TO_∞, which IIR FLL_3 reaches in the stable state. We can calculate TO_∞ = lim TO(k) if k→∞, using TO(z). This is shown in eq. (11). It comes out from eq. (11), that TO_∞=TI if eq. (12) is satisfied. Changing TO(z) given by eq. (10) into eq. (9), τ(z) is calculated and shown in eq. (13), where $S_{ab}= b_2+b_1+a_2+a_1-1$. Based on eqs. (10) and (13), we can define two transfer functions $H_{TO}(z)$ and $H\tau(z)= \tau(z)/TI(z)$, shown in eqs. (14) and (15). Finally, based on eq. (5), we can express the transfer function of the second-order digital filter, shown in eq. (16).

$$\begin{aligned} y_{k+2} &= bd_0 x_{k+2} + bd_1 x_{k+1} + bd_2 x_k - \\ &- ad_1 y_{k+1} - ad_2 y_k , \end{aligned} \quad (5)$$

$$\begin{aligned} TO_{k+3} &= b_1 TI_{k+2} + b_2 TI_{k+1} + b_3 TI_k + \\ &+ a_1 TO_{k+2} + a_2 TO_{k+1}, \end{aligned} \quad (6)$$

$$\tau_{k+1} = \tau_k + TO_k - TI_k , \quad (7)$$

$$\begin{aligned} &z^3TO(z) - z^3TO_0 - z^2TO_1 - zTO_2 = b_1[z^2TI(z) + \\ &-z^2TI_0 - zTI_1] + b_2[zTI(z) - zTI_0] + b_3TI(z) + \\ &+a_1[z^2TO(z) - z^2TO_0 - zTO_1] + a_2[zTO(z) - zTO_0], \end{aligned} \quad (8)$$

$$z\tau(z) - z\tau_0 = \tau(z) + TO(z) - TI(z), \quad (9)$$

$$TO(z) = TI(z)\frac{z^2b_1 + zb_2 + b_3}{z^3 - z^2a_1 - za_2} + R(z), \quad (10)$$

$$TO_\infty = \lim[(z-1)TO(z)]_{z\to 1} = TI\frac{b_1 + b_2 + b_3}{1 - a_1 - a_2}, \quad (11)$$

$$b_1 + b_2 + b_3 + a_1 + a_2 = 1, \quad (12)$$

$$\tau(z) = TI(z)\frac{-z^2 + z(a_1 + b_1 - 1) + S_{ab}}{z^3 - z^2a_1 - za_2} + \frac{R(z) + z\tau_0}{z-1}, \quad (13)$$

$$H_{TO}(z) = \frac{TO(z)}{TI(z)} = \frac{z^2b_1 + zb_2 + b_3}{z^2 - za_1 - a_2} \cdot z^{-1}, \quad (14)$$

$$H_\tau(z) = \frac{-z^2 + z(a_1 + b_1 - 1) + b_2 + b_1 + a_2 + a_1 - 1}{z^3 - z^2a_1 - za_2}, \quad (15)$$

$$H_{DF}(z) = \frac{y(z)}{x(z)} = \frac{z^2bd_0 + zbd_1 + bd_2}{z^2 + zad_1 + ad_2}. \quad (16)$$

Equation (14) is structurally the same like eq. (16) and the next step is to simply change the parameters of FLL_3 in eq. (14) with the corresponding coefficients of the digital filter, shown in eq. (16). This will give $b_1= bd_0$, $b_2= bd_1$, $b_3= bd_2$, $a_1= -ad_1$ and $a_2= -ad_2$. After this substitution, eq. (14) transforms into eq. (17). The transfer functions $H_{DF}(z)$ and $H_{TO}(z)$, given by eqs. (16) and (17), cover the same zeros and poles, but the difference between them is in their denominators. Namely, their ratio can be expressed as $H_{TO}(z)= H_{DF}(z).z^{-1}$. This means that the magnitudes of the frequency responses of $H_{TO}(z)$ and $H_{DF}(z)$ will be the same. But due to one step delay, which refers to factor "z^{-1}", IIR FLL_3 will introduce an additional delay of -2π [rad] on the output signal, in relation to the phase which the digital filter makes on its output signal. Note that if we consider only half of the sample rate, this delay will be $-\pi$ [rad]. Based on the MATLAB rules for definitions of vector "b" and "a" of the IIR digital filters, we can define vectors b_{DF} and b_{TO}, as well as vectors a_{DF} and a_{TO}, using the transfer functions $H_{DF}(z)$ and $H_{TO}(z)$, given by eqs. (16) and (17). The corresponding vectors b_{DF}, b_{TO}, a_{DF} and a_{TO} are shown in eqs. (18), (19), and (20). If we change $b_1=b_{0d}$, $b_2=b_{1d}$, $b_3=b_{2d}$, $a_1=-a_{1d}$ and $a_2=-a_{2d}$ in eq. (15), we can determine vectors b_τ and a_τ, which are shown in eqs. (21) and (22). All of vectors are necessary for the frequency analyses of the described IIR FLL_3 and the digital filter, using MATLAB tools intended for the IIR digital filters.

