THEORY OF CATASTROPHES REGARDING THE OPERATION OF A DC ELECTRIC MOTOR WITH SERIES EXCITATION

CRISTIAN GEORGE DRAGOMIRESCU¹, RADU MIRCEA CIUCEANU¹, MARIA-IULIANA DASCALU¹, IOSIF VASILE NEMOIANU¹

Keywords: Catastrophe theory; Dynamical systems; Discontinuities; Singularities; Equilibrium points; Sudden transitions; Control space; Phase space; Catastrophe cusp; Direct current electric motor with series excitation.

This paper brings into the spotlight the theory of catastrophes and its application for analyzing the equilibrium positions of dynamic electrical systems. A study model was adopted for a dc electric motor with series excitation operating in a transient regime. From the perspective of the theory of catastrophes, starting from the operating equations of the direct current electric motor with series excitation, the functions that describe its behavior in the phase space and the control space were obtained. Then the equilibrium points of the chosen model were determined. Also, for a clearer understanding of the results obtained, the evolution of this behavior in the phase space and the control space was highlighted graphically using MATLAB software.

1. INTRODUCTION

Starting from the daily observations, namely, that, in everything around us, we are dealing with permanent transformations of forms, starting with their generation and continuing with their development and destruction, the theory of catastrophes was initially born as a study dedicated to this issue.

At the same time, with the development of this concept, an essential interest was given to analyzing the singular points of a surface from the perspective of practical purposes. Thus, the name "catastrophe theory" appeared as a reflection of any phenomenological discontinuity. To study these discontinuities, René Thom proposed using the topological theory of dynamical systems developed by Henry Poincaré. E. C. Zeeman further developed the theory, and later Hector J. Sussmann, Raphael S. Zahler and T. Poston, and I. Stewart, respectively, continued to study this problem and its applications [1-9]. The evolution of the parameters determines the functioning of dynamic systems in the control space, whose values can cause slow changes, respectively, fast catastrophic changes, for different critical values. They correspond to bifurcation points in dynamic systems. The system thus loses its stability and suddenly jumps to another state. An edifying example of understanding these catastrophic developments is the classic "Zeeman catastrophe device".

With the development of observation and measurement tools and computing technology, new areas of applicability and specific methods of solving have been identified. As expected, many authors have approached various topics in electrical engineering utilizing the catastrophe theory [15–30] as a promising tool enabling novel and useful theoretical and practical interpretations.

In practical applications, the behavior of a dynamic system can present a continuous or discontinuous evolution. In this last situation, it turns out that the study of a surface's discontinuities (of singular points) sometimes determines a special meaning. For the analysis of the phenomena that may appear, the critical points of a function are calculated (for which the first-order differential is zero), which allows highlighting its character as a Morse function (the secondorder differential calculated at the critical points is nondegenerate). The importance of Morse functions lies in their structural stability, which means that if a disturbance is added to such a function, the function does not change in structure. In the operation of electric machines, we encounter situations characterized by an unexpected evolution of the state parameters that can lead to the catastrophic destruction of the working machine. Stability and control aspects have been addressed over time, and many analyses and proposed solutions have been put forward, such as those in the following reported contribution [31–35].

The article aims to study how the theory of catastrophes can be applied to the study of the dynamic stability corresponding to the operation of an electric motor.

2. STUDY OF SINGULARITIES AND ANALYSIS OF EQUILIBRIUM POSITIONS WITHIN THE CATASTROPHES THEORY

An important aspect of the theory of catastrophes is the types of singularities in one variable (fold catastrophe; cusp catastrophe; swallowtail catastrophe, butterfly catastrophe) and in two variables (elliptical umbilicus catastrophe; hyperbolic umbilicus catastrophe; parabolic umbilicus catastrophe), whose analysis allowed the assessment of the stability of the equilibrium positions, as well as the structural stability (universal relevances).

