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EXPERIMENTAL DETERMINATION OF LINEAR DYNAMIC MODEL PARAMETERS OF THE SEPARATELY EXCITED BRUSHED DC MOTOR BY MODIFIED PÁSEK'S METHOD

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Keywords: Separately excited brushed dc motor; Linear dynamic model parameters; Experimental determination; Modified Pásek's method.

This paper aims to propose a modified version of Pásek's method for determining the linear dynamic model parameters of the separately excited brushed dc motor, *i.e.*, equivalent electrical resistance and self-inductance of a rotor-armature electrical circuit, rotational mutual inductance, viscous friction torque coefficient and moment of inertia referred to the common rotational axis of the rotor, commutator, and motor shaft. The proposed method employs the transient response of rotor-armature electrical current to a sudden step increase in dc voltage at rotor-armature electrical circuit terminals of the motor, which is initially operating at steady-state no-load conditions.

1. INTRODUCTION

Brushed direct-current (dc) motor drives still find wide industrial applications as a preferred choice for variable speed control systems with low-speed developed torque performances. For designing dc motor control systems and fulfilling high-performance criteria, it is paramount to define the dc motor dynamic model and identify its parameters. However, the values for reference of the dc motor dynamic parameters provided by the motor manufacturers are inadequate due to their relatively large tolerances.

In an earlier attempt to specify a testing technique, which would yield the significant dynamic parameters of dc motors, Pásek developed a method of determining linear dynamic model parameters of a separately excited brushed dc motor requiring the only transient response of the rotor-armature electrical current to a step input of rotor-armature voltage for the no-load start-up of the motor, on the assumption of negligible viscous friction torque component [1]. Later, Pásek's method was reviewed and applied to low-inertia dc servomotors [2 – 4].

In the present paper, Pásek's method for experimental determination of second-order linear dynamic model parameters of the separately-excited brushed dc motor is modified by (i) employing the transient response of rotor-armature electrical current to a sudden step increase in dc voltage at rotor-armature electrical circuit terminals of the motor, which is initially operating at steady-state no-load conditions, and (ii) including the viscous friction torque component in the dynamic mechanical equation of the motor.

This paper is organized as follows. In section 2, the linear dynamic model of the separately-excited brushed dc motor is first applied to analyze the motor behavior during the electromechanical transient process due to a sudden step increase in dc voltage at rotor-armature electrical circuit terminals of the motor, which is initially operating under steady-state no-load conditions; then, the transient response of the rotor-armature electrical current is further processed to derive motor dynamic parameter-related equations required by the modified Pásek's method. Section 3 describes in detail the successive steps in the modified Pásek's method and applies them to a sample separately excited brushed dc motor; the values thus obtained for the dynamic parameters of the motor are finally checked by comparison with conventional measurement results. Some conclusions are drawn in the last section.

2. DERIVATION OF BRUSHED DC MOTOR DYNAMIC PARAMETER-RELATED EQUATIONS REQUIRED BY MODIFIED PÁSEK'S METHOD

If the stator-field winding electrical circuit of an unloaded, fully compensated, separately excited brushed dc motor at standstill is supplied by a constant voltage U_f, an excitation electrical current of constant strength I_f is established, after a short while, in the stator-field winding electrical circuit of the motor. By applying further on a step constant voltage U_{a1} (below the rated value U_{aN}) at the terminals of the rotor-armature electrical circuit, the motor is started until reaching a steady-state no-load operating mode characterized by the algebraic electrical and mechanical equations:

$$U_f = R_f I_f \ , \ U_{a1} = R_a I_{a1} + L'_{af} I_f \Omega_1 , \tag{1}$$

$$M_{em1} = L'_{af} I_f I_{a1} = D_M \Omega_1 , \tag{2}$$

where I_{a1}, M_{em1} and Ω_1 stand for the constant strength of the rotor-armature electrical current, the constant value of the electromagnetic developed torque, and the constant rotational mechanical angular speed of the rotor and motor shaft, respectively; R_f , R_a , L'_{af} and D_M represent the electrical resistance of the stator-field winding electrical circuit, the equivalent electrical resistance of the rotor-armature electrical circuit, the rotational mutual inductance and the coefficient of viscous friction torque $D_M\Omega_1$, respectively, all these motor parameters being considered of constant values.