$$H_{TO}(z) = \frac{TO(z)}{TI(z)} = \frac{z^2 bd_0 + z bd_1 + bd_2}{z^2 + z ad_1 + ad_2} \cdot z^{-1}, \quad (17)$$

$$b_{DF} = [bd_0 \;\; bd_1 \;\; bd_2], \quad (18)$$

$$b_{TO} = [0 \;\; bd_0 \;\; bd_1 \;\; bd_2] = [0 \;\; b_{DF}], \quad (19)$$

$$a_{DF} = a_{TO} = [1 \;\; ad_1 \;\; ad_2] \quad (20)$$

$$b_\tau = [0 \;\; -1 \;\; (-ad_1 + bd_0 - 1) \;\; (bd_1 + bd_0 - ad_2 - ad_1 - 1)], \quad (21)$$

$$a_\tau = a_{DF} = a_{TO} = [1 \;\; ad_1 \;\; ad_2]. \quad (22)$$

After we developed vectors "a" and "b", based on the transfer functions of the IIR FLL_3, the further procedure of frequency analysis of the outputs TO and τ, can be performed in a completely identical way. In the following text, we will give the emphasis to the design and analysis of the filter characteristics of IIR FLL_3 using output TO and comparing it with the corresponding digital filter. In order to design an IIR FLL_3 digital filter, we have to first design the corresponding IIR digital filter of the second order (IIR DF_2). Let us design Butterworth low pass IIR DF_2, defined by the cutoff frequency fg=2000 Hz and sampling frequency fs=10000 Hz. Using MATLAB command [b_{DF}, a_{DF}] = butter (N, f_n), where the filter order N=2 and $f_n=f_g/(f_s/2)$, we can get vectors b_{DF}=[0.2066 0.4131 0.2066] and a_{DF}=[1 -0.3695 0.1958]. Note that eq. (12) is satisfied, if we change b_1=0.2066, b_2=0.4131, b_3=0.2066, a_1=-(-0.3695) and a_2=-0.1958. This means that after changing the parameters with the coefficients of IIR DF_2, IIR FLL_3 will stay stable. In order to determine the frequency responses of H_{TO} and H_{DF}, we need vectors b_{TO}, a_{TO}, b_{DF} and a_{DF}, which are defined in eqs. (18), (19) and (20). Based on these vectors and using MATLAB commands freqz (b_{TO}, a_{TO}, 1024, fs) and freqz (b_{DF}, a_{DF}, 1024, fs), the frequency responses of IRR FLL_3 and IRR DF_2, are determined and presented in Fig. 2 for the half of the sample rate. It can seen that the magnitudes of the IIR DF_2 and IIR FLL_3 are identical. Since both of IIR FLL_3 and IIR DF_2 are the IIR digital filters, no one of their phases is linear, but for the half of the sample rate, the phase of IIR FLL_3 is -360° and the phase of IIR DF_2 is -180°. It can be seen in Fig. 2 that the phases, which two systems introduced into the output signals, differ for expected -180°, for the half of the sample rate. This proves that the adaptation of the third-order FLL, with the aim of functioning as a second-order IIR digital filter, has been successfully realized.

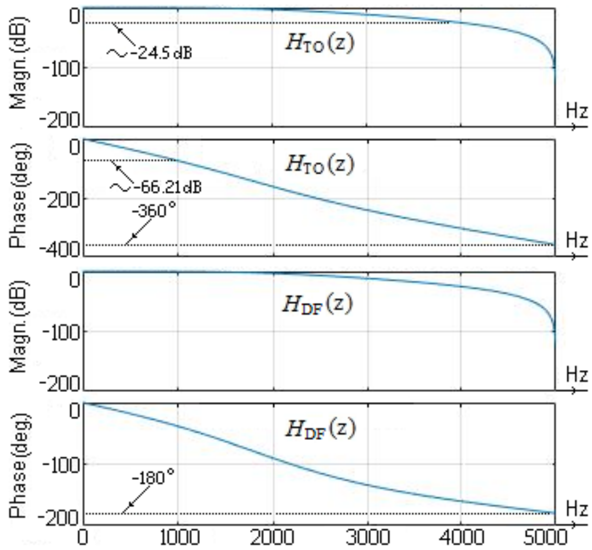

Fig. 2 – Magnitudes and phases of the frequency responses of $H_{TO}(z)$ and $H_{DF}(z)$.

