# OBLONG CORE CROSS-SECTION IMPACT ON POWER TRANSFORMER CHARACTERISTICS 

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On the $1^{\text {st }}$ of July 2021, the second phase of 548/2014 EU Regulation, which implements the 2009/125/EC Directive and imposes the maximum level of losses for transformers, entered into force. Some manufacturers have responded to the challenge of reducing the losses by manufacturing power transformers with an elongated core cross-section. In this context, the paper proposes an algorithm to generalize the steps (stacks of laminations) sizing procedure for the core cross-section of power transformers, an algorithm that allows the transition from a circular to an oblong core cross-section. Thus, the dimensions of the lamination stacks making up the core cross-section could be estimated for any predetermined value of cross-section diameter. An example of using the proposed algorithm and the impact analysis of the oblong cross-section is applied for an oil cooling power three-phase transformer with a 250 kVA rated power. For the oblong cross-section design solution, decreases in the transformer core dimensions and no-load losses were obtained.

## 1. INTRODUCTION

Increased electricity consumption, with a direct correlation to economic progress in any country, has led to the global concern for environmental protection and resource conservation through increased product efficiency. Thus, the 2009/125/EC European Parliament Directive (also known as the Ecodesign Directive) is distinctive as it establishes the framework for defining ecodesign requirements for energy-related products.

The concern for increasing the energy-related product's efficiency is also found in the power transformers domain, which are the most important devices of the power grid and whose losses account for about $5 \%$ of the global electricity consumption [1]. The design requirements for low, medium and high power transformers are imposed by the 548/2014 (EU) Regulation [2]. So, after the $1^{\text {st }}$ of July 2021, transformers that do not meet the minimum requirements cannot be placed on the market or put into service. As a result, the power transformer manufacturers, who are responsible for applying the regulation, have been driven to look for new ways to reduce power transformer losses. One research direction is the analysis of the factors that influence the lamination quality from which the transformer ferromagnetic core is made, with effects on the no-load losses [3, 4]. A different possibility of losses decrease in power transformers is to change the ferromagnetic core cross-section type. Some consulting firms and manufacturing companies are analyzing and announcing the launch of power transformers with an oblong core crosssection (also known as an oval cross-section) [5-8]. There is also a patent for an octagonal section of the core to increase the filling factor up to $99 \%$, which leads to decreased losses than the circular or oblong cross-section constructive solution [9]. Thus, from the practitioner's perspective, it seems that the power transformers with oblong core cross-sections have smaller dimensions, lower mass, and reduced losses [5-9, 11]. However, the literature review revealed very few studies on the oblong core crosssection transformers, mostly an impact analysis of the shape of the oval winding [10-12].

This paper proposes an algorithm to generalize the steps (stacks of laminations) sizing procedure for the core crosssection of power transformers. This algorithm allows the transition from a circular to an oblong core cross-section. Thus, the dimensions of the lamination stacks that the core cross-section consists of could be estimated for any predetermined value of the cross-section diameter.

The paper is structured as follows. Section 2 describes the proposed algorithm. Its testing is done by customizing for a circular core cross-section and comparing the obtained results with those existing in the literature. In Section 3, the impact of changing the core cross-section from circular to oblong on the transformer core dimensions (height, length, and width) and its performance (Joule losses, no-load losses, efficiency) is analyzed. An example of the proposed algorithm is applied for an oil cooling power three-phase transformer with a 250 kVA rated power.

