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PROCEDURES FOR ACCELERATING THE CONVERGENCE OF THE HĂNȚILĂ METHOD FOR SOLVING THREE-PHASE CIRCUITS WITH NONLINEAR ELEMENTS – PART I

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The Hănțilă method is a fixed-point method which treats nonlinearity by constructing a Picard-Banach sequence with assured convergence. Sometimes, the contraction factor of the operator building the iteration sequence is very close to the unit value and thus the convergence is rather slow. We propose and analyze several procedures for accelerating the calculation algorithm in case of utilizing the Hănțilă method for solving nonlinear three-phase circuits.

1. INTRODUCTION

The issues generated by the nonlinear loads present in the electricity networks are becoming more and more important considering the electricity production and consumption trends and the necessity of implementing innovative renewable energy technologies [1–7]. The identification of efficient methods for calculating and modeling three-phase circuits with nonlinear elements has become very important in this context.

The Hănțilă method has been used successfully in solving several electrical engineering problems where nonlinearities may occur: magnetic and electromagnetic fields in nonlinear environments, electrical circuits with nonlinear elements [8]. The Hănțilă method is a fixed-point method treating nonlinearity by constructing a converging Picard-Banach sequence.

For the first time, the method was presented and used in [9] for solving circuits with nonlinear resistive elements, and then developed in a series of articles [10 –12], for periodic circuits. The solution for the three-phase circuits using the Hănțilă method was proposed in [13] and later developed and analyzed in the articles [14, 15] in the case of some practical examples, including non-linear elements with controlled switching (*i.e*., thyristors). The method proved effective in all the analyzed cases. We mention that the analyzed method has several advantages compared to other methods [14, 15]: assured convergence, single-phase solution analyzes, the possibility of solving circuits with nonlinear elements with switched or defined characteristics on branches, the possibility to solve circuits having different circuit element values on harmonics or sequences (for example: generators presenting different reactances on symmetrical components).

The solution is to "linearize" the circuit by replacing the nonlinear elements with generators with controlled sources and internal resistors. The value of the sources is corrected iteratively by constructing an algorithm with assured convergence. The value of the internal resistances is chosen to ensure convergence. The correction of the controlled sources according with the nonlinear characteristics is performed in the time domain. The analysis of the linear circuits connected to the terminals of nonlinear elements is done in frequency domain, separately on harmonics and sequences. Finally, when the value of the controlled source is obtained with sufficient accuracy, the currents and voltages may be calculated in the frequency domain for all the elements present in the circuit [13–15].

An important advantage is the possibility of adopting a large number of harmonics, which is practically impossible with other methods. Note that if a large number of harmonics need to be considered, the data volume and computation time can increase significantly. Depending on the nonlinear characteristic and the circuit connected to the terminals of the nonlinear element, situations in which the contraction factor of the algorithm has values very close to 1 may occur. As a result, the number of iterations and computation time may increase substantially [8].

We analyze several procedures for accelerating the calculation algorithm specific to the application of the method: the optimal choice of the calculation resistance R, the use of overrelaxation, the correction of the controlled source using either the voltage or the current. Considering the limited extent of the present article, other acceleration procedures will be presented in a Part II follow up paper, namely: the use of harmonic selection, hybrid voltage / current correction method, the use of "less harsh" nonlinear characteristics with better suited contraction factors, the use of modified values for the linear circuit elements, respectively the correction to the nonlinear characteristic by including some other existing elements from the circuit.

2. APPLICATION OF THE HĂNȚILĂ METHOD – SHORT DESCRIPTION

A brief description of the method is useful for understanding convergence acceleration procedures. There are two options for using it: voltage correction or current correction of the controlled source. The two variants are dual [13–15]. For simplicity reasons, we choose a three-phase circuit with a single star-balanced nonlinear load, which has identical nonlinear elements on each of the three phases. The method can also be applied if several nonlinear loads are present in the circuit. We briefly present the application of the method detailed in [13–15], and we will enter into details if necessary.

2.1. VOLTAGE CORRECTION OF THE CONTROLLED SOURCE

In this case the method consists in replacing the nonlinear elements with voltage generators having the controlled voltage sources e and internal resistances $\mathbb{R}$.

The u–i characteristic of the nonlinear element on a phase is described in time-domain by the function $f: \mathrm{R} \to R$:

$$i = f(u), \tag{1}$$

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where i and u are the current and respectively the voltage for the nonlinear element.

We substitute each of the nonlinear elements having the characteristic (1) with voltage generators made up of voltage sources e (controlled by the voltage at the terminals of the branch) and the internal resistance R:

$$u = R\, i + e \tag{2}$$

with

$$e = u - R\, f(u) = u\left(1 - R\frac{f(u)}{u}\right) = u\left(1 - \frac{R}{R_u(u)}\right) = g(u). \tag{3}$$

According with [8, 13–15] we chose R such that $g(u)$ be a contraction in the Hilbert space of periodic functions: the contraction factor $\theta_g \in (0, 1)$ exists to ensure the inequality

$$\|g(u_1) - g(u_2)\| \le \theta_g \|u_1 - u_2\| \;\forall\; u_1, u_2. \tag{4}$$

In order to define the norm relative to the voltages, we consider in the Hilbert space of periodic functions of period T the weighted (with the factor $1/R$) scalar product [11, 13], as follows

$$\langle u_1, u_2 \rangle_{1/R} = \int_0^T \frac{u_1 u_2}{R}\, \mathrm{d}t. \tag{5}$$

In this way the function $g(u)$ is defined making the transition in time domain from the voltage u across the nonlinear element to the controlled source e, which ensures the correction according with the nonlinear characteristic.

