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PROCEDURES FOR ACCELERATING THE CONVERGENCE OF THE HĂNȚILĂ METHOD FOR SOLVING THREE-PHASE CIRCUITS WITH NONLINEAR ELEMENTS – PART II

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The Hănțilă method has proven its effectiveness in solving three-phase circuits with nonlinear elements and presents several advantages compared to other methods. It is a fixed-point method, and the solution is obtained by constructing a Picard-Banach sequence with assured convergence. Sometimes the contraction factor of the sequence construction operator is very close to 1, and thus, convergence is slow. To develop the method and increase its efficiency, we propose and analyze several procedures to accelerate the calculation algorithm.

1. INTRODUCTION

The development of power electronics equipment connected to the three-phase network is a source of distorting effects that can have undesirable consequences on the power system operation [1–3]. In this context, identifying efficient calculation and modeling methods of three-phase circuits with non-linear elements is very important.

The usefulness of the Hănțilă method [4–9] for solving three-phase circuits with nonlinear elements was analyzed in [10], including the case of nonlinear elements with controlled switching (thyristors) [11]. The method has demonstrated its efficiency in all the analyzed cases. It presents several advantages [10–12], as follows: convergence is always ensured, the possibility of solving the circuit on a single phase, the possibility of solving circuits with nonlinear elements with switched characteristics, the possibility of solving circuits with different values of circuit elements on harmonics or sequences (for example generators with different reactances on sequences), the easy highlighting of power transfer on harmonics, the possibility of adopting a large number of harmonics, a fact practically impossible in the case of other methods.

Essentially, the Hănțilă method is a fixed-point method. The nonlinearity is treated by constructing a Picard-Banach sequence with assured convergence. The solution consists of "linearizing" the circuit by replacing the nonlinear elements with generators with controlled sources and fixed internal resistances. There are two dual ways of applying the method: voltage correction of the controlled source or current correction [4–12]. Briefly, in the voltage correction case, the solution algorithm (detailed in [12]) can be summarized as:

$$e^{(n)} \xrightarrow{\mathrm{F}} \underline{E}^{(n)} \xrightarrow{h} \underline{U}^{(n)} \xrightarrow{F^{-1}} u^{(n)} \xrightarrow{g} e^{(n+1)}, \qquad (1)$$

where u is the voltage on the nonlinear element, e is the value of the controlled source, F and F^{-1} are the direct and inverse Fourier transforms, (n) the iteration number, h is the linear diagonal operator ensuring the connection with the linear part of the circuit in the frequency domain, and g provides the correction according with the nonlinear characteristic in the time domain.

Sometimes the contraction factor of the sequence construction operator is very close to 1, and convergence is slow. An important development direction of the method is to increase the calculation speed. This can be done by: identifying algorithms that are more efficient in both speed and accuracy for computing forward, inverse Fourier transforms, including non-uniform sampling in the time domain, and by developing speed-up procedures [11,12].

In [12], we analyzed several procedures for accelerating the algorithm specific to the application of the method: the optimal choice of the calculation resistance/conductance, the use of overrelaxation, and the correction of the controlled source in voltage or in current.

The present work is devoted to developing this analysis for the following acceleration procedures: harmonic selection, hybrid voltage/current correction procedure, use of "less harsh" nonlinear characteristics with better contraction factors, the use of modified values for the linear circuit elements, respectively correcting the nonlinear characteristic by including other elements existing in the circuit or being extracted from the equivalent impedance connected to the terminals of the nonlinear element.

2. FASTER COMPUTATION OF AN INTERMEDIATE RESULT

This class of acceleration procedures is based on the property of Picard-Banach sequences to converge to the solution starting from any initial value and on the possibility to faster calculating in a first stage an approximate result, but close enough to the fixed point, and to use this result as an input value for the final calculation with the desired parameters and accuracy.

Several procedures can be developed to reduce the time and the amount of calculation data, depending on the characteristics of the circuit, for example, a) a hybrid correction procedure for the controlled source, first in voltage and then in current, or vice versa, b) the use of linear elements / modified circuit nonlinearities, with the subvariants: b_1) the use of less harsh nonlinear characteristics with better contraction factors and / or b_2) modified values for the linear circuit elements, and finally, c) the use in the first stage of some faster but less accurate computation algorithms, as well as variations and combinations between them.

2.1 THE USE OF LESS ACCURATE ALGORITHMS

In reference [7], an efficient procedure to reduce the data volume and computational time using the harmonic selection and a selection algorithm are proposed.