Thus, for elementary catastrophes, the study is performed in the phase space, whose coordinates are the variables x (or x and y) and the parameters a, b, c ... as well as in the control space, whose coordinates are a, b, c

The significance of universal relevances is, in general, a potential $V(x, a, b, c \dots)$ or $V(x, y, a, b, c \dots)$, and the study of catastrophes is performed by determining the set of equilibrium positions corresponding to this potential, which, written for a single variable, is:

$$\partial V / \partial x = 0. \tag{1}$$

Next, the set of singularities is obtained by eliminating variable *x* in the following equations:

$$\partial V / \partial x = 0, \quad \partial V^2 / \partial x^2 = 0$$
 (2)

According to the theory, a section through the surface of the phase space (Fig. 1) highlights the branch ABC and the segments AE and DB represents the sudden transitions from half branch to another.

Thus, a relation of form is obtained f(a, b, c...) = 0, which in the control space represents a surface. The existence and stability of equilibrium positions are assessed according to the position of the point coordinates (a, b, c...) from the control space as projected to the control surface [10].

¹ University POLITEHNICA of Bucharest, 313 Splaiul Independenței, 060042, Bucharest, Romania, E-mail: c.dragomirescu@upb.ro, radu.ciuceanu@upb.ro, maria.dascalu@upb.ro, iosif.nemoianu@upb.ro.



Fig. 1 - Section through the surface of the phase space.

Therefore, we can study the dynamic behavior of a system by investigating if, in its operation, there are these sudden transitions and, if so, by determining the coordinates of the equilibrium points. The impact of these transitions at the system level must also be evaluated. We can do all this starting from a system of equations that describes the dynamic functioning of the system, the search for the potential as a function, and its equilibrium points.

3. THE MODEL USED AND ITS ANALYTICAL DESCRIPTION

The model in question is that of a dc motor with series excitation. The series motors are robust and easily withstand overloads, torque shocks, and ample supply voltage drops, mainly used in electric traction and heavy electrical duty.

During the dc machine's operation, the rotor's winding sections are successively in contact with the brushes of the collector blades. When the induced electromotive voltages appear in the short-circuited sections, currents will run in these circuit segments. This commutation between two collector blades leads to the initiation of an electric arc, a phenomenon called the switching of the direct current machine. These "sparks" at the collector lead to rapid damage of the brushes and slats, a phenomenon which must be avoided. The windings of the auxiliary poles, together with the compensation windings carried by the main poles, constitute the series excitation of the machine, which has the role of compensating the magnetic reaction field of the rotor, fixing the physical neutral axis, and radically improving the switching of the dc machine. The considered a scheme without field rheostat so that the series excitation winding, arranged on the main poles of the machine, is traversed by the entire rotor current and creates the main magnetic inductor field to which it is added the reaction field from the inductor. In this way, the induced electromotive voltage is load dependent [11].

The equivalent circuit characterizes the model chosen for the in transient operation regime in Fig. 2, for which the stability of the equilibrium positions determined after the irregular change of the mechanical load is analyzed, from the perspective of catastrophe theory.



Fig. 2 – The studied model.

The notations used in the scheme of the chosen model are the following:

- *U* motor supply voltage;
- *R*_A inductor winding resistance (rotor);
- $L_{\rm A}$ inductance of the inductor winding;
- $R_{\rm E}$ excitation winding resistance;
- *L*_E excitation winding inductance;
- $L_{\text{EA}} = L_{\text{AE}}$ mutual inductance of the coupling existing between the inductor and the excitation windings.
- $i_{\rm A}(t)$ instantaneous value of the induced current;
- *i*_E(*t*) = *i*_A(*t*) instantaneous value of the excitation current (the two windings are connected in series);
- m(t) instantaneous value of the torque developed at the motor shaft;
- $m_r(t)$ instantaneous value of the torque applied to the motor shaft by the load.

The transient regime equations for the considered model are [12,13]:

$$\begin{cases} U = [R_{\rm A} + R_{\rm E} + L_{\rm EA}\Omega(t)] i_{\rm A}(t) + (L_{\rm A} + L_{\rm E}) \frac{di_{\rm A}(t)}{dt} \\ m(t) = L_{\rm EA} i_{\rm A}^2(t) , \qquad (3) \\ m(t) - m_{\rm r}(t) = J_{\rm Red} \frac{d\Omega(t)}{dt} \end{cases}$$

or

$$\begin{cases} (L_{\rm A}+L_{\rm E})\frac{di_{\rm A}(t)}{dt} = U - L_{\rm EA}\Omega(t)\,i_{\rm A}(t) - (R_{\rm A}+R_{\rm E})\,i_{\rm A}(t)\\ J_{\rm Red}\frac{d\Omega(t)}{dt} = L_{\rm EA}\,i_{\rm A}^2(t) - m_{\rm r}(t) \end{cases},\qquad(4)$$

where J_{Red} is the total moment of inertia of the reduced drive system at the motor shaft and $\Omega(t)$ angular velocity of the shaft.