A necessary condition for $g: \mathrm{R} \to \mathrm{R}$ be a contraction is that function f is a Lipschitz and uniformly monotone function [8, 11–15],

$$0 < \frac{1}{R_{\max}} \overset{\text{def}}{=} \lambda \le \left\| \frac{f(u_1) - f(u_2)}{u_1 - u_2} \right\| \le \Lambda \overset{\text{def}}{=} \frac{1}{R_{\min}}, \tag{6}$$

$\forall\, u_1, u_2$ and $u_1 \ne u_2$.

By substituting the nonlinear elements with controlled generators, one will get a linear periodic regime circuit allowing a single phase solving, on sequences, as discussed in more detail in [14]. For each harmonic of rank k the linear circuit at the terminals of the nonlinear source can be replaced with the equivalent generator having the complex source emf $\underline{E}_{g_k}$ and impedance $\underline{Z}_{e_k}$. The linear function h_k results immediately, by which it the voltage vector $\underline{U}_k$ is obtained at the terminals of the nonlinear element having the controlled source k-harmonic emf $\underline{E}_k$ [14, 15], given as

$$\underline{U}_k = (\underline{E}_k + \underline{E}_{g_k})\underline{Z}_{e_k} / (\underline{Z}_{e_k} + R) = h_k(\underline{E}_k). \tag{7}$$

From (7) we get

$$h_k\left(\underline{E}'_k\right) - h_k\left(\underline{E}''_k\right) = \left(\underline{E}'_k - \underline{E}''_k\right)\frac{\underline{Z}_{e_k}}{\underline{Z}_{e_k} + R}. \tag{8}$$

In (8) we notice that h_k is non-expansive. The vector $\underline{U}$ of the voltages at the terminals of the nonlinear element has the components given by (7), resulting $\underline{U} = h(\underline{E})$, where h is a linear diagonal operator of components h_k. Vector $\underline{U}$ has the role in frequency domain to connect with the linear part of the circuit. Function h is also non-expansive.

Using the above-defined functions $g(u)$ contractive and $h(\underline{E})$ non-expansive, one can construct the Picard-Banach sequence implementing the following iterative process (from iteration n to $n + 1$) [14]:

$$e^{(n)} \xrightarrow{F} \underline{E}^{(n)} \xrightarrow{h} \underline{U}^{(n)} \xrightarrow{F^{-1}} u^{(n)} \xrightarrow{g} e^{(n+1)}, \tag{9}$$

where F and F^{-1} are the direct and inverse Fourier transforms, respectively.

The iterative process is contractive, being obtained by composing a contraction with non-expansive functions [13–15].

2.2. CURRENT CORRECTION OF THE CONTROLLED SOURCE

If f is a Lipschitz and uniformly monotone function, it is invertible, and its inverse f^{-1} also results as a Lipschitz and monotone function [13]:

$$0 < \frac{1}{G_{\max}} \overset{\text{def}}{=} \lambda_i = \frac{1}{\Lambda} \le \left\| \frac{f^{-1}(i_1) - f^{-1}(i_2)}{i_1 - i_2} \right\| \le \frac{1}{\lambda} = \Lambda_i \overset{\text{def}}{=} \frac{1}{G_{\min}}$$
$$\forall\, i_1, i_2 \text{ and } i_1 \ne i_2. \tag{10}$$

The nonlinear elements of characteristic $u = f^{-1}(i)$ are being replaced this time with controlled current generators i_s and internal conductance G, allowing us to write

$$i = i_s + G\, u, \tag{11}$$

with $$i_s = i - G f^{-1}(i) = i\left(1 - G\frac{f^{-1}(i)}{i}\right) = g_i(i). \tag{12}$$

Similar to Section 2.1, the value of the conductance G is chosen in such a way that the function $g_i(i)$ represents a contraction [14].

In order to define the norm relative to the currents, we consider in the Hilbert space of periodic functions of period T the weighted scalar product (with the factor $1/G$) [11, 13]:

$$\langle i_1, i_2 \rangle_{1/G} = \int_0^T \frac{i_1 i_2}{G}\, \mathrm{d}t. \tag{13}$$

In the same manner as presented in Section 2.1, on each harmonic k, the linear circuit connected to the nonlinear current source is substituted with the equivalent current generator made up of the complex current source $\underline{I}_{g_k}$ and the equivalent complex admittance $\underline{Y}_{e_k}$. The k-harmonic of the current through the nonlinear element becomes

$$\underline{I}_k = h_{i_k}(\underline{I}_{s_k}) = (\underline{I}_{s_k} + \underline{I}_{g_k})\underline{Y}_{e_k} / \left(\underline{Y}_{e_k} + G\right). \tag{14}$$

Function h_{i_k} is non-expansive. Suppose we consider N harmonics for solving the circuit. Then, the vector $\underline{I}$ of the currents through the nonlinear element has its components given by (14) resulting $\underline{I} = h_i(\underline{I}_{s_k})$, where h_i is a linear diagonal operator of components h_{i_k}. Operator h_i assures the link with the linear part of the circuit, and is also non-expansive.

The iterative process utilizing the correction of the current provided by the controlled source can be summarized by the following successive transformations

$$i_s^{(n)} \xrightarrow{F} \underline{I}_s^{(n)} \xrightarrow{h_i} \underline{I}^{(n)} \xrightarrow{F^{-1}} i^{(n)} \xrightarrow{g_i} i_s^{(n+1)}. \tag{15}$$

3. SPECIFIC CONVERGENCE ACCELERATION PROCEDURES FOR THE PROPOSED METHOD

In this category we include the acceleration procedures that implicitly result from the application of the method: the optimal choice of the calculation resistance R, the voltage or current correction for the controlled source, as well as the use of over-relaxation.

