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SLIDING MODE CONTROLLER DESIGN: STABILITY ANALYSIS AND TRACKING CONTROL FOR FLEXIBLE JOINT MANIPULATOR

MAJDI BOUSSOFFARA[1], IKBEL BEN CHEIKH AHMED[2], ZIED HAJAIEJ[3]

Key words: Sliding mode control, Flexible joint manipulator, Lyapunov stability theory, Hurwitz conditions, Trajectory tracking.

Flexible robots are subject of many research-works since their advantages in terms of safety, compliance, low energy consumption, manoeuvrability, high payload to manipulator weight ratio, low cost, and high speed. However, the flexibility of manipulator's links or joints and the under-actuation leads to complexity in the modelling and control. To deal with this problem, a sliding mode control is designed and applied to a presented model of the system. So, this paper presents the modelling of flexible joint manipulator, the design of adequate sliding mode controller which can stabilize the flexible joint manipulator. The robust tracking performance will be proved in the simulation.

1. INTRODUCTION

The high human-robot interaction in different important sectors like medical [1], haptics [2], space [3] and industries [4] leads to development in the design, modelling [5] and control [6] of robotic systems to achieve an efficient, easy and safe interaction [7]. Interactive robots are distinguished by their mechanical properties: lightweight and flexibility in joint or links. There are many references of commercialized lightweight manipulators such as YuMi of ABB, ASSIST of CEA-LIST and LWR of KUKA-DLR.

Compared to rigid manipulator, flexible robots are distinguished by reduced inertia and high dexterity [8, 9], possible integration in small spaces and sensitivity to the environment that allows abnormalities detection and trajectories learning. They ensure more compliance thanks to their flexible structure or actuation. These advantages come at the cost of structure flexibility that leads to complexity in modelling and control [10]. Indeed, mechanical flexibilities cause vibrations [11] that could deteriorate tracking performance of the system. These difficulties make them less expanded at some levels of applications [12].

So, it's mandatory to take them into account during the synthesis of control law. In particular, this work is focusing on robots with flexible joint which have a nonlinear dynamic behavior. Elasticity in the flexible transmission elements is the origin of flexibility and different models and control schemes have been proposed by researchers. Between them, we site the model introduced by M.W Spong [13] for a single rigid link with flexible joint shown in Figure 1. This system is considered as an underactuated system due to the fact that the number of actuations is less than the degree of freedom [14].

The linear control becomes not efficient technique in presence of external disturbances, nonlinearities and uncertainties in the model parameters [15]. So, many nonlinear strategies have been proposed by researchers like back stepping control [16] and linear quadratic regulator [17]. Despite their approved effectiveness, they also present some shortcomings such as the lack of robustness in front of parameter's uncertainties or the need of big amount of energy.

The sliding mode control (SMC) is one of the most robust control techniques of high interest for nonlinear system [18, 19]. It can be applied to the flexible joint systems and provide a robust control in front of disturbances and model uncertainties [20, 21].

SMC offers several assets like high precision and fast dynamic response of the system in feedback loops in tracking or regulation modes, also the robustness to parameter variations and external disturbances [22, 23]. The principle of sliding mode control is to constrain the trajectories of the system to reach a sliding surface and then remain there [24]. The choice of adequate sliding mode function represents a critical part of sliding mode control design to stabilize the trajectories of the system. Then, a switched feedback gain is constructed to drive the states trajectory to the sliding surface and ensure the convergence [25]. So, it's necessary to add a discontinuous term to the control input that may cause the chattering phenomenon characterized with high frequency oscillation of plant trajectories around the sliding surface. To attenuate chattering, we replaced the sign function with the saturation function.

Some works have investigated the sliding mode controller for flexible joint manipulators where transformation of the state coordinate of the system [26] or a transfer from dynamical equations to error domain [27] is required.

The proposed controller is directly synthesized from the dynamic model, the performances of stability and robustness will be approved by simulation results.

In this paper, we present a brief overview of the flexible joint single link manipulator. Then we design the sliding surface and derive the control law on the basis of Lyapunov stability theory and Hurwitz conditions. Stabilization and tracking control are demonstrated by the simulation on MATLAB of the flexible joint manipulator and the SMC controller in closed loop.