Let us demonstrate the filter characteristics of the Butterworth low pass IIR FLL_3 digital filter, based on the third-order FLL. Suppose that the input period TI_{k+1} is defined as TI(k+1) = 6+S_1(k) + S_2(k) [time units], where $S_1(k)=5\cdot\sin(2\pi/fs\cdot f_1\cdot k)$ and $S_2(k)=5\cdot\sin(2\pi/f_s\cdot f_2\cdot k)$. The input periods are continuously changing under effects of two sinusoidal signals S_1 and S_2. Suppose that the values of frequencies are f_1=1000 Hz and f_2=4000 Hz. Note that the frequency f_1 is less than the cutoff frequency f_g=2000 Hz and the frequency f_2 is greater than f_g. The time unit [t.u.] can be, μsec, msec or any other, but assuming the same time units for all time variables. It was more suitable to omit [t.u.] in the diagrams. The first step in this presentation is to form vector TI of 10000 values of TI, using the above equation for TI_{k+1}. Based on the vector TI, the output period vector TO = filter (b_{TO}, a_{TO}, TI) is determined. This vector was also formed in simulations on the basis of eqs. (6) and (7). After that, using the "fft" command, the input and output vectors of IIR FLL_3 are formed as X = fft (TI) and Y = fft (TO). Finally, using the command "stem", stem (abs (X)) and stem (abs (Y)), the spectrums of the input and output periods are presented in Fig. 3. These spectrums present the absolute values of the amplitudes, covering the whole sample rate. They appear as positive values in the symmetric second half of the sample rate. It is visible in Fig. 3 that signal S_1 at 1000 Hz, is only slightly attenuated, since f_1 is less than cutoff frequency f_g=2000 Hz. This agrees with magnitude of the IIR FLL_3 frequency response shown in Fig. 2, since at f_1=1000 Hz, the attenuation is close to zero. t the same time signal S_2 at 4000 Hz is suppressed for about -24.5 dB in Fig. 2, because f_2=4000 Hz is greater than cutoff frequency f_g. It can be seen in Fig. 3, that the zero component at the frequency close to zero is not attenuated, what is also in agreement with the magnitude of IIR FLL_3, shown in Fig. 2. A more complete description regarding the zero component are presented in [1,2].

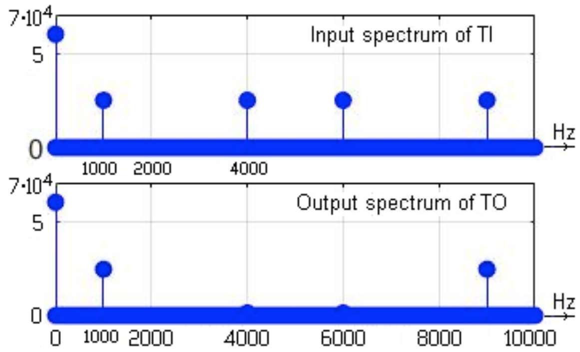

Fig. 3 – The input spectrum of TI and the output spectrum of TO.

In order to gain additional insight and understanding of the physical process of IIR FLL_3, we will now present the inputs and outputs of the IIR FLL_3 in the time domain, which is shown in Figs. 4 and 5. These presentations will also allow us to check the mutual agreements between the frequency and time analyzes and then to check the agreement of these analyzes with the frequency response of the IIR FLL_3 and with the math simulation of the IIR FLL_3 functioning, which was made using eqs. (6) and (7). Their complete compliance will be a full guarantee of the complete correctness of this entire article.