3.1. OPTIMAL CHOICE OF COMPUTATIONAL RESISTANCE R

The choice of the calculation resistance R (internal resistance of the generator with which we replaced the nonlinear element) must be made in a certain interval to ensure the convergence of the method. This choice can influence the contraction factor and the convergence speed of the algorithm.

3.1.1. The choice of parameter r in case of voltage correction

Assuming that $g(u)$ is a contraction, it follows from equations (3) and (4) that

$$\|u_1 - R f(u_1) - (u_2 - R f(u_2))\| \le \theta_g \|u_1 - u_2\| \text{ for each } u_1, u_2. \quad (16)$$

If we consider that $f : \mathrm{R} \to \mathrm{R}$, we have

$$\left|1 - R\frac{f(u_1) - f(u_2)}{u_1 - u_2}\right| = \left|1 - \frac{R}{\Delta R_{12}}\right| \le \theta_g < 1 \quad (17)$$

with $\Delta R_{12} \in [R_{\min}, R_{\max}]$.

We have in view that the argument and the value of f are the functions $u, i : [0, T] \to \mathrm{R}$. Function g is contractive if $\forall\, t \in [0, T]$, $g(u(t))$ is a contraction.

To ensure that $g(u)$ is a contraction, R must be chosen in the range $(0, 2R_{\min})$ [13, 14].

The correction factor is given by

$$\theta_g = \max\left[\left(1 - \frac{R}{R_{\max}}\right), \left(\frac{R}{R_{\min}} - 1\right)\right]. \quad (18)$$

Reference [13] proposes a solution for minimizing θ_g, by choosing $R = R_{\text{opt}} = \frac{2R_{\min}R_{\max}}{R_{\min} + R_{\max}}$, which leads to the lowest contraction factor $\theta_{g_{\text{opt}}} = \frac{R_{\max} - R_{\min}}{R_{\min} + R_{\max}}$. Note that it is not mandatory for the convergence rate to be higher when we use the optimal resistance, because this rate depends on the values of the arguments u and implicitly on the moduli of the applied coefficients u that correct the voltage sources as $e = g(u)$. The optimal contraction has the advantage that the evaluation of the error in the iterative construction of the Picard-Banach sequence is more efficient, considering that:

$$\left\|e^{(n+1)} - e^*\right\| \le \frac{\theta}{1-\theta}\left\|e^{(n+1)} - e^{(n)}\right\|, \quad (19)$$

where e^* is the fixed point and $\theta < \theta_g$, the correction factor of the mapping (9) [8, 13–15].

From (3), it can be noticed that the contraction factor θ_g is a majorant of the modulus of the coefficients applied to u within the function

$$e = g(u) = u\left(1 - \frac{R}{R_u}\right). \quad (20)$$

A minimum of the modulus of these coefficients for the choice interval of $R \in (0, R_{\min}]$ is obtained for $R = R_{\min}$, with $\theta_g = \left(\frac{R_{\max} - R_{\min}}{R_{\max}}\right)$. In order to ensure a better convergence rate, the selection range of R can thus be restricted to the interval $[R_{\min}, 2R_{\min})$.

For $R > R_{\min}$, part of the coefficients will be lower in absolute value than in the case of $R = R_{\min}$, but there are values of R_u (values close to $R_{\min}$) for which the coefficients will have a greater absolute value. Without knowing the distribution of arguments u, one cannot narrow the selection range any further.

The non-expansive function h is determined by the properties of the circuit connected to the terminals of the nonlinear element. By truncating the Fourier series, h is practically a contraction. From (7) and (8) it is observed that for a harmonic of rank k the correction coefficient of $\underline{E_k}$ becomes

$$\underline{Z_{e_k}} / (\underline{Z_{e_k}} + R) = 1/(1 + R/\underline{Z_{e_k}}). \quad (21)$$

For a given circuit and for a finite number of harmonics, there is a maximum subunit absolute value for the expression (21). In this case a contraction factor can be defined for the function h in the following manner

$$1 > \theta_h = \max_k \left|1 / (1 + R/\underline{Z_{e_k}})\right|. \quad (22)$$

A higher value of R ensures a lower value contraction factor for h, as well as lower values for the modulus of the coefficients of $\underline{E_k}$.

From (9) it can be seen that the contraction factor is a composition of the contraction factors of the component functions: g and h, as well as the contraction factor generated by F as a result of the performed series truncation.

To conclude, in order to ensure the best possible convergence speed, the choice for R must be made in the interval $[R_{\min}, 2R_{\min})$. Most probably, the choice of R close to R_{opt} is likely to ensure a very good convergence rate. The selection interval of R could be narrowed in some particular cases depending on: the values of the circuit elements, the nonlinear characteristic, as well as the properties of the signal we are trying to determine.

It should be noted that in the case of a "hard" nonlinearity ($R_{\min}$ negligible *vs.* $R_{\max}$), R_{opt} is very close to $2R_{\min}$, and the process may become divergent or with "oscillating convergence" due to computation errors. We will analyze this situation in the chapter dedicated to the illustrative example.

3.1.2. The choice of parameter g in case of current correction

Similar to the approach of Subsection 3.1.1, we impose that $g_i(i)$ is a contraction, and we get:

$$\left\|i_1 - Gf^{-1}(i_1) - (i_2 - Gf^{-1}(i_2))\right\| \le \theta_{g_i}\|i_1 - i_2\| \text{ for each } i_1, i_2, \quad (23)$$

respectively

$$\left|1 - G\frac{f^{-1}(i_1) - f^{-1}(i_2)}{i_1 - i_2}\right| = \left|1 - \frac{G}{\Delta G_{12}}\right| \le \theta_{g_i} < 1, \quad (24)$$

with $\Delta G_{12} \in [G_{\min}, G_{\max}]$.

For $g_i(i)$ to be contractive, G must be chosen within the interval $(0, 2G_{\min})$ [13,14].