2. DESCRIPTION OF THE FLEXIBLE JOINT SINGLE LINK MANIPULATOR AND ASSYMPTIONS

We consider in this work the manipulator in Fig. 1 with single rigid link and flexible revolute joint actuated by a dc motor [29]. This type of arm is of high interest by modern

[1] University of Tunis El Manar, Faculty of Sciences of Tunis (FST), Laboratory Analysis and Processing of Electrical and Energy Systems, Tunisia, E-mail Majdi.boussoffara@yahoo.fr
[2] University of Tunis El Manar, National Engineering School of Tunis, Automatic Research Laboratory L.A.R.A Tunisia

researchers recently since its spread in industry. The elasticity of the joint is modelled as a linear torsional spring with stiffness K. I and J represent respectively the link and motor inertias. x_1 and x_3 are respectively angular positions of the link and motor and l is the height of the center of mass of the link.

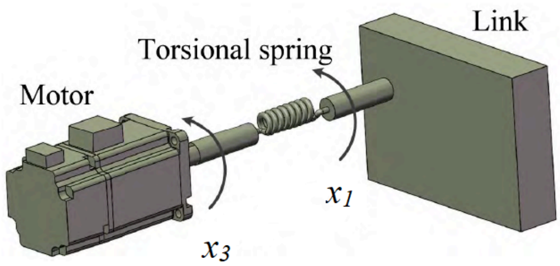

Fig. 1 – Single-link flexible joint robot.

The equations of motion of this system are obtained thanks to the Euler-Lagrange equation (3) where L is the total kinetic and potential energies noted K_{tot} and P_{tot} respectively.

$$K_{tot} = \frac{1}{2} J\dot{x}_3^{\,2} + \frac{1}{2} I\dot{x}_1^{\,2}, \tag{1}$$

$$P_{tot} = \frac{1}{2} k\left(x_1 - x_3\right)^2 + mgl\cos\left(x_1\right), \tag{2}$$

$$L = K_{tot} + P_{tot}. \tag{3}$$

Then, the Euler-Lagrange's equation of motion (4) is used to pick up the rotational acceleration of the motor and the link given by (6.1) and (6.2) respectively. In (4), u represents the torque or the control input, and x_i is the variable of differentiation *i.e.* x_1 and x_3.

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = u. \tag{4}$$

Then

$$I\ddot{x}_1 + mgl\sin\left(x_1\right) + k\left(x_1 - x_3\right) = 0, \tag{5.1}$$

$$J\ddot{x}_3 - k\left(x_1 - x_3\right) = u. \tag{5.2}$$

So

$$\ddot{x}_1 = -\frac{mgl}{I}\sin\left(x_1\right) - \frac{k}{I}\left(x_1 - x_3\right), \tag{6.1}$$

$$\ddot{x}_3 = \frac{k}{J}\left(x_1 - x_3\right) + \frac{u}{J}. \tag{6.2}$$

The system can be written into the following cascade state space model as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{mgl}{I}\sin\left(x_1\right) - \dfrac{k}{I}\left(x_1 - x_3\right) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{k}{J}\left(x_1 - x_3\right) + \dfrac{u}{J} + d, \end{cases} \tag{7}$$

where x_i, $i = 1..4$ are system states, u is the control input and d is a disturbance added to evaluate its influence on the control such as $|d| \le D$.

The first control goal is the stabilization of all the states of the system to zero. The second one is to ensure a tracking error that tends to zero as the time tends to infinity.