Under these no-load steady-state initial conditions of motor operation, a step increase ΔU_a in the constant voltage U_{a1} is suddenly applied at rotor-armature electrical circuit terminals of the motor, causing an electromechanical transient process that consists of two simultaneous and coupled transient processes: (i) one in the rotor-armature electrical circuit, described by voltage differential equation,

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$$U_{a1}+\Delta U_a=R_a\left(I_{a1}+\Delta i_a(t)\right)+$$
$$+L_a\frac{\mathrm{d}(I_{a1}+\Delta i_a(t))}{\mathrm{d}t}+L'_{af}I_f\left(\Omega_1+\Delta\Omega(t)\right), \qquad (3)$$

where L_a represents the equivalent self-inductance of the rotor-armature electrical circuit; (ii) the other one at the motor shaft, described by the mechanical differential equation,

$$M_{em1}+\Delta m_{em}(t)=L'_{af}I_f\left(I_{a1}+\Delta i_a(t)\right)=$$
$$=J_M\frac{\mathrm{d}(\Omega_1+\Delta\Omega(t))}{\mathrm{d}t}, \qquad (4)$$

where J_M defines the moment of inertia of the motor referred to as the rotational axis of the rotor, commutator, and motor shaft.

By solving with the aid of Laplace transform the linear differential equations (3)–(4) – in which the voltage drop at brush-commutator contact, as well as the magnetic saturation and iron losses in the motor magnetic circuit, are neglected, and relations (1)–(2) are taken into account – one obtains the analytical expression of the transient variation $\Delta i_a(t)$ of rotor-armature electrical current strength with respect to its initial value I_{a1} [5]:

$$\Delta i_a(t)=\frac{\Delta U_a}{L_a}\cdot\frac{T_1T_2}{T_M}\cdot$$
$$\cdot\left(1+T_M\frac{1-T_1/T_M}{T_1-T_2}\mathrm{e}^{-t/T_1}-T_M\frac{1-T_2/T_M}{T_1-T_2}\mathrm{e}^{-t/T_2}\right), \qquad (5)$$

where

$$1/T_{1,2}=(1/2)(1/T_a+1/T_M)\cdot$$
$$\cdot\left[1\mp\sqrt{1-(4/T_a)(1/T_{emec}+1/T_M)/(1/T_a+1/T_M)^2}\right],$$

the time constant of the rotor-armature electrical circuit, the mechanical time constant**,** and the electromechanical (or inertial) time constant of the motor, are, respectively

$$T_a=L_a/R_a\,,\ T_M=J_M/D_M\ \text{, and}\ T_{emec}=J_MR_a/(L'_{af}I_f)^2\,.$$

For a separately excited brushed dc motor with sufficiently high inertia, $T_a<T_2<T_1<T_{emec}<T_M$, thus enables the introduction of suitable approximations in (5):

$$1-T_1/T_M\cong 1,\qquad 1-T_2/T_M\cong 1,$$
$$1/T_a+1/T_M=1/T_a(1-T_a/T_M)\cong 1/T_a, \qquad (6)$$

so that it becomes

$$\Delta i_a(t)\cong\frac{\Delta U_a}{L_a}T_1T_2\left[\frac{1}{T_M}+\frac{\left(\mathrm{e}^{-t/T_1}-\mathrm{e}^{-t/T_2}\right)}{T_1-T_2}\right], \qquad (7)$$

with

$$1/T_{1,2}\cong(1/2T_a)\left(1\mp\sqrt{1-4T_a(1/T_{emec}+1/T_M)}\right)=$$
$$=(1/2T_a)(1\mp a), \qquad (8)$$

and the non-dimensional quantity $a=\sqrt{1-4T_a(1/T_{emec}+1/T_M)}$.