## 2. POWER TRANSFORMER CROSS-SECTION CORE DESIGN

The power transformer magnetic core consists of thin laminations (to reduce eddy currents) of cold-rolled grained oriented electrical steel with $0.28 \div 0.35 \mathrm{~mm}$ thickness, $g$. In terms of construction, the magnetic core cross-section is rectangular (for low power transformers, $S_{\mathrm{n}}<1 \mathrm{kVA}$ ) or stepped (for $S_{\mathrm{n}}>1 \mathrm{kVA}$ ) [13-17]. The reason why the core cross-section is built-in steps (stacks of laminations) is as follows. The lamellar structure of the core would lead to the use of a very large number of laminations to fill the surface of the core cross-section because, in theory, each lamination should have a different width for a perfect filling of the circular cross-section (circular cross-section being the most economical in terms of conductive material consumption). In practice, the number of lamination widths available is limited, and the cross-section consists of a finite number of stacks or steps. The number of steps and the dimensions of each step are standardized elements for each manufacturer [14-17].
In the literature on the power transformers' analytical design, there are some coefficients that relate the diameter of the core cross-section to the step widths for the circular

[^0]core cross-section. These coefficients give the maximum possible coverage of the circular cross-section (Fig.1) for up to 6 steps [16] or 10 steps [14]. This makes it possible to determine the dimensions of the lamination stacks (or steps) that fill up the core cross-section.


Fig. 1 - The section diameter-step width coefficients, for circular core cross-section [16].

As far as the oblong cross-section is concerned, no procedure has been identified to determine steps sizes. To fill this gap, the paper proposes an algorithm for generalizing the stepped core cross-section sizing of the power transformer. It allows computing the dimensions of each step, $\left(a_{i}, b_{i}\right)$, of the core cross-section, for several cases: the circular or the oblong cross-section, different predetermined diameter crosssection values, and a various number of steps. The proposed algorithm is described below:

Stage 1. Choosing the core cross-section type based on the ellipse equation (the mathematical equivalent of the oblong cross-section), equation (1):
a) for circular cross-section: $D_{c x}=D_{c y}=D_{c}$ (Fig. 2a);
b) for oblong cross-section: $D_{c x} \neq D_{c y}$ (Fig. 2b);
$D_{\text {cx }}$ is the diameter corresponding to the first radius of the ellipse, and $D_{\text {cy }}$ is the diameter corresponding to the second radius of the ellipse (Fig. 2). The values of these diameters are analytically predetermined according to the classical design.

$$
\begin{equation*}
\frac{x^{2}}{\left(\frac{D_{c x}}{2}\right)^{2}}+\frac{y^{2}}{\left(\frac{D_{c y}}{2}\right)^{2}}=1 \tag{1}
\end{equation*}
$$


a)

b)

Fig. 2 - Core cross-section types: a) circular; b) oblong: $a_{1}-$ the width of the $1^{\text {st }}$ step (first stacks of laminations width) ; $b_{1}$ - the height of the $1^{\text {st }}$ step (first stacks of laminations thickness)

Stage 2. Setting the number of steps, $n_{\mathrm{i}}$, to maximize the core cross-section area filling; the number of steps corresponds to the size of the core section diameter. For high power transformers and therefore large cross-section sizes, a large number of steps is chosen (in the literature, for a circular section, $n_{\mathrm{i}}$ is between $3 \div 11$ steps [16]);

Stage 3. Width computation of the steps, $a_{\mathrm{i}}$, to maximize the core cross-section area filling. To do this, generate and solve the equation system (2) [14-15]:

$$
\begin{equation*}
\frac{\partial A_{c g}}{\partial a_{i}}=0, \quad i=1, \ldots n \tag{2}
\end{equation*}
$$

where:
$A_{c g}$ - geometric core area;

$$
\begin{equation*}
A_{c g}=2 \cdot \sum_{i=1}^{n} a_{i} \cdot b_{i} \tag{3}
\end{equation*}
$$

$n$ - the number of steps; $a_{i}$ - the width of the steps; $b_{i}-$ the height of the steps (the stacks of laminations thickness).

The geometric core area, $A_{\mathrm{cg}}$, will only be written according to the $a_{\mathrm{i}}$. The link between $a_{\mathrm{i}}$ and $b_{\mathrm{i}}$ results by solving the equation (1). For $n=6$ steps and using the symbolic and numeric computing environment, Maple 12 [18], were obtained:
a) for circular cross-section, $\left(D_{c x}=D_{c y}=D_{c}\right)$ : the (4.1)(4.6) equations and the (3) system conversion in (5);
b) for oblong cross-section, $\left(D_{c x} \neq D_{c y}\right)$ : the (6.1)-(6.6) equations and the (3) system conversion in (7).