For the system (7), let:

$$f_1\left(x_1, x_2, x_3\right) = -\frac{mgl}{I}\sin\left(x_1\right) - \frac{k}{I}\left(x_1 - x_3\right), \tag{8.1}$$

$$f_2\left(x_1, x_2, x_3\right) = \frac{k}{J}\left(x_3 - x_1\right), \tag{8.2}$$

$$b\left(x_1, x_2, x_3\right) = \frac{1}{J}. \tag{8.3}$$

So, the system can be written as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1\left(x_1, x_2, x_3\right) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2\left(x_1, x_2, x_3\right) + b\left(x_1, x_2, x_3\right)u + d \end{cases}. \tag{9}$$

$f_1\left(x_1, x_2, x_3\right)$ must satisfy some assumptions:

Assumption 1

$$\frac{\partial f_1}{\partial x_3} \text{ is invertible.} \tag{10.1}$$

Assumption 1 is guaranteed as $\dfrac{\partial f_1}{\partial x_3} = \dfrac{k}{I}$.

Assumption 2

$$\text{If } f_1\left(0, 0, x_3\right) \to 0 \text{ then } x_3 \to 0 \tag{10.2}$$

Assumption 2 is satisfied as $f_1\left(0, 0, x_3\right) = -kx_3$.

Assumption 3

$$\left|\frac{\partial f_1}{\partial x_3}\right| \le \beta,\ i = 1,2,3 \text{ and } \beta > 0. \tag{10.3}$$

Assumption 3 is validated as $\left|\dfrac{\partial f_1}{\partial x_3}\right| = \dfrac{k}{I}$ is bounded

These assumptions satisfied by the system (7) are requirements to develop a sliding mode controller that stabilizes all the states of the system.

3. SMC DESIGN AND IMPLEMENTATION FOR THE FLEXIBLE JOINT SINGLE LINK MANIPULATOR

The design of the sliding surface depends on the class of the system and its dynamics. For the flexible joint manipulator, the nonlinearity appears into first equation and the number of actuations is less than the degree of freedom of the system.

3.1. SMC CONTROLLER DESIGN

Let's design the sliding mode function as:

$$\sigma = \sum_{i=1}^{n-1} \alpha_i e_i + e_n \,. \tag{11}$$

where n = 4, α_i, i = 1,2,3 are positive constant numbers chosen to ensure an asymptotically stable dynamics of the system on the sliding manifold $\sigma = 0$ and $e_i, i = 1..4$ are such:

$$\begin{aligned} e_1 &= x_1 \\ e_2 &= x_2 \\ e_3 &= \ddot{e}_1 = f_1(x_1, x_2, x_3) \\ e_4 &= \dddot{e}_1 = \frac{\partial f_1}{\partial x_1} x_2 + \frac{\partial f_1}{\partial x_2} f_1 + \frac{\partial f_1}{\partial x_3} x_4 \,. \end{aligned} \tag{12}$$

So, the sliding mode function can be written as:

$$\sigma = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + e_4 \,. \tag{13}$$

The sliding mode controller u consists of two terms: the equivalent control part u_e that maintain the system states on the sliding surface and a switching control part u_s that ensure the convergence of the system trajectories to the sliding manifold $\sigma = 0$. Then:

$$u = u_e + u_s \,. \tag{14}$$

From the equation $\dot{\sigma} = 0$, we conclude the equivalent control part u_e .

Since

$$\begin{aligned} \dot{\sigma} &= \alpha_1 \dot{e}_1 + \alpha_2 \dot{e}_2 + \alpha_3 \dot{e}_3 + \dot{e}_4 \\ &= \alpha_1 x_2 + \alpha_2 f_1 + \alpha_3 \left(\frac{\partial f_1}{\partial x_1} x_2 + \frac{\partial f_1}{\partial x_2} f_1 + \frac{\partial f_1}{\partial x_3} x_4 \right) + \\ & \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_1} x_2 \right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_2} \right) f_1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_3} \right) x_4 + \\ & \frac{\partial f_1}{\partial x_3} \left(f_2 + bu + d \right). \end{aligned} \tag{15}$$

Then, we can extract the equivalent control part as:

$$\begin{aligned} u_e = -\left(\frac{\partial f_1}{\partial x_3} b \right)^{-1} & \left\{ \alpha_1 x_2 + \alpha_2 f_1 + \alpha_3 \left(\frac{\partial f_1}{\partial x_1} x_2 + \frac{\partial f_1}{\partial x_2} f_1 + \frac{\partial f_1}{\partial x_3} x_4 \right) \right. \\ & \left. + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_1} x_2 \right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_2} \right) f_1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_3} \right) x_4 + \frac{\partial f_1}{\partial x_3} f_2 \right\}. \end{aligned} \tag{16}$$