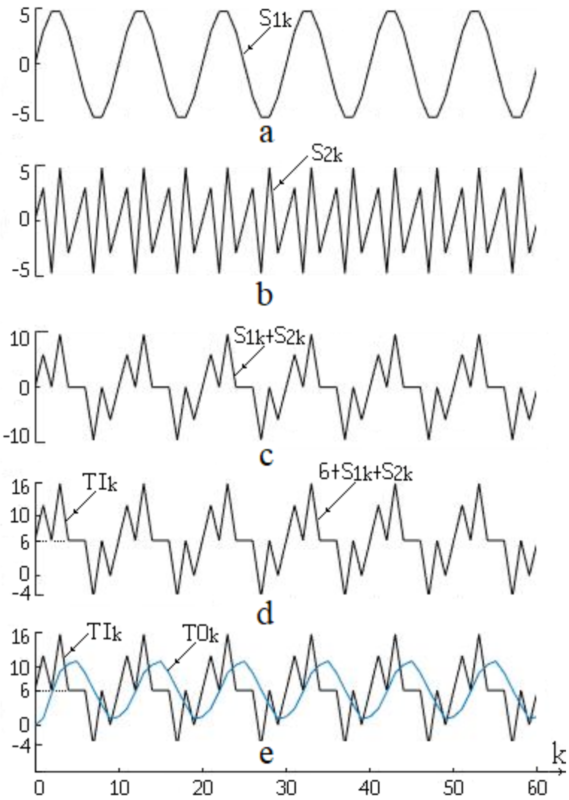

Fig. 4 – The simulation of the input and output signals of IIR FLL_3, using eq. (6).

All signals in Fig. 4 are generated by simulation of the input and output signals using eq. (6). All signals are presented in 60 steps. The initial conditions, used for all signals in Fig. 4, are TO_0=0 t.u., τ_0=0 t.u. and TI_0=6 t.u. Signal S_{1k} is presented in Fig. 4a. Since the frequency of S_{1k} is f_1=1000 Hz and the sampling frequency fs=10000 Hz, it means that signal S_{1k} is sampled 10000/1000= 10 times per period. This can be noticed in Fig. 4a. Signal S_{2k} is presented in Fig. 4b. Since the frequency of S_{2k} is f_2=4000 Hz, it means that signal S_{2k} is sampled 10000/4000=4.5 times per period. Both S_{1k} and S_{2k} in Figs. 4a and 4b are deformed sinusoidal signals. However, the number of samples per period of S_{2k} is significantly smaller, so the S_{2k} signal is highly deformed into needle-like shapes, which create a wider range of higher frequency components in the frequency domain. The sum of S_{1k} and S_{2k} is shown in Fig. 4c. The input TI_k, as the sum of 6 t.u, S_{1k} and S_{2k} is presented in Fig. 4d. At last, the input TI(k) as well as TO(k) are shown in Fig. 4e. Figure 4e shows that the IIR FLL_3 generates a slightly deformed S_{1k} signal at its output, while the S_{2k} signal is practically eliminated. This is in agreement with Fig. 3, where we can see that, in the output spectrum of TO_k, the component of 4000 Hz, belonging to S_{2k}, has almost completely disappeared. The identical results of the simulations in the time domain, shown in Fig. 4, with the results of the analysis in the frequency domain are proof, at the same time, that the entire Z transform mathematical analysis of IIR FLL_3 is correct.

It is of interest to check whether the time presentations from Fig. 4 corresponds to the magnitude and phase of the frequency response of IIR FLL_3, shown in Fig. 2. To do that it is necessary to determine from Fig. 4 how much the phase and magnitude of TIk are changed, passing through IIR FLL_3. Of course, the changes introduced by the IIR FLL_3 depend on the frequency of the input signal. Therefore, we will adopt to perform this check for the signal S_{1k}, whose frequency is f_1=1000 Hz. Let's note that the given task will be realized with quality only if there is no admixture of other signals in the signal S_{1k}. The presence of a part of the spectrum from another signal in the signal S_{1k}, will affect the overall phase and magnitude of the output signal. Therefore, in order to determine the phase and magnitude which IIR FLL_3 enters into the input signal, it is necessary to compare the original signal S_{1k} with the output signal TO_k which contains frequency of 1000 Hz. These two signals are taken from Figs. 4a and 4e, enlarged and shown in Fig. 5a. It can be seen in Fig. 5a that the signal S_{1k} which appears in the output signal TO_k is partially deformed, due to the presence of a smaller part of the spectrum of the signal S_{2k}, whose frequency is f_2 = 4000 Hz. Therefore, this signal is not suitable for accurate determination of the change in phase and magnitude of S_{1k} at the output of IIR FLL_3. A better solution is shown in Fig. 5b, which is obtained by completely eliminating the signal S_{2k} from the input of IIR FLL_3. It can be seen in Fig. 5b, that the signal which appears in the form of the output signal TOk, is identical in amplitude and shape to the signal S_{1k}, but it is phase delayed. The signal S_{1k}, as consisting part of TO_k in Fig. 5b, is also dc leveled up by 6 t.u, because this has already been done in the input signal TI_k.