It is easy to observe that (4.1)-(4.6) equations and the (5) system corresponding to the circular core cross-section are obtained if the $D_{c x}=D_{c y}=D_{c}$ is given in (6.1)-(6.6) equations and in (7) system. Therefore, the (6.1)-(6.6) and (7) equations represent the generalized formulas and can be applied no matter the transformer core cross-section type.

$$
\begin{align*}
& b_{1}=\frac{1}{2} \cdot \sqrt{D_{c}^{2}-a_{1}^{2}} \cdot  \tag{4.1}\\
& b_{2}=\frac{1}{2} \cdot \sqrt{D_{c}^{2}-a_{2}^{2}}-b_{1}  \tag{4.2}\\
& b_{3}=\frac{1}{2} \cdot \sqrt{D_{c}^{2}-a_{3}^{2}}-\left(b_{1}+b_{2}\right) . \tag{4.3}
\end{align*}
$$

$b_{4}=\frac{1}{2} \cdot \sqrt{D_{c}^{2}-a_{4}^{2}}-\left(b_{1}+b_{2}+b_{3}\right)$.
$b_{5}=\frac{1}{2} \cdot \sqrt{D_{c}^{2}-a_{5}^{2}}-\left(b_{1}+b_{2}+b_{3}+b_{4}\right)$.
$b_{6}=\frac{1}{2} \cdot \sqrt{D_{c}^{2}-a_{6}^{2}}-\left(b_{1}+b_{2}+b_{3}+b_{4}+b_{5}\right)$.

$$
\left\{\begin{array}{l}
\frac{D_{c}^{2}-2 \cdot a_{1}^{2}+a_{2} \cdot a_{1}}{\sqrt{D_{c}^{2}-a_{1}^{2}}}=0 \\
\frac{D_{c}^{2}-2 \cdot a_{2}^{2}-\sqrt{D_{c}^{2}-a_{1}^{2}} \cdot \sqrt{D_{c}^{2}-a_{2}^{2}}+a_{3} \cdot a_{2}}{\sqrt{D_{c}^{2}-a_{2}^{2}}}=0 \\
\frac{D_{c}^{2}-2 \cdot a_{3}^{2}-\sqrt{D_{c}^{2}-a_{2}^{2}} \cdot \sqrt{D_{c}^{2}-a_{3}^{2}}+a_{4} \cdot a_{3}}{\sqrt{D_{c}^{2}-a_{3}^{2}}}=0 \\
\frac{D_{c}^{2}-2 \cdot a_{4}^{2}-\sqrt{D_{c}^{2}-a_{3}^{2}} \cdot \sqrt{D_{c}^{2}-a_{4}^{2}}+a_{5} \cdot a_{4}}{\sqrt{D_{c}^{2}-a_{4}^{2}}}=0 \\
\frac{D_{c}^{2}-2 \cdot a_{5}^{2}-\sqrt{D_{c}^{2}-a_{4}^{2}} \cdot \sqrt{D_{c}^{2}-a_{5}^{2}}}{\sqrt{D_{c x}^{2}-a_{5}^{2}}}=a_{6} \cdot a_{5} \\
\frac{D_{c}^{2}-2 \cdot a_{6}^{2}-\sqrt{D_{c}^{2}-a_{5}^{2}} \cdot \sqrt{D_{c}^{2}-a_{6}^{2}}}{\sqrt{D_{c x}^{2}-a_{6}^{2}}}=0
\end{array} .\right.
$$