In three-phase electrical networks, the values for the amplitudes of the harmonics are generally decreasing with the harmonic order. In this situation, a rudimentary harmonic selection method can be used, which does not require a lot of

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calculations: progressively increasing the number of harmonics and using the result in the next iterative cycle. For example, truncating the series up to 25, 100, 500, 1000, ...

In most cases, both the number of iterations required, and the calculation time are expected to decrease, depending on the evolution of the coefficients h / h_i and the better contraction given by the truncation of the Fourier series to a reduced number of terms. h and h_i are the functions that ensure the correction according to the linear part of the circuit in the case of voltage correction or current correction.

In the case of an inductive circuit connected to the terminals of the non-linear element, truncation to a smaller number of harmonics for voltage correction also ensures a better contraction factor, along with reducing the calculation volume.

Similarly, a capacitive circuit provides a better contraction factor by truncating to a lower number of harmonics if the current correction is performed.

The selection of harmonics is a solution for calculating an intermediate result that uses, in the first phase, a faster but less accurate calculation algorithm. Similarly, other faster calculation algorithms can be used, but with a higher error for the direct and/or inverse Fourier transforms.

Another example is the use of a small number of sampling points in the first phase. Then, the so-obtained solution can be used with the reduction of harmonics.

For example, an initial calculation can be made for the first 25 harmonics with 50 sampling points, and the result obtained to be used for the calculation using 250 harmonics with 8,000 sampling points. Additional intermediate steps may also be introduced.

2.2 HYBRID VOLTAGE / CURRENT CORRECTION PROCEDURE

Starting from the evaluation of the contraction factors described and exemplified in [12], concerning the controlled source, in certain cases, it may be useful to successively use the voltage correction and then current correction (or *vice-versa*).

For example, in the case of inductive circuits, it may be beneficial to apply the truncation method for a smaller number of harmonics (up to the rank where the coefficients h are favorable compared to those of h_i) by using the voltage correction, followed by the current correction as the number of harmonics increases. In such cases, a hybrid solution may be faster.

Similarly, in the case of capacitive circuits, it may be useful, in certain cases, to apply the truncation method for a reduced number of harmonics using the current correction, then, for an increasing number of harmonics, the voltage correction might be used.

The decision to apply the hybrid procedure or directly the voltage or current correction can only be made following the evaluation of the contraction factors and the correction coefficients, as described in [12].

2.3 THE USE OF NONLINEAR CHARACTERISTICS WITH BETTER CONTRACTION FACTORS

A sufficient condition for g to be a contraction is that f – the characteristic function of the nonlinear element in the time domain is Lipschitz and uniformly monotonic [12]:

$$0 < \frac{1}{R_{\max}} \stackrel{\text{def}}{=} \lambda \le \left\| \frac{f(u_1) - f(u_2)}{u_1 - u_2} \right\| \le \Lambda \stackrel{\text{def}}{=} \frac{1}{R_{\min}} , \quad (2)$$

$\forall\, u_1, u_2$ and $u_1 \neq u_2$.

The contraction factor for the function $g(u)$ is in the case of voltage correction [12]:

$$\theta_g = \text{Max}\left[\left(1 - \frac{R}{R_{\max}}\right), \left(\frac{R}{R_{\min}} - 1\right)\right] . \quad (3)$$

It can be seen from (3) that a “very hard” nonlinear characteristic can cause contraction factors very close to 1, as proven in [9]. In such cases, the circuit can be solved in the first phase using a “less steep” characteristic with $\lambda_2 > \lambda$, respectively $\Lambda_2 < \Lambda$, ensuring a better contraction factor. For example, instead of $\lambda = 10^{-10}$ and $\Lambda = 10^{10}$, the circuit can be solved using $\lambda_2 = 10^{-4}$ and $\Lambda_2 = 10^4$. The obtained result becomes the initial value for the calculation with the correct characteristic. The solution is efficient in many cases, thus significantly reducing total computation time.

By reducing the ratio

$$\frac{\Lambda}{\lambda} = \frac{R_{\max}}{R_{\min}} = \frac{G_{\max}}{G_{\min}} , \quad (4)$$

one can notice a reduced value of the contraction factor θ_g for the function $g(u)$. Similarly, it can be noticed a decrease of the contraction factor θ_{g_i} corresponding to the function g_i, in the case of the current correction.

2.4 THE USE OF MODIFIED VALUES FOR LINEAR CIRCUIT ELEMENTS

By changing the values of certain circuit elements, one can change the values for $\underline{Z}_{e_k}$ and $\underline{Y}_{e_k}$, thus influencing the contraction factors and coefficients for the functions $h(\underline{E})$ and $h_i(\underline{I}_s)$. We denote by $\underline{Z}_{e_k}$ the equivalent impedance connected at the terminals of the nonlinear element, for the harmonic of rank k, if the voltage correction procedure is considered. Similarly, $\underline{Y}_{e_k}$ is the harmonic order k equivalent admittance connected to the terminals of the nonlinear element in the case of current correction use.