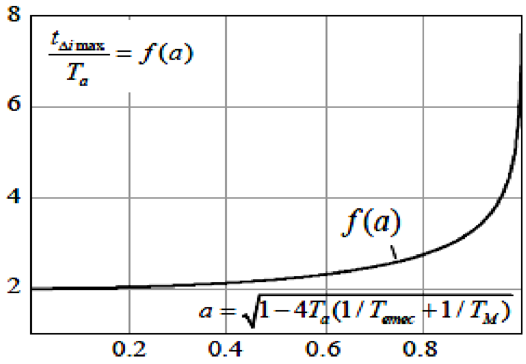

Fig. 1 – Nomogram (in Mathcad 15.0) for function $f(a)$.

During the electromechanical transient process, the aperiodic time function (7) attains its maximum value at an instant $t_{\Delta i\max}$, which is obtained by setting to zero the time derivative of relation (7),

$$\frac{\mathrm{d}(\Delta i_a(t))}{\mathrm{d}t}\cong\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{\Delta U_a}{L_a}T_1T_2\left[\frac{1}{T_M}+\frac{1}{T_1-T_2}\left(\mathrm{e}^{-t/T_1}-\mathrm{e}^{-t/T_2}\right)\right]\right\}=0,$$

with the solution

$$t_{\Delta i\max}\cong\frac{T_1T_2}{T_1-T_2}\ln\left(\frac{T_M/T_2-1}{T_M/T_1-1}\right)\cong\frac{T_a}{a}\ln\left(\frac{1+a}{1-a}\right), \qquad (9)$$

from which,

$$\frac{t_{\Delta i\max}}{T_a}\cong\frac{1}{a}\ln\left(\frac{1+a}{1-a}\right)=f(a), \qquad (10)$$

where the function $f(a)$ is simply represented by the nomogram of Fig. 1.

At the end of the electromechanical transient process, the separately excited brushed dc motor reaches a final steady-state no-load operating mode characterized by the algebraic electrical and mechanical equations:

$$U_f=R_fI_f\,,U_{a2}=U_{a1}+\Delta U_a=R_aI_{a2}+L'_{af}I_f\Omega_2, \qquad (11)$$

$$M_{em2}=L'_{af}I_fI_{a2}=D_M\Omega_2\,, \qquad (12)$$

where according to relation (5),

$$I_{a2}=I_{a1}+\Delta i_a(t\to\infty)=$$
$$=I_{a1}+\frac{\Delta U_a}{L_a}\cdot\frac{T_1T_2}{T_M}=I_{a1}+\Delta I_a. \qquad (13)$$

Using the above relations (7), (8), (9), and (13), one can derive the approximate analytical expression of the ratio:

$$\frac{\Delta i_a(2t_{\Delta i\max})-\Delta I_a}{\Delta i_a(t_{\Delta i\max})-\Delta I_a}\cong\frac{\mathrm{e}^{-2t_{\Delta i\max}/T_1}-\mathrm{e}^{-2t_{\Delta i\max}/T_2}}{\mathrm{e}^{-t_{\Delta i\max}/T_1}-\mathrm{e}^{-t_{\Delta i\max}/T_2}}=$$
$$=\mathrm{e}^{-\frac{T_a}{aT_1}\ln\left(\frac{1+a}{1-a}\right)}+\mathrm{e}^{-\frac{T_a}{aT_2}\ln\left(\frac{1+a}{1-a}\right)}=$$
$$=\left(\frac{1+a}{1-a}\right)^{-\frac{T_a}{aT_1}}+\left(\frac{1+a}{1-a}\right)^{-\frac{T_a}{aT_2}}= \qquad (14)$$
$$=\left(\frac{1+a}{1-a}\right)^{-\frac{1+a}{2a}}\left[1+\left(\frac{1+a}{1-a}\right)^{\frac{T_a}{a}\cdot\frac{1+a-(1-a)}{2T_a}}\right]=$$
$$=\frac{2}{1-a}\cdot\left(\frac{1+a}{1-a}\right)^{-\frac{1+a}{2a}}=g(a),$$

with the representation of function $g(a)$ provided by the nomogram in Fig. 2.