To satisfy $\sigma\dot{\sigma} \le 0$ the switching control is designed as:

$$u_s = -\left(\frac{\partial f_1}{\partial x_3} b \right)^{-1} \left\{ \Gamma \,\mathrm{sat}(\sigma) + \delta\sigma \right\}, \delta > 0 \,, \tag{17}$$

where $\mathrm{sat}(\sigma)$ is the saturation function chosen instead of the sign function to reduce the chattering of the control input that can deteriorate the actuator [28] and $\delta\sigma$ is a proportional rate term that force the state to reach the switching manifold faster when σ is large

$$\mathrm{sat}(\sigma) = \begin{cases} \dfrac{\sigma}{\varepsilon} & if \left| \dfrac{\sigma}{\varepsilon} \right| \le 1 \\ \mathrm{sign}(\sigma) > 0 \end{cases} . \tag{18}$$

$\varepsilon > 0$ is the boundary layer and Γ is set as:

$$\Gamma = \beta D + \rho, \ \rho > 0 \,. \tag{19}$$

Since the assumption 1 is verified, so the control law is effective. Substituting (14) into (15) we obtain:

$$\dot{\sigma} = -\Gamma \,\mathrm{sat}(\sigma) - \delta\sigma \,. \tag{20}$$

3.2. STABILITY ANALYSIS

To prove the stability of the system, let's design the Lyapunov function as $V = \frac{1}{2}\sigma^2$, then

$$\begin{aligned} \dot{V} &= \sigma\dot{\sigma} \\ &= \sigma \left\{ -(\beta D + \rho)\mathrm{sign}(\sigma) - \delta\sigma + \frac{\partial f_1}{\partial x_3} d \right\} \\ &= -(\beta D + \rho)|\sigma| - \delta\sigma^2 + \sigma \frac{\partial f_1}{\partial x_3} d \\ &\le -\rho\sigma \le 0. \end{aligned} \tag{21}$$

Thus, the system converge to the manifold $\sigma = 0$ in a finite time and stay on it, *i.e.* there exists t_0 such as for $t > t_0$ we have $\sigma = 0$ then $e_4 = -\alpha_1 e_1 - \alpha_2 e_2 - \alpha_3 e_3$

From (12), we have: $\dot{e}_1 = e_2$, $\dot{e}_2 = e_3$ and $\dot{e}_3 = e_4$.

Let $\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_1 & -\alpha_2 & -\alpha_3 \end{bmatrix}$ and $\boldsymbol{\Lambda} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$, then we can obtain the reduced system:

$$\dot{\boldsymbol{\Lambda}}_1 = \boldsymbol{A}\boldsymbol{\Lambda} \,, \tag{22}$$

such as $\boldsymbol{A}$ is Hurwitz.

Design the Lyapunov function of system (22) as:

$$\dot{\boldsymbol{V}}_{\boldsymbol{1}} = \boldsymbol{\Lambda}_{\boldsymbol{1}}^{\mathrm{T}} \boldsymbol{F} \boldsymbol{\Lambda}_{\boldsymbol{1}} \,. \tag{23}$$

Then, we can obtain this equation:

$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{F} + \boldsymbol{F}\boldsymbol{A} = -\boldsymbol{Q}$, where $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} > 0$ and $\boldsymbol{F} = \boldsymbol{F}^{\mathrm{T}} > 0$ is the unique solution. So, the derivate $\dot{\boldsymbol{V}}_{\boldsymbol{1}}$ of $\boldsymbol{V}_{\boldsymbol{1}}$ is such:

$$\begin{aligned} \dot{V}_1 &= \dot{\boldsymbol{\Lambda}}_{\boldsymbol{1}}^{\mathrm{T}} \boldsymbol{F}\boldsymbol{\Lambda}_{\boldsymbol{1}} + \boldsymbol{\Lambda}_{\boldsymbol{1}}^{\mathrm{T}} \boldsymbol{F}\dot{\boldsymbol{\Lambda}}_{\boldsymbol{1}} \\ &= (\boldsymbol{A}\boldsymbol{\Lambda}_{\boldsymbol{1}})^{\mathrm{T}} \boldsymbol{F}\boldsymbol{\Lambda}_{\boldsymbol{1}} + \boldsymbol{\Lambda}_{\boldsymbol{1}}^{\mathrm{T}} \boldsymbol{F}(\boldsymbol{A}\boldsymbol{\Lambda}_{\boldsymbol{1}}) \\ &= \boldsymbol{\Lambda}_{\boldsymbol{1}}^{\mathrm{T}} (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{F} + \boldsymbol{F}\boldsymbol{A})\boldsymbol{\Lambda}_{\boldsymbol{1}} \\ &= -\boldsymbol{\Lambda}_{\boldsymbol{1}}^{\mathrm{T}} \boldsymbol{Q}\boldsymbol{\Lambda}_{\boldsymbol{1}} \le -a_{\min}(\boldsymbol{Q})\|\boldsymbol{\Lambda}\|^2 \le 0, \end{aligned} \tag{24}$$

where $a_{\min}(\boldsymbol{Q})$ is the minimum eigenvalue of $\boldsymbol{Q}$.

Since $\dot{\boldsymbol{V}}_{\boldsymbol{1}}$ is negative definite, so the transformed system is asymptotically stable and the variables e_1, e_2 and e_3 converge to zero. So, from assumption 2, the state x_3 will also converge to zero, and then from (12) e_4 will converge to zero.

3.3. POSITION TRACKING

For the position tracking, if we choose x_d and $\dot{x}_d$ as desired position and velocity respectively of the manipulator's terminal link, then the variables e_i, $i = 1..4$ are set as:

$$\begin{aligned} e_1 &= x_1 - x_d \\ e_2 &= x_2 - \dot{x}_d \\ e_3 &= \dot{e}_1 = f_1(x_1, x_2, x_3) - \ddot{x}_d \\ e_4 &= \ddot{e}_1 = \frac{\partial f_1}{\partial x_1} x_2 + \frac{\partial f_1}{\partial x_2} f_1 + \frac{\partial f_1}{\partial x_3} x_4 - \dddot{x}_d . \end{aligned} \tag{25}$$

So:

$$\begin{aligned} \dot{\sigma} &= \alpha_1 \dot{e}_1 + \alpha_2 \dot{e}_2 + \alpha_3 \dot{e}_3 + \dot{e}_4 \\ &= \alpha_1 \dot{x}_1 + \alpha_2 \dot{x}_2 + \alpha_3 \ddot{x}_2 + \alpha_3 \left(\frac{\partial f_1}{\partial x_1} x_2 + \frac{\partial f_1}{\partial x_2} f_1 + \frac{\partial f_1}{\partial x_3} x_4 \right) \\ &+ \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_1} x_2 \right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_2} \right) f_1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_3} \right) x_4 \\ &+ \frac{\partial f_1}{\partial x_3} (f_2 + bu + d) - \alpha_1 \dot{x}_d - \alpha_2 \ddot{x}_d - \alpha_3 \dddot{x}_d - x_d^{(4)} . \end{aligned} \tag{26}$$

Then from $\dot{\sigma} = 0$, we get the equivalent control

$$\begin{aligned} u_e = -\left(\frac{\partial f_1}{\partial x_3} b \right)^{-1} &\left\{ \alpha_1 x_2 + \alpha_2 f_1 + \alpha_3 \left(\frac{\partial f_1}{\partial x_1} x_2 + \frac{\partial f_1}{\partial x_2} f_1 \right.\right. \\ &\left. + \frac{\partial f_1}{\partial x_3} x_4 \right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_1} x_2 \right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_2} \right) f_1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_3} \right) x_4 \\ &\left. + \frac{\partial f_1}{\partial x_3} f_2 \right\} - \alpha_1 \dot{x}_d - \alpha_2 \ddot{x}_d - \alpha_3 \dddot{x}_d - x_d^{(4)} . \end{aligned} \tag{27}$$

To demonstrate the performance of the proposed control law, SMC and flexible joint system will be simulated for both stabilization and position tracking.