An intermediate result can thus be readily calculated to be further used as an input value for the calculation utilizing the correct (initial) values.

3. MODIFYING THE NON-LINEAR CHARACTERISTIC BY INCLUDING SOME EXISTING ELEMENTS IN THE CIRCUIT

Let us analyze a solution to increase the convergence speed by using elements already present in the circuit that we embed in the nonlinear characteristic.

Starting from the observation that in many applications, the nonlinear element has a resistor in series, we propose embedding this resistor R_s in the nonlinear characteristic.

Including the aforementioned resistor as part of the nonlinear element conveniently modifies the $u - i$ characteristic. Initially, we dispose of the dependency $i \rightarrow u$ having the minimum and maximum slope values bounded by $R_{\max}$ and $R_{\min}$. After the inclusion of the resistor of resistance R_s, these slopes change to $R_s + R_{\max}$ and $R_s + R_{\min}$, respectively.

Sometimes it is possible that, in this way, an initially non-monotonic (non-increasing) $u - i$ relationship becomes monotonically increasing. By including the series resistor R_s in the nonlinear characteristic, a "hard" nonlinear characteristic becomes less "harder" by decreasing the ratio (4), which becomes:

$$\frac{\Lambda^S}{\lambda^S} = \frac{(R_{max} + R_s)}{(R_{min} + R_s)} < \frac{R_{max}}{R_{min}}. \quad (5)$$

If we use the voltage correction of the nonlinearly controlled source, following this operation of inclusion in the nonlinear characteristic of the series resistor R_s we have a new characteristic function f_2 instead of the initial nonlinear characteristic $i = f(u)$:

$$i = f_2(u_2) \quad (6)$$

with

$$u_2 = u + R_s i \quad (7)$$

or

$$f_2^{-1}(i) = f^{-1}(i) + R_s\, i. \quad (8)$$

We thus get

$$u_2 = R^s\, i + e \quad (9)$$

with

$$e = u_2 - R^s f_2(u_2) = g_2(u_2), \quad (10)$$

where u_2 is the voltage across the new nonlinear element comprising the initial nonlinear element and the series resistance R_s. R^s denotes the new resistance calculation.

If f is uniformly monotonic and Lipschitz, then it is invertible, and its inverse f^{-1} is also uniformly monotonic and Lipschitz [8]. f_2^{-1} is a sum of uniformly monotone and Lipschitz functions and is in turn, uniformly monotone and Lipschitz. Implicitly, f_2 is uniformly monotone and Lipschitz.

The contraction factor corresponding to the function $g_2(u_2)$ becomes

$$\theta_g^{\,s} = \text{Max}\left[\left(1 - \frac{R^s}{R_s + R_{max}}\right), \left(\frac{R^s}{R_s + R_{min}} - 1\right)\right], \quad (11)$$

with the selection range for R^s defined as $[R_s + R_{min}, 2(R_s + R_{min})]$ (considering the optimal selection interval recommended in [12]).

Having in view that if we choose $R = x\,R_{min}$ and $R^s = x\left(R_s + R_{min}\right)$ with $x \in (0, 2)$, we always get: $\frac{x(R_s + R_{min})}{R_s + R_{max}} > \frac{x\,R_{min}}{R_{max}}$ for $R_{max} > R_{min}$ and $\frac{x(R_s + R_{min})}{R_s + R_{min}} = \frac{x\,R_{min}}{R_{min}}$. It follows that: $\theta_g^{\,s} \leq \theta_g$. It is observed that a higher value of R_s ensures a better contraction factor and better (contraction) coefficients for $g_2(u_2)$.

With the inclusion of resistor R_s as part of the nonlinear element, the equivalent complex impedance of the circuit seen at the terminals of the newly formed element becomes

$$\underline{Z}_{e_{k2}} = \underline{Z}_{e_k} - R_s. \quad (12)$$

Like [12], we have:

$$h_k\left(\underline{E}'_k\right) - h_k\left(\underline{E}''_k\right) = \left(\underline{E}'_k - \underline{E}''_k\right)\frac{1}{1 + R/\underline{Z}_{e_k}}. \quad (13)$$

Considering the new function $h_2(\underline{E})$, for the harmonic of order k, the correction coefficient becomes

$$1/(1 + R^s/(\underline{Z}_{e_k} - R_s)), \quad (14)$$

with

$$\left|1/(1 + R^s/(\underline{Z}_{e_k} - R_s))\right| < \left|1/(1 + R/\underline{Z}_{e_k})\right|. \quad (15)$$

A better contraction factor is also ensured in this case, $\theta_h^{\,s} < \theta_h$. The inequality is also valid for the moduli of the coefficients on the harmonics, ensuring better values here.