Next, we will consider for simplicity reasons:

 $L = L_{\rm A} + L_{\rm E},$

$$R = R_{\rm A} + R_{\rm E},$$
 (5)
$$m_{\rm r}(t) = k \,\Omega(t),$$

where k is a proportionality constant, accounting for a

resistant torque proportional to the rotor's speed.

The system of eq. (4) will be written as:

$$L \frac{di_{\rm A}(t)}{dt} = U - L_{\rm EA} \Omega(t) i_{\rm A}(t) - R i_{\rm A}(t)$$

$$J_{\rm Red} \frac{d\Omega_{(t)}}{dt} = L_{\rm EA} i_{\rm A}^2(t) - k \Omega(t) .$$
(6)

Assuming a small perturbation field, which produces, in turn, a very small additional current $i_{1A}(t)$, equations (6) become:

$$\begin{cases} L \frac{di_{A}(t)}{dt} = U - L_{EA} \ \Omega(t) \left[i_{A}(t) + i_{1A}(t) \right] - R \ i_{A}(t) \\ J_{Red} \frac{d\Omega(t)}{dt} = L_{EA} i_{A}(t) \left[i_{A}(t) + i_{1A}(t) \right] - k \ \Omega(t) \ , \end{cases}$$
(7)

with which the equilibrium positions are rewritten as:

$$\begin{cases} 0 = U - L_{EA} \,\Omega(t) \left[i_A(t) + i_{1A}(t) \right] - R \, i_A(t) \\ 0 = L_{EA} i_A(t) \left[i_A(t) + i_{1A}(t) \right] - k \, \Omega(t). \end{cases}$$
(8)

To determine the current function $i_A(t)$, the angular velocity $\Omega(t)$ is eliminated between the equations of system (8).

We thus obtain the single equation:

$$L_{\rm EA}{}^{2}i_{\rm A}(t)\left[i_{\rm A}(t)+i_{1\rm A}(t)\right]^{2}+kR\,i_{\rm A}(t)-k\,U=0\tag{9}$$

or using notations:

$$u = i_{A}(t) \sqrt{L_{EA}},$$

$$\xi = i_{1A}(t) \sqrt{L_{EA}},$$

$$v = \frac{kR}{L_{EA}},$$

$$\gamma = k U \sqrt{\frac{1}{L_{EA}}},$$

(10)

equation (9) has the form

$$u^{3}+2\xi u^{2}+(\xi^{2}+\nu)u-\gamma=0.$$
(11)

Using the variable change

$$u = x - \frac{2}{3}\xi,$$
 (12)

equation (11) is written:

$$x^{3} - \left(\frac{1}{3}\xi^{2} - \nu\right)x - \left(\frac{2}{27}\xi^{3} + \frac{2}{3}\xi\nu + \gamma\right) = 0, \qquad (13)$$

respectively

$$x^3 - ax - b = 0, (14)$$

where:

$$a = \frac{1}{3}\xi^2 - v,$$
 (15)

$$b = \frac{2}{27} \xi^3 + \frac{2}{3} \xi v + \gamma.$$

If we derive once again the relation (14), we obtain

$$3 x^2 - a = 0. (16)$$

In according with (2), by removing variable x between equation (14) and its partial derivative (16) concerning x, the implicit equation is obtained, which is the semicubical parabola, symmetric to the axis Oa

$$4 a^3 - 27 b^2 = 0, (17)$$

that is the so-called cusp catastrophe [14].

4. NUMERICAL STUDY AND ANALYSIS OF THE RESULTS

For the numerical data proposed to be considered for our numerical example (see Table 1), it should be noted that for the actual values of the currents considered in the simulation, the magnetic cores are unsaturated so that the inductances mentioned above can be approximated as constant, regardless of current level through the two windings.