The contraction factor becomes

$$\theta_{g_i}=\max\left[\left(1-\frac{G}{G_{\max}}\right),\left(\frac{G}{G_{\min}}-1\right)\right]. \qquad (25)$$

One can use the conductance value $G_{\text{opt}}=\frac{2G_{\min}G_{\max}}{G_{\min}+G_{\max}}$, which produces the smallest contraction factor $\theta_{g_{i\,\text{opt}}}=\frac{G_{\max}-G_{\min}}{G_{\min}+G_{\max}}$.

Similar to Subsection 2.1, the function $h_i(\underline{I_s})$ has for a harmonic of rank k the correction coefficient for $\underline{I_{s_k}}$ given by

$$\underline{Y_{e_k}}/\left(\underline{Y_{e_k}}+G\right)=1/(1+G/\underline{Y_{e_k}}) \qquad (26)$$

and, as a result of the truncation of the Fourier series, it becomes a contraction with a contraction factor given by

$$1>\theta_{h_i}=\max_k\left|1/(1+G/\underline{Y_{e_k}})\right|. \qquad (27)$$

A greater value of parameter G ensures a lower value contraction factor for h, as well as lower values for the modulus of the coefficients of $\underline{I_{s_k}}$.

The overall contraction factor is a composition of the contraction factors of the component functions g_i and h_i, as well as the contraction factor generated by F as a result of the series truncation.

The previous subsection conclusions are maintained: in order to ensure the best possible convergence speed, the choice of G must be made in the interval $[G_{\min}, 2G_{\min})$. Therefore, most probably, the choice of G close to G_{opt} is likely to ensure a very good convergence rate.

3.2. VOLTAGE OR CURRENT CORRECTION FOR THE CONTROLLED SOURCE

As we have shown, the Hănțilă method allows the solving of the nonlinear three-phase circuits utilizing two dual variants, namely voltage or current correction of the controlled source. Depending on the properties of the circuit connected to the terminals of the nonlinear element, one of the variants may provide a shorter computation time and a smaller iteration number, compared to the other.

The moduli of the correction and contraction factors can be readily evaluated for the voltage correction θ_g, θ_h and for the current correction θ_{g_i}, θ_{h_i}, respectively. These factors depend on parameters $R_{\min}$ ($G_{\max}$), $G_{\min}$ ($R_{\max}$), on the way R and G are respectively chosen, as well as on $\underline{Z_{e_k}}$, $\underline{Y_{e_k}}$.

If parameters R and G are chosen as $R=xR_{\min}$ and $G=xG_{\min}$ with $x \in (0, 2)$, then for the same value of factor x, we will get $\theta_g=\theta_{g_i}$. Additionally, we will also have $\theta_{g_{\text{opt}}}=\theta_{g_{i\,\text{opt}}}$.

The evaluation must practically be performed for θ_h and θ_{h_i}, and for the moduli of the correction coefficients of h and h_i on the harmonics.

Based on this evaluation, one can opt for the faster calculation option between the two.

In the case of inductive circuits ($|\underline{Z_{e_k}}|$ increases with increasing k), the modulus of harmonic coefficients of h $\left|1/(1+R/\underline{Z_{e_k}})\right|$ increase and approach 1 as k increases, while those of h_i, $\left|1/(1+G/\underline{Y_{e_k}})\right|$, decrease and approach zero. If a large number of harmonics are used, the current correction may prove to be faster. Conversely, in the case of a capacitive circuit, the voltage correction could be faster. However, the limit values between which the coefficient moduli increase or decrease are also important. As we will further discuss in the illustrative example, a decision can only be made by analyzing the modulus of the coefficients on harmonics.

3.3. OVER-RELAXATION

Being based on a Picard-Banach type sequence, an assured convergent procedure, the method allows the use of over-relaxation, which in many cases may significantly reduce the number of iterations, hence the computation time as discussed in [8, 10, 14, 15]:

$$\underline{E'^{(n)}}=\underline{E^{(n-1)}}+\omega\,(\underline{E^{(n)}}-\underline{E^{(n-1)}}) \text{ with } \omega>1. \qquad (28)$$

A useful criterion to evaluate the choice for the over-relaxation factor ω is to assess the evolution of the error (distance) between two successive iteration values defined as

$$\varepsilon^{(n)}\stackrel{\text{def}}{=}\left\|\underline{E^{(n)}}-\underline{E^{(n-1)}}\right\|. \qquad (29)$$

We will always have $\quad \varepsilon^{(n)}<\varepsilon^{(n-1)}$. (30)

For the over-relaxation applied to iteration n to be effective, the error obtained at the next iteration $n+1$ must be smaller compared to the case when over-relaxation was not applied,

$$\varepsilon'^{(n+1)}<\varepsilon^{(n+1)}. \qquad (31)$$

Note that, as indicated by (30), the error at iteration $n+1$ is always smaller than that at iteration n, regardless over-relaxation is applied or not.

The algorithm can be used with a fixed value for ω, or a procedure for searching the values of ω can be developed by testing the inequality (31).

If the contraction factor is significantly less than 1, the use of over-relaxation may not bring a significant improvement.

For solving electromagnetic field problems in nonlinear media, in [8], a procedure for determining a “dynamic” over-relaxation factor is proposed. This one is calculated in two steps so that the next iteration error, given by (29), is minimal. In reference [8], the procedure is used only to speed up the obtaining of the first harmonic. Unlike the problem of a nonlinear resistive element, in the case of the electromagnetic field, the nonlinear function f has vectorial variables and values, providing intricate partial derivatives relationships, in the case when the harmonics are taken into account. In a follow-up paper, we intend to develop such a solution for evaluating a “dynamic” over-relaxation factor in the case of electrical circuits with nonlinear elements.