To conclude this section, including the resistor as part of the nonlinear element is beneficial for speeding up the iterative procedure.

It is also observed that a higher value of R_s provides a better contraction factor and better harmonic coefficients.

In this present case R_s is a series resistance part of $\underline{Z}_{e_k}$ and implicitly $R_s \leq \min_k(\text{Re}(\underline{Z}_{e_k}))$.

The proposed solution also has the advantage that the value of the controlled source e is calculated directly, without requiring additional calculations.

If an existing series resistor is included in the nonlinear characteristic and current correction is used, the acceleration effect by improving the contraction factor is only sometimes ensured. One of the contraction factors (and the correction coefficients corresponding to the function g) decreases $\theta_{g_i}^{\,S} < \theta_{g_i}$, and the other contraction factor (and the coefficients corresponding to the function h) increases $\theta_{h_i}^{\,S} > \theta_{h_i}$.

According [12], if the same weight x is maintained when choosing $R^s = x\left(R_s + R_{min}\right)$ with $x \in (0, 2)$, respectively G^s we will have $\theta_g^{\,s} = \theta_{g_i}^{\,S}$. Considering the increase of the contraction factor and the corresponding coefficients of h_i, such a procedure will be slower than he one analyzed above.

The current correction is the dual procedure to voltage correction [10–12]. A parallel resistor can be included in the non-linear characteristic if we use current correction. The proofs and conclusions are dual to those presented above. In this case, including a parallel resistor in the nonlinear characteristic always ensures a better convergence, benefiting from the already presented advantages.

Similarly, if we include a parallel resistor in the nonlinear characteristic and use voltage correction, the acceleration effect by improving the contraction factor is only sometimes ensured. The procedure will also be always slower than the one using the current correction.

The existence of a series or parallel resistor to the nonlinear element is a new decision criterion regarding the calculation solution option concerning voltage or current correction, in addition to those discussed in [12].

4. MODIFYING THE NONLINEAR CHARACTERISTIC BY INCLUDING ELEMENTS EXTRACTED FROM THE IMPEDANCE OF THE EQUIVALENT CIRCUIT

The solution of modifying the non-linear characteristic by including existing resistive circuit elements has proven its effectiveness. The possibility of transferring such resistances from the impedance of the circuit connected at the nonlinear element terminals would allow the generalization of this procedure application.

A possible solution would be to insert an additional resistor (in series or parallel) aiming to calculate an intermediate step (like the procedures described in subsection 2.3), and then to solve the circuit with the correct values using the so-obtained intermediate result as a starting value.

Alternatively, one can initially transfer a resistor from the equivalent impedance connected to the terminals of the nonlinear element. The solution is like that presented in Section 3, with the choice for the values of the computational resistances so that the functions g'_2 and h'_2 remain contractive and thus to ensure a better convergence speed.

For the voltage correction procedure, if we transfer a series resistance R'_s from the equivalent impedance $\underline{Z_{e_k}}$ connected to the terminals of the non-linear element, we will have the change of function g to g'_2, like the approach discussed in Section 3, such that $\theta'^{s}_{g} < \theta_g$.

A larger value of R'_s provides a better contraction factor (and coefficients) for g'_2 and a better convergence speed if the selection range of R is respected.

For changing h to h'_2, to ensure a better contraction, we should have:

$$\left|\frac{\underline{Z_{e_k}} - R'_s}{\underline{Z_{e_k}} + R}\right| < \left|\frac{\underline{Z_{e_k}}}{\underline{Z_{e_k}} + R}\right| < 1 \,. \tag{16}$$

The condition is fulfilled if

$$R'_s < 2 \min_k \left(\mathrm{Re}(\underline{Z_{e_k}})\right), \tag{17}$$

with an optimal value for

$$R'_s = \min_k \left(\mathrm{Re}(\underline{Z_{e_k}})\right). \tag{18}$$

Using the value given by (18) produces a significant speedup, provides increased convergence speed for both g'_2 and h'_2 functions, and avoids potential problems with solving the circuit using negative resistor values.

Like those discussed in Section 3, the inclusion of a parallel resistor in the current correction case is dual to those presented above.

The equivalent impedance can be brought into the desired form using the series-parallel impedance transformation formulas. Starting from the property that a resistance can be decomposed into two higher value parallel resistances (and one of the two resistances values can be arbitrarily imposed), the parallel resistance with the highest value on the harmonics can be selected to be extracted.