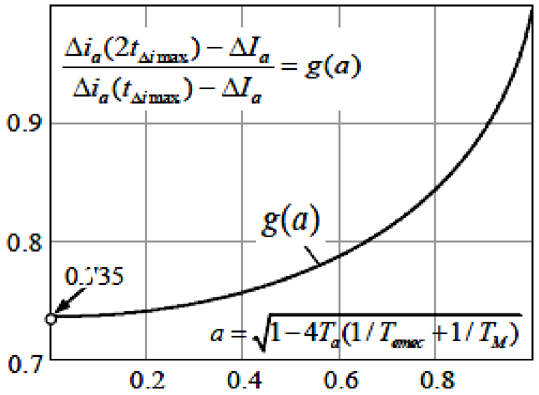

Fig. 2 – Nomogram (in Mathcad 15.0) for function $g(a)$.

The typical time variation of rotor-armature electrical current strength during the electromechanical transient process – due to a sudden step increase ΔU_a in the constant voltage U_{a1} at rotor-armature electrical circuit terminals of the separately-excited brushed DC motor, which is initially operating under steady-state no-load conditions – is shown in Fig. 3, where all quantities involved in the ratio expression (14) are revealed.

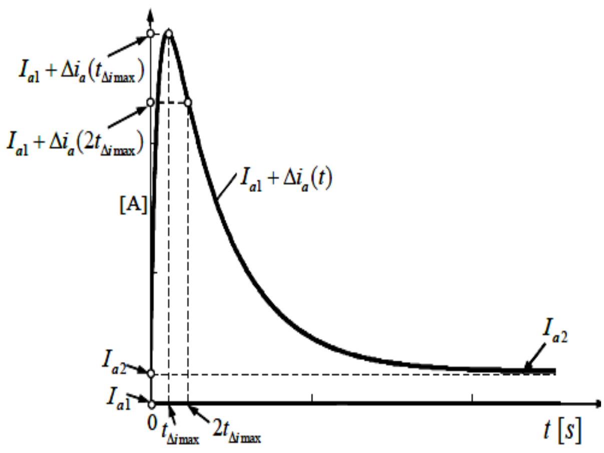

Fig. 3 – Typical time variation of the rotor-armature electrical current strength during the electromechanical transient process occurred in the separately-excited brushed dc motor.

3. MODIFIED PÁSEK'S METHOD-BASED DETERMINATION OF SEPARATELY EXCITED BRUSHED DC MOTOR DYNAMIC PARAMETERS. EXPERIMENTAL RESULTS

The sample separately excited brushed dc motor used for experimental determination of its linear dynamic model parameters by modified Pásek's method has the nameplate data listed in Table 1.

Table 1

Nameplate data of the sample separately excited brushed dc motor

Rated mechanical output power, P_{mecN}	8.1 kW
Rated mechanical rotational speed, n_N	2 720 rpm
Stator-excitation rated voltage, U_{fN}	190 V
Stator-excitation rated electrical current strength, I_{fN}	1.4 A
Rotor-armature rated voltage, U_{aN}	440 V
Rotor-armature rated electrical current strength, I_{aN}	20 A

The experimental steps of the proposed modified Pásek's method for determining the electromagnetic and mechanical dynamical parameters, namely, R_a, L_a, L'_{af}, J_M and D_M, of the sample separately-excited brushed dc motor are the following:

(i) the stator-excitation electrical circuit of the unloaded motor at standstill is supplied by a constant voltage U_f = $= U_{fN}$ = 190 V; hence, after a short while, an excitation electrical current of steady-state strength $I_f = I_{fN}$ = 1.4 A is established in the stator-excitation electrical circuit of the motor;

(ii) constant voltage U_{a1} = 178 V is applied at the terminals of the rotor-armature electrical circuit for the start-up of the motor under no-load conditions until a steady-state operating mode of the motor – defined by rotor-armature electrical current strength I_{a1} = 0.96 A and mechanical rotational speed n_1 = 1 128 rpm – is reached.