4. SIMULATION RESULTS AND DISCUSSIONS

SMC controller (14) and flexible joint system (7) with the parameters of Table 1 [30] were introduced in MATLAB. Let the initial states of the plant are set as $[0.3 \;\; 0.5 \;\; 0.3 \;\; 0.5]$, design Γ from (19) with $\sigma = 1$, $\delta = 1$ and set the boundary layer of the saturation function as $\varepsilon = 0.5$.

A is Hurwitz, so: $\left| A - aI \right| = \begin{vmatrix} -a & 1 & 0 \\ 0 & -a & 1 \\ -\alpha_1 & -\alpha_2 & -\alpha_3 \end{vmatrix} = 0$, then,

we can choose $\alpha_1 = 27$, $\alpha_2 = 27$ and $\alpha_3 = 9$.

Let's the disturbance $d = 0.4 \sin(3t)$, so we can set $D = 0.4$.

Table 1

System parameters

Parameter	Symbol	Value (model 1)	Unit
Mass	m	1	kg
Stiffness	k	100	Nm/rad
Length	L	1	m
Gravity	g	9.8	m/s^2
Inertia of link	I	1	$kg\ m^2$
Inertia of motor shaft	J	1	$kg\ m^2$

4.1. STABILIZATION

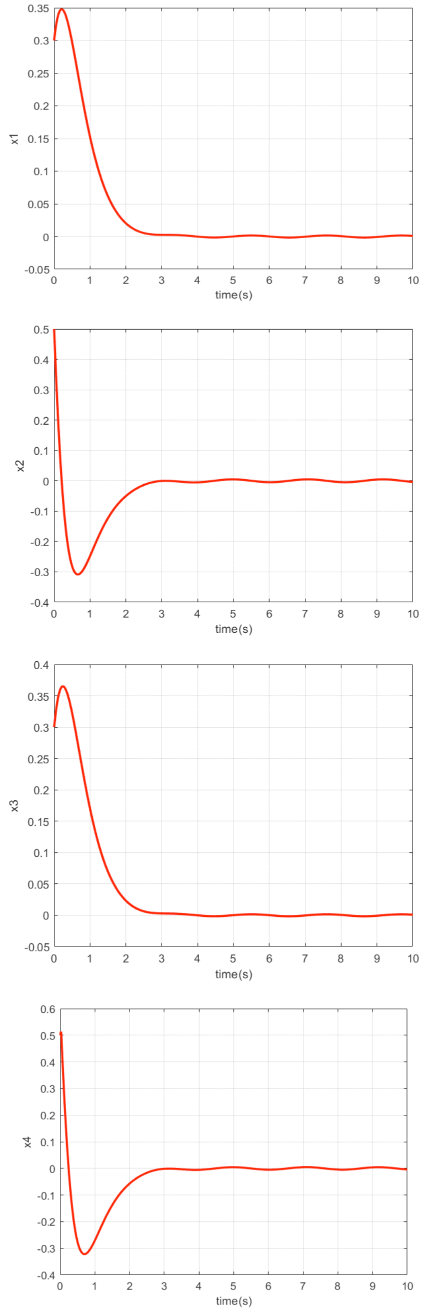

Fig. 2 – System states response.

To demonstrate the stability of the system around the equilibrium point, we consider the equations (12), so the control goal is that all the states x_i, i = 1 to 4 converge to zero. From the model (9), we have:

$$\frac{\partial f_1}{\partial x_1} = \frac{-k}{I} - \frac{mgl}{I} \cos(x_1), \; \frac{\partial f_1}{\partial x_2} = 0, \; \frac{\partial f_1}{\partial x_3} = \frac{k}{I},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_1} x_2 \right) = -\frac{k}{I} f_1 + \frac{mgl}{I} \sin(x_1) x_2^2 - \frac{mgl}{I} \cos(x_1) f_1 ,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_2}\right) f_1 = 0\,, \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial f_1}{\partial x_3}\right) x_4 = 0\,, \frac{\partial f_1}{\partial x_3} f_2 = \frac{k}{I} f_2\,.$$

From $\frac{\partial f_1}{\partial x_3} = \frac{k}{I}$ and Assumption 3, we can choose:

$$\beta = \frac{k}{I} + 0.1\,.$$

Simulation results are shown in Figs. 2 and 3 which represent respectively the system states responses and the control input which brought back all the states to the equilibrium point zero despite of the presence of a disturbance and justify the asymptotic stability of the system.