Pulling a resistor in parallel and using voltage correction or pulling a series resistor and using current correction has the disadvantage mentioned in section 3. If one factor (and the contraction coefficients) decreases, the other increases.

Like [12], an analysis can be made for the functions g'_2 and h'_2, respectively $g_i{}'_2$ and $h_i{}'_2$, concerning the contraction factors and the moduli of the correction coefficients resulting from the extraction of the series and parallel resistances. Hence, a faster procedure can be chosen.

5. ILLUSTRATIVE EXAMPLE

In the present section, we analyze the solution of the same circuit analyzed in [12], as shown for convenience in Fig. 1.

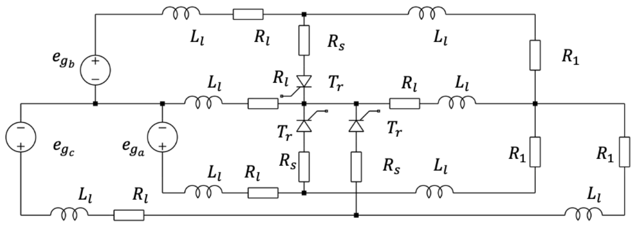


Fig. 1 – Proposed three-phase circuit to be solved.

We consider the same values for the circuit elements, namely: for the three-phase generator with symmetrical sources of 325 V amplitude, frequency of 50 Hz, $R_l = 1\ \Omega$, $R_1 = 2\ \Omega$, $R_s = 10\ \Omega$, $L_l = 5\times10^{-4}$ H, $L_1 = 5\times10^{-2}$ H. For the three identical thyristors T_r: blocking resistance $R_b = 1/G_b = 10^4\ \Omega$, conduction $R_c = 1/G_c = 0.05\ \Omega$, $V_f = 5$ V and $\alpha = \pi/5$ ($t_\alpha = T/10$).

Also, for the thyristors T_r we consider the same linearized characteristic depicted in [11–13]. Evidently, some different nonlinear elements, and implicitly characteristic dependences, may be used.

Let us modify the nonlinear characteristic by including the series resistor R_s, as proposed in Section 3. Using the voltage correction, for period T, yields

$$i = f_2(u_2) = \begin{cases} \dfrac{u_2}{R_c + R_2} + \dfrac{V_f\,(R_c - R_b)}{R_b\,(R_c + R_s)} & \text{for } t \in \begin{cases} [t_\alpha, t_b), & t_b < T \\ [0, t_b) \cup [t_\alpha, T], & t_\alpha > t_b \end{cases} \\ \dfrac{u_2}{R_b + R_s} & \text{for the rest of period } T \end{cases} . \tag{19}$$

The blocking condition becomes

$$u_2 \le V_f\left(1 + \frac{R_s}{R_b}\right). \tag{20}$$

Function $g_2(u_2)$ can be expressed as

$$g_2(u_2) = \begin{cases} u_2\left(1 - \dfrac{R^s}{R_c + R_s}\right) - \dfrac{R^s\, V_f\,(R_c - R_b)}{R_b\,(R_c + R_s)}, & t \in \begin{cases} [t_\alpha, t_b), & t_b < T \\ [0, t_b) \cup [t_\alpha, T], & t_\alpha > t_b \end{cases} \\ u_2\left(1 - \dfrac{R^s}{R_b + R_s}\right) & \text{for the rest of period } T \end{cases} . \tag{21}$$

If we choose $R^s = R^S_{\min} = R_{\min} + R_s = R_c + R_s$ (the new conduction resistance value), we get

$$g_2(u_2) = \begin{cases} \dfrac{V_f\,(R_b - R_c)}{R_b} & \text{for } t \in \begin{cases} [t_\alpha, t_b), & t_b < T \\ [0, t_b) \cup [t_\alpha, T], & t_\alpha > t_b \end{cases} \\ u_2\left(1 - \dfrac{R_c + R_s}{R_b + R_s}\right) & \text{for the rest of period } T \end{cases} . \tag{22}$$

By replacing in (22) the numerical values, we will obtain on one branch a correction coefficient of 0. On the second 0.998996, respectively, a contraction factor $\theta_g = 0.998996$, a value significantly better (lower) than the 0.999995 obtained in the case of the function $g(u)$ (calculated in [12]).