(iii) a step increase ΔU_a = 239 V is suddenly applied in the constant voltage U_{a1} = 178 V at the rotor-armature electrical circuit terminal of the motor, causing the experimental time variation of the rotor-armature electrical current strength during the electromechanical transient process occurred in the motor (Fig. 4);

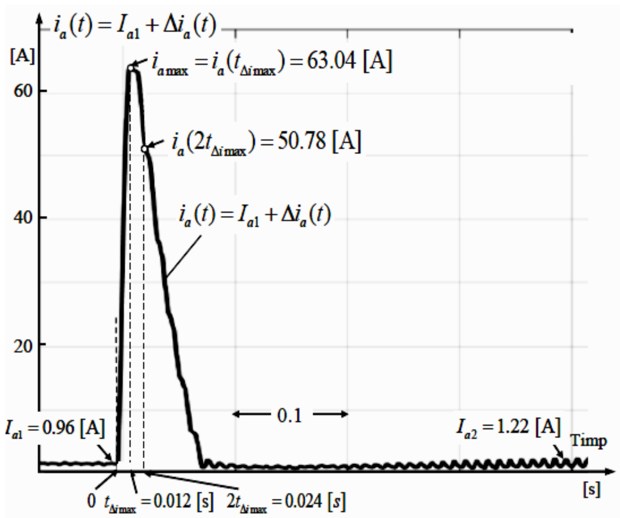

Fig. 4 – Experimental time variation (retrieved from the oscilloscope using MATLAB GUI) of the rotor-armature electrical current strength during the electromechanical transient process occurred in the sample separately-excited brushed dc motor.

(iv) the final no-load steady-state operating mode of the motor, at the end of the electromechanical transient process, is characterized by rotor-armature terminal voltage U_{a2} = = 417 V and electrical current strength I_{a2} = 1.22 A (Fig. 4), and mechanical rotational speed n_2 = 2 660 rpm;

(v) using the recorded values of quantities defining the initial and final steady-state no-load operating modes of the motor, before and after the electromechanical transient process, and introducing these values in relations (1), (2) (11), and (12), one can determine, successively, the rotational mutual inductance L'_{af}, the equivalent electrical resistance of rotor-armature electrical circuit R_a and the coefficient of viscous friction torque D_M :

$$L'_{af} = \frac{U_{a2} - U_{a1} I_{a2} / I_{a1}}{I_f(\Omega_2 - \Omega_1 I_{a2} / I_{a1})} = \frac{417 - 178 \cdot 1.22 / 0.96}{1.4(278.5 - 118.1 \cdot 1.22 / 0.96)} =$$

$$= \frac{190.79}{1.4 \cdot 128.41} = 1.061 \text{ H},$$

$$R_a = \frac{\Delta U_a - L'_{af} I_f (\Omega_2 - \Omega_1)}{\Delta I_a} = \frac{239 - 1.061 \cdot 1.4(278.5 - 118.1)}{0.26} =$$
$$= \frac{239 - 238.26}{0.26} = 2.85 \ \Omega,$$

$$D_M = \frac{\Delta I_a (L'_{af} I_f)^2}{\Delta U_a - R_a \Delta I_a} = \frac{0.26 \cdot (1.061 \cdot 1.4)^2}{239 - 2.85 \cdot 0.26} =$$
$$= \frac{0.574}{238.26} = 2.41 \cdot 10^{-3} \ \mathrm{Nm \cdot s/rad},$$

where $\Delta I_a = I_{a2} - I_{a1} = 1.22 - 0.96 = 0.26$ A, $\Omega_1 = \pi n_1/30 = 1128\pi/30 = 118.1$ rad/s, $\Omega_2 = \pi n_2/30 = 2\,660\pi/30 = 278.5$ rad/s;

(vi) the experimental chronogram of Fig. 4 yields the following numerical data: $t_{\Delta i\max} = 0.12$ s, $2\, t_{\Delta i\max} = 0.024$ s, $i_a(t_{\Delta i\max}) = I_{a1} + \Delta i_a(t_{\Delta i\max}) = 63.04$ A, $i_a(2t_{\Delta i\max}) = I_{a1} + \Delta i_a(2t_{\Delta i\max}) = 50.78$ A, from which, it results

$$\frac{\Delta i_a(2t_{\Delta i\max}) - \Delta I_a}{\Delta i_a(t_{\Delta i\max}) - \Delta I_a} = \frac{[I_{a1} + \Delta i_a(2t_{\Delta i\max})] - I_{a2}}{[I_{a1} + \Delta i_a(t_{\Delta i\max})] - I_{a2}} =$$

$$== \frac{50.78 - 1.22}{63.04 - 1.22} = 0.8017;$$

(vii) using the nomogram of Fig. 2 for function $g(a)$ having the expression (14) with the above value 0.8017, one obtains the abscissa: $a = 0.663$; employing then the nomogram of Fig. 1 for function $f(a)$ having the expression (10), one gets on the ordinate, for $a = 0.663$: $f(a) = t_{\Delta i\max}/T_a = 2.407$;