4. ILLUSTRATIVE EXAMPLE

The present section is devoted to the analysis of a three-phase circuit, quite similar to the one in solved in [15], as shown in Fig. 1. Let us consider the following characteristics for the circuit elements: sinusoidal three-phase generator of

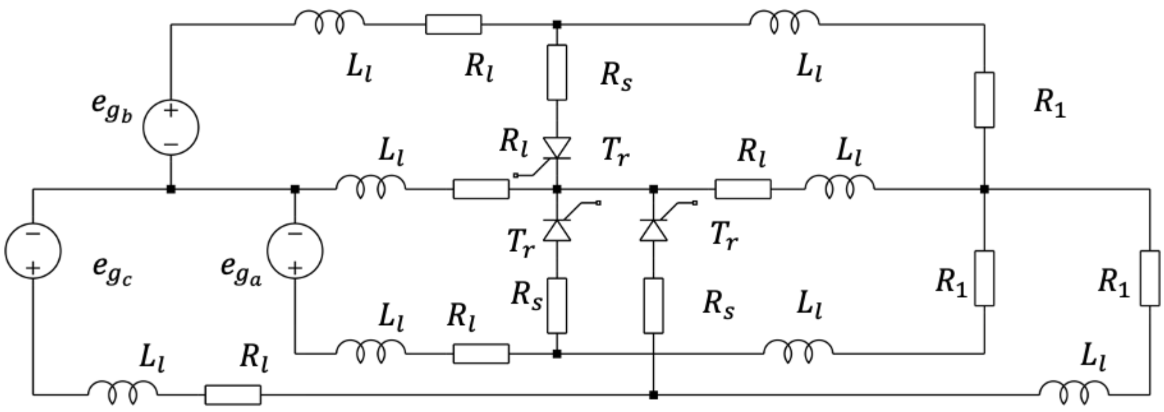

Fig. 1 – The proposed three-phase circuit to be solved.

symmetrical voltage sources with 325 V in amplitude and frequency 50 Hz, $R_l = 1\ \Omega$, $R_1 = 2\ \Omega$, $R_s = 10\ \Omega$, $L_l = 5\times10^{-4}$ H, and $L_1 = 5\times10^{-2}$ H.

For the identical thyristors T_r, we will consider the linearized characteristic described in [15, 16], as depicted in Fig. 2. Obviously, one can use some other nonlinear circuit elements and different current-voltage characteristics.

As shown in Fig. 2a, below the threshold value $u < V_f$, there is the blocking conductance G_b. Above the threshold $u \geq V_f$, the thyristor is characterized by the conduction conductance G_c. The dependence shown in Fig. 2b is valid before the control signal is applied to the gate and after the voltage u drops below the threshold value V_f, being described by the blocking conductance G_b.

The characteristic function of (1) is in this particular case the following piecewise function spanning signal period T:

$$i = f(u) = \begin{cases} uG_c + V_f(G_b - G_c), & t \in \begin{cases} [t_\alpha, t_b), & t_b < T \\ [0, t_b) \cup [t_\alpha, T], & t_\alpha > t_b \end{cases} \\ uG_b, & \text{for the rest of period } T \end{cases} \qquad (32)$$

where t_b is the blocking time at which the condition $u < V_f$ is fulfilled for the first time after the disappearance of the control signal applied to the gate. The inequality $t_\alpha > t_b$ occurs when the conduction started in the previous period and is maintained in the present period until the condition $u < V_f$ is met.

For the thyristors T_r depicted in Fig. 1, the following parameters are considered: blocking resistance $R_b = 1/G_b = 10^4\ \Omega$, conduction resistance $R_c = 1/G_c = 0.05\ \Omega$, $V_f = 5$ V and $\alpha = \pi/5$ (corresponding in terms of time to $t_\alpha = T/10$).

We will truncate the Fourier series up to the 100th harmonic rank, inclusively, and divide the period T also by a number of 40,000 equally spaced points, using the computation algorithm for F and F^{-1} described in [15]. We will thus ensure a sufficiently good calculation accuracy for higher harmonics as well. We will stop the iterations when the relative distance (relative error) $\varepsilon^{(n)}/\|e_g\|$ falls below 10^{-8}. We use the relative distance to be able to compare the calculated results with different metrics (different values of R, G). To simulate the computational algorithm, we used GNU Octave 6.2.0 environment [17]. After calculating the value of the controlled voltage source, it is possible to calculate the currents and voltages for all circuit elements [13– 15].

According to the Section 3.2, we analyze the modulus of the coefficients h and h_i for each harmonic, as well as for the contraction factors θ_h and θ_{h_i}.

For $R = R_{\min} = R_c$ and $G = G_{\min} = G_b$, the values of the modulus of the coefficients resulting from (21) for the function h are decreasing on Fortescue sequences, with the increase of harmonic order k, exhibiting a minimum value of 0.99546 and a maximum value $\theta_h = 0.99982$.

Following the analysis of the modulus of the coefficients on harmonics we have 77 lower values for the function h and only 24 lower values for the function h_i (only on the harmonics multiple of 3 and of high frequency cases).