Let us maintain the calculation parameters used in [12] to be able to compare the calculation speeds: we truncate the Fourier series up to and including the 100th order harmonic and divide the period T also into 40,000 equidistant points. We use the same calculation algorithm for F and F^{-1}, and we stop the iterations when the relative distance (relative error) $\varepsilon^{(n)}/\|e_g\|$ drops below the value of 10^{-8}. To simulate the calculation algorithm, we used the GNU Octave 6.2.0 environment [14].

Table 1

Solving using voltage correction for different values of $\boldsymbol{R^s}$ with /without overrelaxation, by varying the blocking resistance and the number of sampling points

Experiment no.	Voltage corrections	No. of iterations	Time [s]	$\theta_g^{\,s} \times \theta_h^{\,s}$
1	$R^s = 0.8\,(R_s + R_{\min})$	119	23.69	0.98237
	overrelaxation μ = 2	105	40.64	
	with R_b modification	115	22.93	
	of no. of points	140	20.21	
2	$R^s = (R_s + R_{\min})$	94	19.10	0.97548
	overrelaxation μ = 2	88	34.57	
	with R_b modification	91	18.77	
	of no. of points	99	16.42	
3	$R^s = 1.5\,(R_s + R_{\min})$	61	13.47	0.95453
	overrelaxation μ = 1.5	58	23.59	
	with R_b modification	59	12.99	
	of no. of points	77	11.98	
4	$R^s = 1.8\,(R_s + R_{\min})$	51	11.60	0.93976
	overrelaxation μ = 1.5	50	21.28	
	with R_b modification	48	11.01	
	of no. of points	56	10.10	
5	$R^s = 1.9\,(R_s + R_{\min})$	**48**	11.13	0.93452
	overrelaxation μ = 1.2	47	19.76	
	with R_b modification	46	10.65	
	of no. of points	61	9.74	
6	$R^s = R_{\text{opt}}^{\;S}$	**48**	11.12	0.92925
	modification of no. of points	63	**9.58**	

Table 1 shows the times and number of iterations obtained using different R^s values, including overrelaxation and two other speed-up procedures: changing the value of R_b (as described in Subsection 2.3.1) and changing the number of sampling points (as described in Subsection 2.1).

In this case, a higher value resistor R^s also reduces the time and number of iterations. It is observed that the fastest solution was obtained for the value $R^s = R_{\text{opt}}^{\;S}$. A close computation time and the same number of iterations was also obtained for $R^s = 1.9\,(R_s + R_{\min})$.

Comparing the time values and the number of iterations shown in Table 1 with those obtained in [12], a spectacular shortening of both calculation times and the number of iterations can be observed.

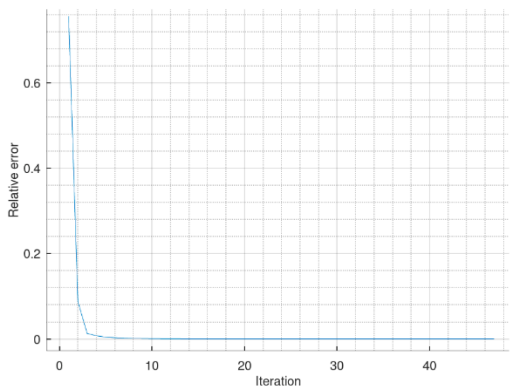
Once again, in this case we used fixed valued overrelaxation factors. Compared to [12] the values of overrelaxation factors that could be adopted following the proposed procedure are significantly lower. Parameter μ being less than or equal to 2, we can see a decrease in the number of iterations but an increase in the calculation time. The time required to perform the additional calculations is greater than the savings achieved.

An overrelaxation factor could not be adopted for $R^s = R_{\text{opt}}^{\;S}$.

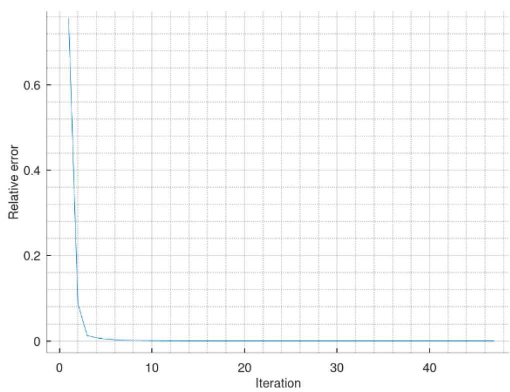
In the present case, the contraction factor being good enough, only small value overrelaxation factors can be adopted, and overrelaxation does not significantly improve the computation time.

Figure 2 shows the evolution of the relative error for $R^s = (R_s + R_{\min})$ and for $R^s = R_{\text{opt}}^{\;S}$. The disappearance of the oscillating phenomenon reported in [12] is observed. A better contraction factor avoids the oscillating phenomenon and can even compensate for possible calculation errors while maintaining convergence.