Table 1	
Operating parameters for the chosen model	
Property	Value
$U(\mathbf{V})$	220
$R_{ m A}\left(\Omega ight)$	0.8
$L_{\rm A}$ (H)	0.02
$R_{ m E}\left(\Omega ight)$	0.4
$L_{\rm E}$ (H)	0.563
$L_{\text{EA}}=L_{\text{AE}}(\text{H})$	0.0796
$J_{ m Red}(m kg{\cdot}m^2)$	4.5

$i_{\rm E}(t)=i_{\rm A}(t)$ (A)	4
Ω (rad/s)	676.07
k (J·s/rad)	0.066

Using the Matlab software, figures show the phase space determined by equation (14) and the parameter space determined by equation (17), respectively.





Among the results obtained by representation in the phase

space, the phenomenology in Fig. 3 was interpreted, in the sense of the approached theory, resulting in the graphical variations in Figs. 4 and 5.

Thus, Fig. 4 represents, for different parameter values (a = -2, a = -1, a = 1), the phase space sectioned with a vertical plane in the cuspidian surface. For these values, Fig. 5 was made in which the sections from the phase space were represented. These highlight the functional changes of the electric motor chosen for the study.







Fig. 4 – Phase space with the coordinates of the interest points space for a=-2; a=-1; a=1.



Fig. 5 – Sections in the representation of the phase space for a = -2; a = -1; a = 1.





Fig. 6 – Section details in the phase space representation for a = -2.



Fig. 7 – Section details in the phase space representation for a = -1 and a = 1.



Fig. 8 - Bifurcation in the parameter space.

According to the theory, a section through the surface of the phase space (Fig. 6), represents the sudden transitions from a half branch to another, which, when v is close to the unit, can be done for small parameter values b, respectively of the additional current $i_{1A}(t)$.

Otherwise, the manifestation of some possible phenomenon is observed by solving equation (14) and analyzing its roots following the location inside or outside the surface delimited by the bifurcation in the space of the parameters in Fig. 8:

- 3 real roots if the coordinate point (a, b) is inside a region;

- only one real root if the coordinate point (*a*, *b*) is outside this surface;

- a single real root and a real double root if the coordinate point (a, b) is located on the semi cubic parabola (cusp).

In the case of the considered application, the above observations are illustrated in Fig. 8.

5. CONCLUSIONS

Currently, the theory of catastrophes is applied in both technical and economic or social scenarios, the results being remarkable so that its introduction in the study of the operation of electrical engineering equipment can only be beneficial.

The study refers to the stability of DC motor operation under linearly increasing mechanical load. According to the catastrophe theory, equation (14) and its partial derivative concerning x, allows us to establish the set of singularities for the case of a single variable. After substituting for the variable x in the two relationships, we get an equation of form f(a,b, ...) = 0. The function f defines a surface in the control space (parameters a and b). Depending on the position of the coordinate point (a, b, ...) concerning this surface, one can determine the number of possible equilibrium positions for the considered dynamic system and the stability specific to these equilibrium positions.

Functions can be structurally stable (*e.g.*, $f(x) = x^2$) or structurally unstable (*e.g.*, $f(x) = x^3$). The latter can be transformed into structurally stable functions if polynomial terms of degree less than *n* are conveniently added.

For instance, according to the theory of catastrophes (theoretically illustrated in Fig. 1), the operation of the motor in the case of a = -2 puts in evidence the extremum points A and D, corresponding to the catastrophic transitions toward the new functioning points A' and D' respectively, as shown in Fig. 5a.

That allows us to notice a transition from the functioning point A (characterized by b = 0.55 – placed at the interior of the surface delimited by the bifurcation in the phase space of Fig. 8) to the point A' (b = 7.9 at the exterior domain described by the phase space bifurcation – Fig. 8). That means that the dynamic functionating is changing its coefficients and transforms such as from three real solutions it will remain one. Therefore, point A in the control space (determined by the parameters a = -2 and b = 0.55), located inside the domain bounded by the bifurcation, corresponds to three stable equilibrium positions in the phase space. Each may transition to a single stable equilibrium position from the phase–space corresponding to point A' in the control space (determined by the parameters a = -2 and b = 7.9), located outside the domain bounded by the bifurcation.