Similarly, for $R = R_{\text{opt}}$ and $G = G_{\text{opt}}$ the values of the modulus of the coefficients resulting from (21) for the function h are decreasing on Fortescue sequences with the increase of k with a minimum value of 0.99097 and a maximum value $\theta_h = 0.99964$. The moduli of the coefficients

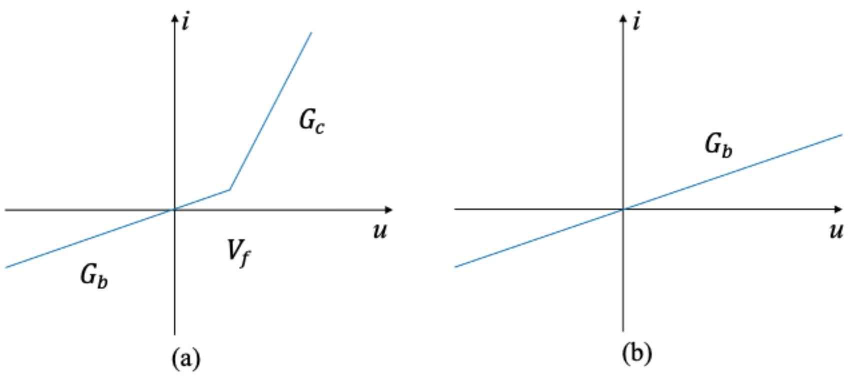

Fig. 2 – Linearized characteristic of the thyristor: a) with gate control applied signal; b) without gate control signal.

of the function h_i resulting from (26) are increasing on sequences, with a minimum value of 0.99719 and a maximum value $\theta_{h_i}= 0.99781$. Following the analysis of the modulus of the coefficients on harmonics we have 77 lower values for the function h and only 24 lower values for the function h_i (only on the harmonics multiple of 3 and high frequency cases).

Although the overall contraction factor is favorable to the current correction given that $\theta_{h_i} < \theta_h$, due to the distribution of the modulus of the coefficients for the two functions h and h_i favorable to the voltage correction, we expect the voltage correction to be faster than the current correction one.

If truncation were to be done for a larger number of harmonics, it might be useful to use of a hybrid voltage / current correction procedure that we are going to discuss in Part II of the article.

4.1. VOLTAGE CORRECTION CASE

If we use the voltage correction for the nonlinear element source, from (3) and (33) we get the function $g(u)$:

$$g(u)=\begin{cases} u\left(1-\frac{R}{R_c}\right)+\frac{RV_f(R_b-R_c)}{R_bR_c},\ t\in\begin{cases}[t_\alpha,t_b),\ t_b<T\\ [0,t_b)\cup[t_\alpha,T],\ \ t_\alpha>t_b\end{cases} \\ u\left(1-\frac{R}{R_b}\right),\ \text{for the rest of period } T.\end{cases} \tag{33}$$

To be sure that $g(u)$ is a contraction, parameter R must be chosen in the interval $(0, 2R_c)$. The interval can be reduced to $[R_c, 2R_c)$, according to Section 3.1.

By choosing $R = R_c$, we will get:

$$g(u)=\begin{cases} \frac{V_f(R_b-R_c)}{R_b}\ ,\ \ t\in\begin{cases}[t_\alpha,t_b),\ t_b<T\\ [0,t_b)\cup[t_\alpha,T],\ \ t_\alpha>t_b\end{cases} \\ \frac{u(R_b-R_c)}{R_b},\ \text{for the rest of period } T.\end{cases} \tag{34}$$

By replacing the numerical values, we obtain from (34): for the first branch a coefficient of u equal to 0, for the second branch 0.999995; the contraction factor θ_g for the function $g(u)$ becomes 0.999995.

Table 1 shows the computation times and the number of iterations obtained using different values of R, including for the use of over-relaxation. We chose for the present study a fixed value for the over-relaxation factor, namely ω = 30. It was found that the benefits achieved even in this case are significant. In a future paper we will present detailed algorithms for determining the values of ω.

Table 1
Voltage correction solution for different values of R with and without over-relaxation

	Voltage correction method	No. of iterations	Computation time [s]	$\theta_g\times\theta_h$
1	$R = 0.8\ R_{\min}$	9305	1664	0.99985
	Over-relaxation factor 30	2193	801	
2	$R = R_{\min}$	7748	1361	0.99981
	Over-relaxation factor 30	1767	675	
3	$R = 1.5\ R_{\min}$	5534	961	0.99971
	Over-relaxation factor 30	**1362**	**506**	
4	$R = 1.8\ R_{\min}$	4750	831	0.99966
	Over-relaxation factor 30	1943	730	
5	$R = 1.9\ R_{\min}$	4539	801	0.99964
	Over-relaxation factor 30	2685	1014	
6	$R = R_{\text{opt}}$	4535	809	0.99962

We mention that simulations were carried out utilizing a MacBook Pro laptop with the following setup: 2.3 GHz 8-Core Intel Core i9 processor and 16 GB 2667 MHz DDR4 memory.

According to the results shown in Table 1, a higher resistance value R reduces the computation time and number of iterations. In the case of the use of overrelaxation, the minimum number of iterations was obtained for $R = 1.5\ R_{\min}$, being also the fastest obtained result. For $R = 1.9\ R_{\min}$, using overrelaxation, there was a decrease in the number of iterations, but a significant increase in computation time. The time saving generated by the introduction of over-relaxation is here less than the time required to perform additional calculations. Note that, an over-relaxation factor could not be adopted for $R = R_{\text{opt}}$.

Starting with $R = 1.8\ R_{\min}$, in the beginning of the iterations, we experienced “oscillating convergence” due to calculation errors and approaching the limit of the interval in which the convergence is ensured. The oscillating behavior is more pronounced with the increase of the R value and the approach of $2R_{\min}$. This is probably one of the reasons why there was no value greater than 1that could be adopted for ω in the case of $R = R_{\text{opt}}$.

Figure 3 shows some details of the evolution graph of the relative error in the cases mentioned above.

Note that the model with linearized characteristic is an extreme case in which the modulus of the applied coefficients u is equal to the contraction factor, being very close to 1. In the case of a real characteristic the values of the modulus of the applied coefficients u are distributed in the range [0, 1). The phenomenon could be reduced by using a larger number of significant digits when performing calculations (but with the disadvantage of increasing the data volume and computation time). Other solutions would be reducing the R value or correcting the nonlinear characteristic by using a less harsh dependence. We will analyze these solutions in the second part of our study.