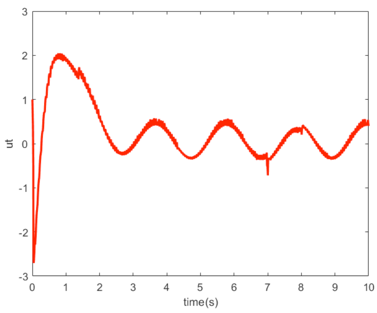

Fig. 3 – Control input for f=0.5 Hz.

4.2. POSITION TRACKING SIMULATION

For position and speed tracking goals, the states $x_1$and x_2 have to reach the desired position and velocity x_d and $\dot{x}_d$ respectively. To demonstrate the performance of the system, we change the frequency noted f of the applied set point as follows: $f = 0.5$, 1 and 5 Hz.

The Figs. 4, 5 and 6 illustrate the control input, position and speed response respectively of the flexible joint system at $f = 0.5$ Hz. The sinusoidal tuned control input allowed the tracking of the set point and the error reach zero in a finite short time.

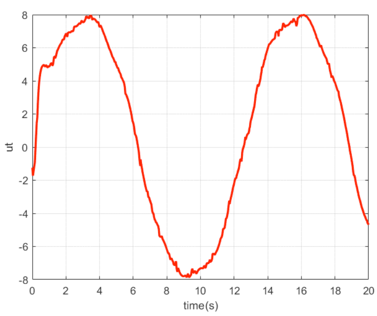

Fig. 4 – Control input for f = 5 Hz.

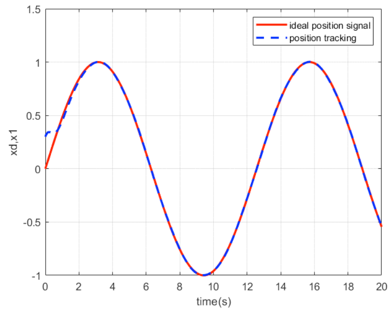

Fig. 5 – Position tracking for f =0.5 Hz.

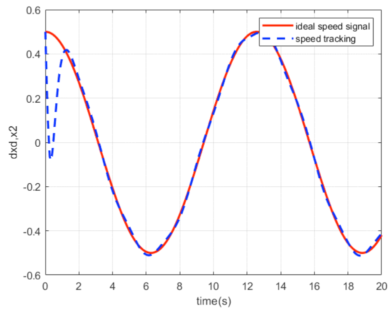

Fig. 6 – Speed tracking for f = 0.5 Hz.

The Figs. 7, 8 and 9 illustrate the control input, position and speed response respectively of the flexible joint system at $f = 1$ Hz. The amplitude of the control system signal is higher and the controlled states reach the desired trajectories in a short finite time and justify the asymptotic convergence.

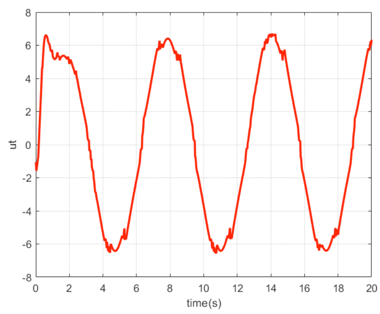

Fig. 7 – Control input for f =1 Hz.

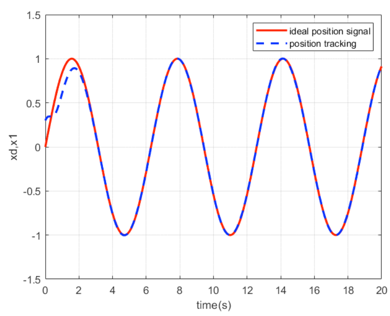

Fig. 8 – Position tracking for f =1 Hz.