(viii) considering the experimentally obtained values, $R_a = 2.85\,\Omega, t_{\Delta i\max} = 0.012\,\mathrm{s},\ T_a = t_{\Delta i\max}/2.407\,\mathrm{s}$, one can determine the equivalent self-inductance of the rotor-armature electrical circuit: $L_a = R_a T_a = 2.85 \cdot 0.012/2.407 = 14.2 \cdot 10^{-3}$ H;

(ix) from relations (8) and (13), after algebraic manipulations, one can finally determine the moment of inertia of the motor, referred to the common rotational axis of the rotor, commutator, and motor shaft:

$$\Delta I_a = I_{a2} - I_{a1} = \frac{\Delta U_a}{L_a} \cdot \frac{T_1 T_2}{T_M} \cong \frac{\Delta U_a}{L_a} \cdot \frac{D_M}{J_M} \frac{4T_a^2}{1 - a^2},$$

from which,

$$J_M = \frac{4}{1 - a^2} \cdot \frac{D_M L_a \Delta U_a}{R_a^2 \Delta I_a} =$$
$$= \frac{4 \cdot 2.41 \cdot 10^{-3} \cdot 14.2 \cdot 10^{-3} \cdot 239}{(1 - 0.663^2) 2.85^2 \cdot 0.26} =$$
$$= \frac{32.72 \cdot 10^{-3}}{1.183} = 27.66 \cdot 10^{-3} \ \mathrm{kg \cdot m^2}.$$

Table 2 displays comparative experimental results for three dynamic parameters of the sample separately excited brushed dc motor, obtained by modified Pásek's method and by conventional measurements, respectively.

Table 2

Comparative results for three linear dynamic model parameters of the sample separately excited brushed dc motor, obtained by modified Pásek's method and by conventional measurement

Linear dynamic model parameter of the sample separately excited brushed dc motor	Experimental determination by modifying Pásek's method	Determination by conventional measurements
Equivalent electrical resistance of the rotor-armature electrical circuit, R_a	2.85 Ω	2.76 Ω*
Equivalent self-inductance of the rotor-armature electrical circuit, L_a	$14.2 \cdot 10^{-3}$ H	$13.8 \cdot 10^{-3}$ H**
Rotational mutual inductance, L'_{af}	1.061 H	1.063 H***

* Value measured with the ohmmeter.
** value obtained (using the storage oscilloscope) by evaluating the time constant of a rotor-armature electrical circuit, whose terminals are connected to a square-wave voltage generator.
*** value obtained from nameplate data with prior measurement of the rated electromagnetic torque value, *i.e.*, $L'_{tf} = M_{emN}/I_{fN}I_{aN} = 29.76/1.4 \cdot 20 = 1.063$ H.

Good agreement can be seen in Table 2 between the linear dynamic model parameter values of the sample separately excited brushed dc motor, experimentally determined by the proposed modified Pásek's method and by conventional measurements.

4. CONCLUSIONS

The modified Pásek's method proposed in this paper for experimental determination of (second order) linear dynamic model parameters of the separately excited brushed dc motor emphasizes the following benefits:

(i) it resides only in the analytical study and experimental test of the transient response of rotor-armature electrical current to a sudden step increase in dc voltage at rotor-armature electrical circuit terminals of the motor, which is initially operating at steady-state no-load conditions,

(ii) it allows determining, at once, all linear dynamic model parameters of the motor;

(iii) it also includes the determination of the viscous friction torque coefficient of the motor;

(iv) it is sufficiently accurate and easy to apply as an in-line quality control technique available to brushed dc motor manufacturers and users alike.

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