Let us now calculate the phase, which IIR FLL_3 adds to S_{1k}, using Fig. 5b. The time difference between original signal S_{1k} and S_{1k} belonging to TO_k in Fig. 5b, is marked with τ_∞. The period of S_{1k} is T_{S1}. The phase difference between S_{1k} and TO_k is Ph=-(τ_∞/T_{S1})·360°. Note that, according to Fig. 1 and eq. (7), τ_k is mathematically defined as positive in case when TO_k is delayed in comparison to TI_k.

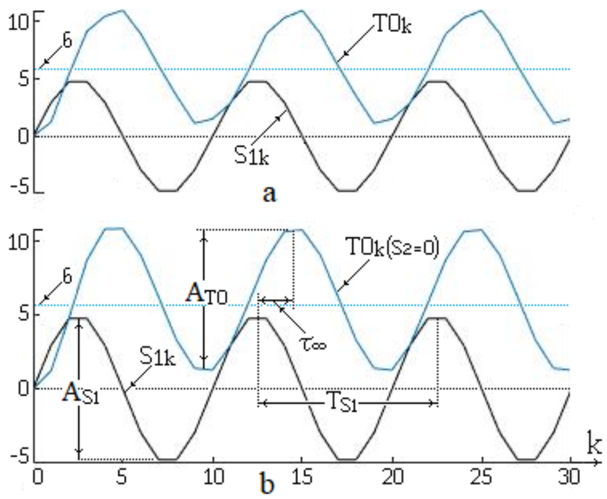

Fig. 5 – The time presentation of the input signal S_{1k} and the output signal TO_k: a. Signal S_{2k} is present in the input signal TI_k b. Signal S_{2k} is not present in the input signal TI_k.

Any positive increment of τ_k will represent the corresponding phase delay of TO_k. In definition of phase difference in MATLAB math, the phase difference is negative if an output signal is delayed in comparison to an input signal. Because of that, sign "-" is used in the above expression of Ph. If we magnify Fig. 5b, we can measure, that relation τ_∞/T_{S1}=21/114, so that Ph=-(21/114)·360° = -66.3°. Let us now determine the same phase difference by the frequency response of H_{TO}. Using the proportionality of the magnified phase of H_{TO} frequency response, we can calculate that the phase at frequency of 1000 Hz is -66.21°, as shown in Fig. 2. These two results agree each to other. At last, let us compare the magnitude of the frequency response in Fig. 2, at the frequency of 1000 Hz, with the time presentation of S_{1k}, belonging to TO_k in Fig. 5b. Namely, if we magnify Fig. 5b and measure A_{S1} and A_{TO}, it can be found that 20 log (A_{TO}/A_{S1}) = 20 log (87/88) = -0.09 dB, what is approximately close to negative zero. It is also visible in Fig. 2, that the magnitude of the frequency response at f=1000 Hz is close to 0 dB. This result agrees with the attenuation of S_{1k} at 1000 Hz, which is calculated using Fig. 5b.

4. **CONCLUSION**

Unlike [1,2] which describe the new kind of FIR digital filters based on the processing of the input periods only, this article presents the design of a new kind of an IIR digital filter, based on the processing of the input and output periods. Both of them use the theory, respectively, of the classical FIR and IIR digital filters. They are both of them intended for the filtering of impulse signal periods.

This article represents an important contribution to the theory and application of new kind of IIR FLL digital filters, based on FLL. The shown adaptation for the third order FLL to function as an IIR digital filter, using the theory of the classical IIR digital filters, can be applied to a FLL of any order.

This article opened the wide possibilities for the usage of IIR FLL digital filters widely in electronics, telecommunications, control and measurements, which use the different forms of periodic and non-periodic pulse signals. There is an obvious need to filter them in some of the applications.

The article contains a wide range of different presentations and analyzes such as mathematics, usage of the Z transform for the discrete linear system analyses, simulation, time presentation of signals, the presentations of the frequency responses of the transfer functions and the presentation of the frequency spectrums of the input and output signals. Therefore, just like in [1], it was made the corresponding effort to connect in logical whole all segments of the different presentations and analyzes. This helped, not only to proof the correctness of all presented materials, but to facilitate the understanding of the physical process described. It is also of interest to emphasize that for the realization of any IIR FLL digital filter, it is necessary to use a microprocessor to perform numerous calculations. If we respect the described principles of hardware control of FLL functioning, described in [3–10], all parts of an IIR FLL filter can be realized by a microprocessor.

However, the presented mathematical process of finding the transfer functions and their corresponding vectors can be a very long and complex procedure, especially for very high-order FLLs, which are expected to be used in filtering. Therefore, in the next step, it is necessary to develop all the necessary equations, used in this adaptation, for the FLL of any order. This will enable a short, simple and safe adaptation, which will almost be reduced to the development of a classical IIR digital filter.

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