Analogously, for the second transition, we will have a leap from functioning point D (characterized by b = 5.54 – placed at the exterior of the surface delimited by the bifurcation in the phase space) to point D' (b = -1.9) – also placed at the exterior of the surface mentioned above), meaning that the dynamic functioning equation remains under a form exhibiting a single real-valued solution.

Comparing the results obtained for a = -1 and a = 1 with the theoretical curve shown in Fig. 1, we conclude that we have no such extreme points, which leads to the conclusion that the electric motor has an operation without sudden and catastrophic transitions.

The results obtained with the catastrophe theory can help analyze and predict the resulting transient regime and size the automation circuitry in case of successive, short-term power outages, which may lead to a catastrophic malfunction for the electric machine.

In this sense, the authors intend to assess some other electrical machines under several faulty operating conditions as a follow-up to this paper.

ACKNOWLEDGMENT

This work was supported by the Ministry of Research, Innovation, and Digitization grant, CNCS–UEFISCDI, project number TE 151 from 14/06/2022, within PNCDI III: "Smart Career Profiler based on a Semantic Data Fusion Framework".

Received on 12 December 2022

REFERENCES

- R. Thom, Stabilité structurelle et morphogenèse, W. A. Benjamin, New York, 1972.
- E.C. Zeeman, Catastrophe theory. Selected papers, 1972–1977. Addison–Wesley Publishing Co., Reading, Mass.–London– Amsterdam, 1977.
- 3. E.C. Zeeman, Catastrophe Theory, Scientific American, April, pp. 65-83 (1976).
- H.J. Sussmann, R.S. Zahler, Catastrophe Theory: Mathematics Misused, The Sciences, 17, 6 (1977).
- H.J. Sussmann, R.S. Zahler, Catastrophe Theory as Applied to the Social and Biological Sciences: A Critique, Synthese–Mathematical Methods in the Social Sciences, Part III, 37, 2, pp. 117–216 (1978).
- T. Poston, I. Stewart, *Catastrophe Theory and its Applications*, Pitman Publishing Ltd., London, San Francisco, Melbourne (1978).
- I. Stewart, *Elementary catastrophe theory*, IEEE Transactions on Circuits and Systems, 30, 8, pp. 578–586 (1983).
- I. Stewart, Non-Elementary Catastrophe-Theory, IEEE Transactions On Circuits And Systems, 30, 9, pp. 663–670 (1983).
- I. Stewart, Applications of nonelementary catastrophe-theory, IEEE Transactions On Circuits And Systems, 31, 2, pp. 165–174 (1984).
- R. Voinea, D. Voiculescu, F.P. Simion, *Introducere în mecanica solidului cu aplicații în inginerie*, Editura Academiei Române, Bucureşti (1989).
- R.M. Ciuceanu, I.V. Nemoianu, C.P. Mihai, *Maşini electrice fundamente* teoretice şi funcționale, Editura Politehnica Press (2020).
- A. Moraru, Maşini electrice. Teorie, încercari şi exploatare, Editura AGIR, Bucureşti (2010).
- 13. N. Galan, *Maşini electrice*, Editura Academiei Române, București (2011).
- 14. R. Voinea, I. Stroe, *Introducere în teoria sistemelor dinamice*, Editura Academiei Române, București (2000).
- A.A. Sallam, J.I. Dineley, Catastrophe-theory as a tool for determining synchronous power system dynamic stability, IEEE Trans. on Power Apparatus and Systems, **102**, 3, pp. 622–630 (1983).
- A.A. Sallam, Power-Systems Transient Stability Assessment Using Catastrophe-Theory, IEEE Proceedings - C Generation Transmission and Distribution, 136, 2, pp. 108–114 (1989).