Figure 4 shows the voltage across the nonlinear element, respectively for the controlled source ($R = R_{\min}$) in the time domain and the harmonic spectrum up to the 50$^{\text{th}}$ harmonic. The appearance of the Gibbs phenomenon is observed.

4.2. CURRENT CORRECTION CASE

For period T we define the following piecewise functions:

$$u=f^{-1}(i)=\begin{cases} i/\,G_c-V_f(G_b-G_c)/G_c\ \ t\in\begin{cases}[t_\alpha,t_b),\ t_b<T\\ [0,t_b)\cup[t_\alpha,T],\ \ t_\alpha>t_b\end{cases} \\ i/G_b\,,\qquad\qquad \text{for the rest of period } T\end{cases} \tag{35}$$

and

$$i_s=g_i(i)=\begin{cases} i\left(1-\frac{G}{G_c}\right)+V_f\frac{G(G_b-G_c)}{G_c},\ t\in\begin{cases}[t_\alpha,t_b),\ t_b<T\\ [0,t_b)\cup[t_\alpha,T],\ \ t_\alpha>t_b\end{cases} \\ i\left(1-\frac{G}{G_b}\right),\ \text{for the rest of period } T.\end{cases} \tag{36}$$

To ensure that $g_i(i)$ is a contraction, G must be chosen in the range $(0, 2G_b)$. The range can be narrowed to $[G_b, 2G_b)$ to optimize the convergence rate according to Subsection 3.2.

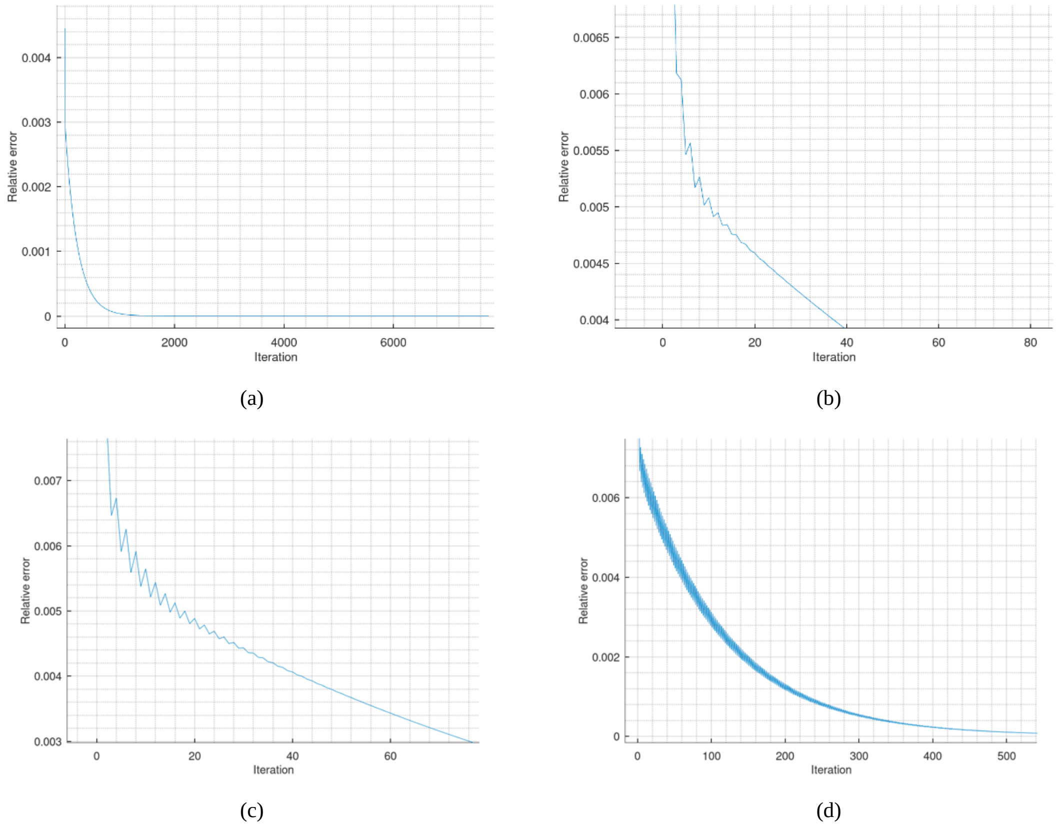

Fig. 3 – Evolution graph of the relative error $\varepsilon^{(n)}/\|e_g\|$ (detail) *vs.* number of iterations for: a) $R = R_{\min}$; b) $R = 1.8\ R_{\min}$; c) $R = 1.9\ R_{\min}$, (d) $R = R_{\text{opt}}$.