To change the value of R_b, we solved the initial circuit with $R_{b2} = 10^3\,\Omega$ which provides a contraction factor and better coefficients. The obtained result was used as an input value for the calculation with the correct value $R_b = 10^4\,\Omega$.



(a)



(b)

Fig. 2 – Relative error evolution *vs*. number of iterations $\varepsilon^{(n)}/\lVert e_g \rVert$ for: (a) $R^s = (R_s + R_{\min})$ and (b) $R^s = R_{\text{opt}}^{\;S}$.

To change the number of sampling points, we initially calculated using 2,000 sampling points. The obtained result was then used as an input value for the calculation with 40,000 sampling points. The number of harmonics was kept the same.

The duration of iterations with smaller sampling points is significantly shorter. In this case, the total computation time and not the number of iterations should be compared.

Among the analyzed compound acceleration procedures, the fastest in terms of calculation time proved to be the inclusion of a series resistance in the nonlinear characteristic along with the choice of $R^s = R_{\text{opt}}^S$ and the use as an intermediate step a faster, but less accurate, calculation algorithm (*e.g.*, reducing the number of sampling points).

Even though in the present example, the number of harmonics considered seems small, it is 11 times higher than the number of harmonics considered by some methods that use models in the frequency domain and which consider only harmonics up to the 25th order, also with elimination of the ones multiple of 2 or 3.

Generally, the calculation of high-order harmonics is also sufficiently accurate when using several sampling points 8 or 16 times higher than the maximum order of the harmonic considered for truncation. In this case, the time and calculation volume reduction are substantial compared to the analyzed example.

Figure 3 shows the voltage across the nonlinear element, respectively, across the voltage sources for the calculation resistances $R^s = (R_s + R_{\min})$ and $R^s = R_{\text{opt}}^S$ in the time domain as well as the harmonic spectrum (in detail up to the 50th harmonic). The appearance of the Gibbs phenomenon is observed.