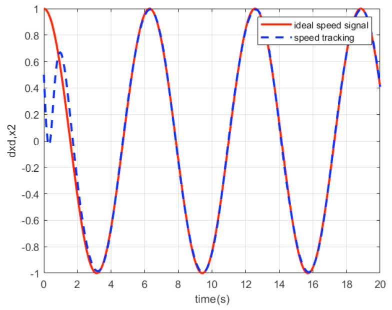

Fig. 9 – Speed tracking for f =1 Hz.

At $f = 5$ Hz, the amplitude of the control system signal is even higher as shown in the figure 10. The Figs. 11 and 12 show that the position and speed responses reach the desired trajectories in a brief delay.

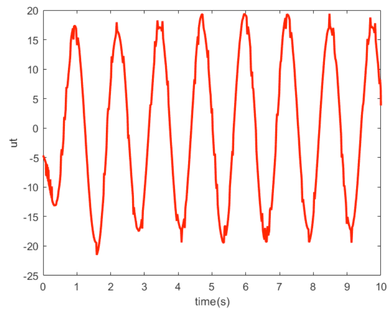

Fig. 10 – Control input for f = 5 Hz.

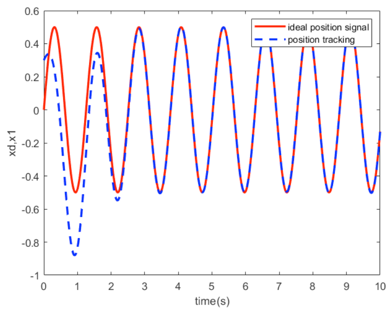

Fig. 11 – Position tracking for f = 5 Hz.

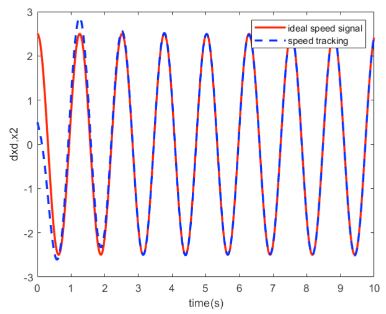

Fig. 12 – Speed tracking for f = 5 Hz.

The proposed controller can be tested on a real target system such as the Quanser model of rotary flexible joint srv02 [31].

The QUARC (Quanser Real Time Control) software, integrated with MATLAB, allows the real time control in closed loop (Fig. 13) directly from Simulink .The code is deployed on the hardware and the system responses are obtained via Q8_USB board from Quanser hardware to MATLAB .

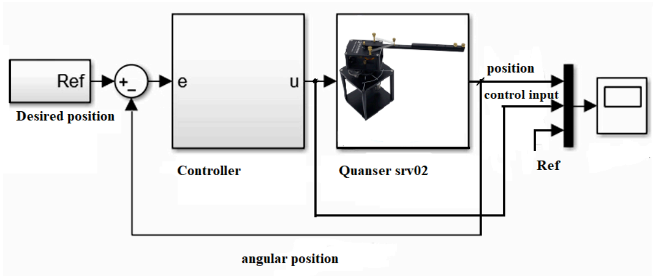

Fig. 13 – Control loop.

4. CONCLUSION

In this paper, a sliding mode controller design is proposed to control a flexible joint single link manipulator. The state model of the system is obtained using the dynamics equation of Euler-Lagrange. The sliding surface and the SMC controller are derived for both stabilization - according to Lyapunov and Hurwitz conditions- and tracking study. The simulation results prove that the calculated control law allowed the system to achieve correctly the tracking setpoints and justify the performance of stability, robustness of this control technique in front of disturbance and internal parameters variation. Although, the choice of control law parameters is sensitive and delicate, like the boundary layer and the parameter D that have to be adjusted with the disturbance and also the proportional rate that have to be adjusted with the frequency. The positive constant δ can be adjusted using fuzzy controller. The high speed of the joint requires high amplitude of the control input signal. So, an adaptive SMC is required.

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