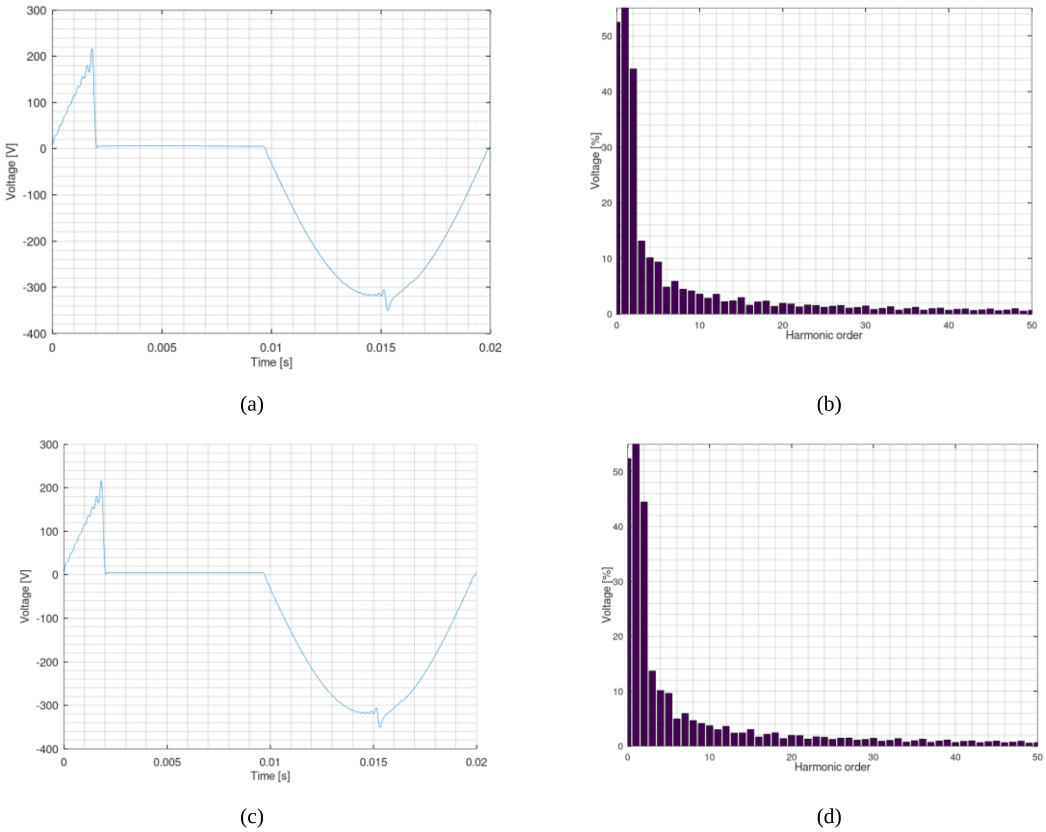

Fig. 4 – Voltage across thyristor T_r: a) in time domain; b) detail of the harmonic spectrum; the controlled voltage source voltage for $R = R_{\min}$; c) in time domain; d) detail of the harmonic spectrum.

Table 2 shows the computation times and the number of iterations obtained using different values of G, including for the use of over-relaxation.

Even in the current correction case, the conclusions drawn at the end of the previous subsections are still valid: a greater value conductance G leads to a reduction of the computation time and of the number of performed iterations. If over-relaxation is used, the fastest solution corresponded to $G = 1.5\ G_{\min}$.

Table 2
Current correction solution for different values of G with and without over-relaxation

	Current correction method	No. of iterations	Computation time [s]	$\theta_{g_i} \times \theta_{h_i}$
1	$G = 0.8\ G_{\min}$	19559	3520	0.99912
	Over-relaxation factor 30	4484	1734	
2	$G = G_{\min}$	15648	2809	0.99889
	Over-relaxation factor 30	3482	1317	
3	$G = 1.5\ G_{\min}$	10435	1881	0.99834
	Over-relaxation factor 30	**2429**	**895**	
4	$G = 1.8\ G_{\min}$	8698	1550	0.99802
	Over-relaxation factor 30	3399	1279	
5	$G = 1.9\ G_{\min}$	8240	1459	0.99791
	Over-relaxation factor 30	4715	1768	
6	$G = G_{\text{opt}}$	8437	1494	0.99791
	Over-relaxation factor 1.1	8287	2955	

For the case $G = G_{\text{opt}}$ we could adopt an over-relaxation factor of just 1.1. For $G = 1.9\ G_{\min}$ and $G = G_{\text{opt}}$ the use of over-relaxation reduces the number of iterations, but significantly increases the computation time.

Starting with $G = 1.5\ G_{\min}$, we experienced again "oscillating convergence" in the early stage of the iterations, the phenomenon being more pronounced with an increasing G value, becoming even more pronounced for $G = G_{\text{opt}}$.

That confirms the decision taken following the analysis of the coefficient moduli, namely that the voltage correction method is (for this particular example) more rapid than that of current correction.

5. CONCLUSIONS

The Hănțilă method provides a useful calculation tool that can be easily utilized from the dimensioning and design stage of three-phase circuits and power distribution networks to effectively address the distorting effect of nonlinear elements and to facilitate the power transfer evaluation on harmonics. Consequently, it allows the implementation and check the effectiveness of the possible adopted corrective measures. The method brings a spectacular reduction of the computation volume because it allows a single-phase solving approach. Convergence is ensured and it is not necessary to use relaxation (as it is the case of other methods). In the case characterized by contraction factors close to 1, over-relaxation can be used effectively and computation time-saving can be significant. If a large number of harmonics needs to be taken into account, the data volume and computation time are expected to increase significantly. Depending on the actual nonlinear characteristic and the circuit connected at the terminals of the nonlinear element, situations may occur in which the contraction factor used in the algorithm has values very close to 1. In such cases, acceleration procedures may prove very useful.

The analyzed acceleration procedures proved to be useful from the perspective of the easiness to implement in circuit analysis software. Other acceleration procedures will be analyzed in Part II, *i.e.*, overrelaxation, harmonic selection, hybrid voltage / current correction method, the use of "less harsh" nonlinear characteristics with better contraction factors, the use of modified values for the linear circuit elements, and last but not least the correction of the nonlinear characteristic by incorporating some other existing elements from the circuit. For the use of over-relaxation, we consider useful to consider the possibility to introduce a "dynamic" over-relaxation factor, similar to the solution presented in [8] for the electromagnetic field in nonlinear media. As mentioned earlier, we intend to develop such a solution in a follow-up paper. We consider that an important direction for the development of the method is the increase of the computation speed. This can be done by: identifying more efficient algorithms, as well as by developing acceleration procedures.

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