- V. Ajjarapu, Application Of Catastrophe-Theory To Describe Voltage Collapse In Power-Systems, Proceedings Of The 32nd Midwest Symposium On Circuits And Systems, 1, 2, pp. 248–251 (1990).
- J. Deng, C. Zhang, Steady-State Stability Assessment Of Multimachine Power-Systems Using Catastrophe-Theory, TENCON '93: 1993 IEEE Region 10 Conference On Computer, Communication, Control And Power Engineering, 5, pp. 195–199 (1993).
- R. Parsiferaidoonian, X.-L. Sun, Catastrophe-Theory Model Of Multimachine Power-Systems For Transient Stability Studies, TENCON '93: 1993 IEEE Region 10 Conference On Computer, Communication, Control, And Power Engineering, 5, pp. 187–190 (1993).
- Y. Huang, S. Tsukao, Y. Tamura, S. Iwamoto, *Application of catastrophe theory to power systems*, Electrical Engineering In Japan, 115, 7, pp. 40–48 (1995).
- S.C. Tripathy, S. Prasad, T.S. Bhatti, A new swallowtail catastrophe model for power system transient stability assessment, Electric Machines And Power Systems, 25, 2, pp. 121–140 (1997).
- S. Prasad, S.C. Tripathy, *Transient stability analysis of power system* using catastrophe theory including field flux decay effect, Electric Machines, and Power Systems, 26, 5, pp. 453–464 (1998).
- S.I. Glukhova, E. A. Palkin, Application of catastrophe theory for multimodal distributions statistical analysis, Theory Of Probability And Its Applications, 49, 3, pp. 414–428 (2004).
- 24. J. Guo, X.L. Chen, H.Z Jin, D.J. Wu, Analyzing brittleness of complex system based on the catastrophe theory, Proceedings Of The International Conference On Intelligent Systems and Knowledge Engineering (ISKE 2007), Book SeriesAdvances in Intelligent Systems Research, Article Number 1377 (2007).
- 25. Z. H. Wang, A. Girgis, V. Aravnthan, E. Makram, Wide Area Power System Transient Stability Assessment Using Catastrophe Theory Method and Synchrophasors, 2011 IEEE Power And Energy Society General Meeting, Book Series IEEE Power and Energy Society General Meeting PESGM (2011).
- G.A. Mahmoud, Voltage stability analysis of radial distribution networks using catastrophe theory, IET Generation Transmission & Distribution, 6, 7, pp. 612–618 (2012).
- 27. J.J. Li, Q. Zhang, S. Guo, G. Wang, F. Sun, T. Zhang, Q.S. Zhao, C. Wang, J. Yu, X.K. Cheng, An Evaluation Method of Multi-source Control Capacity on Transient Angle stability, 2016 IEEE International Conference On Power System Technology (POWERCON) (2016).
- Z. Zeng, H. Li, S.O. Tang, H. Yang, R. X. Zhao, Multi-objective control of multi-functional grid-connected inverter for renewable energy integration and power quality service, IET Power Electronics, 9, 4, pp. 761–770 (2016).
- M.M. Eladany, A.A. Eldesouky, A.A. Sallam, Power System Transient Stability: An Algorithm for Assessment and Enhancement Based on Catastrophe Theory and FACTS Devices, IEEE Access, 6, pp. 26424–26437 (2018).
- M.T. Kenari, M.S. Sepasian, M.S. Nazar, Probabilistic assessment of static voltage stability in distribution systems considering wind generation using catastrophe theory, IET Generation Transmission & Distribution, 13, 13, pp. 2856-2865 (2019).
- N. Ali, W. Alam, M. Pervaiz, J. Iqbal, Nonlinear adaptive backstepping control of permanent magnet synchronous motor, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., 66, 1, pp. 9–14 (2021).
- E. Boudissa, F. Habbi, N. El Houda Gabour, M. Bounekhla, New dynamic genetic selection algorithm: application to induction machine identification, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., 66, 3, pp. 145–151 (2021).
- E. Benyoussef, S. Barkat, Direct torque control based on space vector modulation with balancing strategy of dual star induction motor, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., 67, 1, pp. 15–20 (2022).
- E. Benyoussef, S. Barkat, *Three-level direct torque control based on balancing strategy of five-phase induction machine*, Rev. Roum. Sci. Techn. Électrotechn. et Énerg., 67, 2, pp. 93–98 (2022).
- M.M. Rădulescu, A.A. Pop, Experimental determination of linear dynamic model parameters of the separately excited brushed dc motor by modified Pásek's method, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., 67, 3, pp. 271–274 (2022).