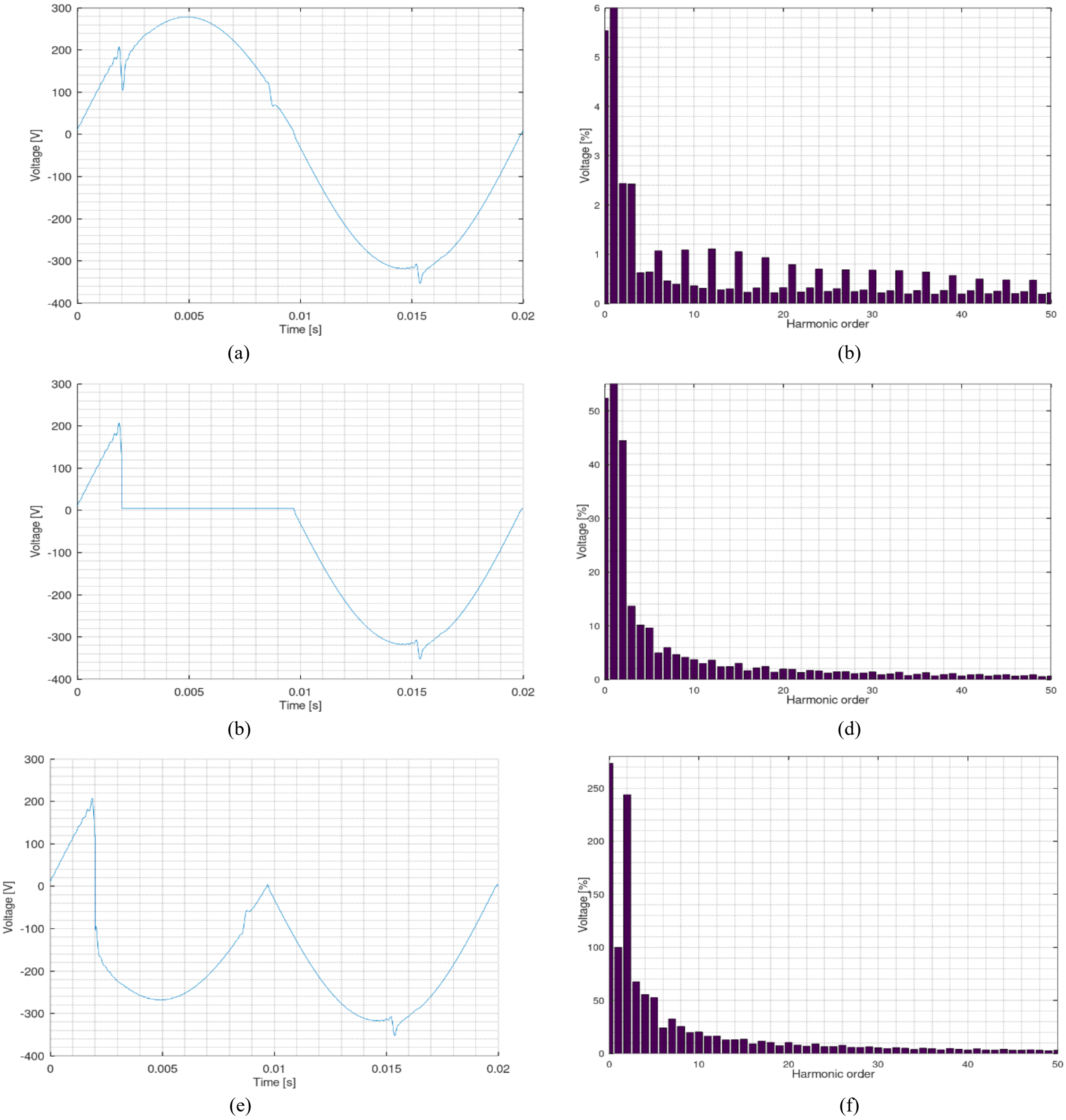


Fig. 3 – Relative error evolution *vs*. the number of iterations $\varepsilon^{(n)}/\|e_g\|$ for: (a) $R^s = (R_s + R_{\min})$ and (b) $R^s = R_{\text{opt}}^S$.

We solved the same circuit shown in Fig. 1, considering 2,000 harmonics, and using 2,000 sampling points (to avoid the Gibbs phenomenon according to the solution proposed in [11]), 4,000 and 8,000 sampling points. We introduced an intermediate step, namely calculating the first 1,000 harmonics. In this case, the acceleration effect given by the harmonics' selection procedure is visible. The results are summarized in Table 2.

In the case of solving using a large number of harmonics, one possibility to reduce the computational effort is to compute for the low-rank harmonics (in the first quarter or first half out of the total considered ones), with a smaller number of sampling points than for the high-rank harmonics, the latter being calculated anyway with very good precision with sampling in a very large number of points for the duration of a period.

Table 2

Solving with voltage correction for $\boldsymbol{R}^s = (\boldsymbol{R}_s + \boldsymbol{R}_{\mathbf{min}})$ with / without intermediate calculation up to harmonic 1,000

Experiment no.	No of sampling points	No. of iterations	Time [s]
1	2000	179	33.50
	stop at intermediate step $\varepsilon^{(n)}/\|e_g\| = 0.001$	174	32.52
	stop at intermediate step $\varepsilon^{(n)}/\|e_g\|=0.0001$	178	30.96
2	4000	269	100.28
	stop at intermediate step $\varepsilon^{(n)}/\|e_g\|=0.0001$	262	89.18
3	8000	494	360.98
	stop at intermediate step $\varepsilon^{(n)}/\|e_g\|=0.0001$	477	334.51

In the case of solving using a large number of harmonics, one possibility to reduce the computational effort is to compute for the low-rank harmonics (in the first quarter or first half out of the total considered ones), with a smaller number of sampling points than for the high-rank harmonics, the latter being calculated anyway with very good precision with sampling in a very large number of points for the duration of a period.

The acceleration solution discussed in [7] and presented in Subsection 2.1 concerning using a reduced number of harmonics for the initial calculation as an intermediate result did not give satisfactory results for the present example.

The total calculation time must be compared and not the number of iterations because the iterations with a smaller number of harmonics are being computed faster. The calculation time can be shortened even more drastically by introducing additional intermediate steps or by using other acceleration procedures.

6. CONCLUSIONS

Numerical computation procedures requiring a significant number of harmonics may lead to increased data volume and running time computation burden.

Depending on the values and characteristics of the circuit elements, situations may arise where the contraction factor of the algorithm has values very close to 1, and the convergence is slow. In such situations, acceleration procedures become very useful.

The analyzed procedures proved useful and easy to implement as calculation programs. In the situation where the contraction factor is already satisfactory (significantly lower than 1) or if the truncation is not done at a very large number of harmonics, their impact can be, in some cases, somewhat reduced.

One direction of developing procedures based on a faster calculation of an intermediate result is to identify a procedure for establishing and optimizing the intermediate steps from the beginning.

Modifying the nonlinear characteristic by including existing resistive circuit elements proved extremely effective in the analyzed example. The effect of reducing the computation time and the number of iterations was spectacular.

An important development direction for the Hănțilă method is increasing calculation speed and acceleration procedures.

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