$b_{1}=\frac{1}{2} \cdot \frac{D_{c y} \cdot \sqrt{D_{c x}^{2}-a_{1}^{2}}}{D_{c x}}$.
$b_{2}=\frac{1}{2} \cdot \frac{D_{c y} \cdot \sqrt{D_{c x}^{2}-a_{2}^{2}}}{D_{c x}}-b_{1}$.
$b_{3}=\frac{1}{2} \cdot \frac{D_{c y} \cdot \sqrt{D_{c x}^{2}-a_{3}^{2}}}{D_{c x}}-\left(b_{1}+b_{2}\right)$.
$b_{4}=\frac{1}{2} \cdot \frac{D_{c y} \cdot \sqrt{D_{c x}^{2}-a_{4}^{2}}}{D_{c x}}-\left(b_{1}+b_{2}+b_{3}\right)$.
$b_{5}=\frac{1}{2} \cdot \frac{D_{c y} \cdot \sqrt{D_{c x}^{2}-a_{5}^{2}}}{D_{c x}}-\left(b_{1}+b_{2}+b_{3}+b_{4}\right)$.
$b_{6}=\frac{1}{2} \cdot \frac{D_{c y} \cdot \sqrt{D_{c x}^{2}-a_{6}^{2}}}{D_{c x}}-\left(b_{1}+b_{2}+b_{3}+b_{4}+b_{5}\right)$
The number of laminations stacks widths, $a_{\mathrm{i}}$, is limited and is determined by each manufacturer. The width of each step must be rounded to the nearest standard value.

Stage 4. The steps height computation (the stacks of laminations thickness), $b_{\mathrm{i}}$, is obtained using the (6.1)-(6.6), equations for the circular cross-section, setting only the $D_{c x}=D_{c y}=D_{c}$ condition. For technological considerations, the laminations stacks thickness, $b_{\mathrm{i}}$, are rounded to a multiple of $2 \cdot g$, where $g$ is the thickness of a thin layer.

The proposed generalization for computing the dimensions steps that transformer core cross-section consist, for any predetermined cross-section diameter, involves determining the coefficients that relate the core cross-section diameter $(s)$ to the width of the steps, $a_{\mathrm{i}}$, so that the core cross-section is maximum (as shown in Fig. 1, for circular cross-section).

$$
\left\{\begin{array}{l}
\frac{D_{c y} \cdot\left(D_{c x}^{2}-2 \cdot a_{1}^{2}+a_{2} \cdot a_{1}\right)}{D_{c x} \cdot \sqrt{D_{c x}^{2}-a_{1}^{2}}}=0 \\
\frac{D_{c y} \cdot\left(D_{c x}^{2}-2 \cdot a_{2}^{2}-\sqrt{D_{c x}^{2}-a_{1}^{2}} \cdot \sqrt{D_{c x}^{2}-a_{2}^{2}}+a_{3} \cdot a_{2}\right)}{D_{c x} \cdot \sqrt{D_{c x}^{2}-a_{2}^{2}}}=0 \\
\frac{D_{c y} \cdot\left(D_{c x}^{2}-2 \cdot a_{3}^{2}-\sqrt{D_{c x}^{2}-a_{2}^{2}} \cdot \sqrt{D_{c x}^{2}-a_{3}^{2}}+a_{4} \cdot a_{3}\right)}{D_{c x} \cdot \sqrt{D_{c x}^{2}-a_{3}^{2}}}=0  \tag{7}\\
\frac{D_{c y} \cdot\left(D_{c x}^{2}-2 \cdot a_{4}^{2}-\sqrt{D_{c x}^{2}-a_{3}^{2}} \cdot \sqrt{D_{c x}^{2}-a_{4}^{2}}+a_{5} \cdot a_{4}\right)}{D_{c x} \cdot \sqrt{D_{c x}^{2}-a_{5}^{2}}}=0 \\
\frac{D_{c y} \cdot\left(D_{c x}^{2}-2 \cdot a_{5}^{2}-\sqrt{D_{c x}^{2}-a_{4}^{2}} \cdot \sqrt{D_{c x}^{2}-a_{5}^{2}}+a_{6} \cdot a_{5}\right)}{D_{c x} \cdot \sqrt{D_{c x}^{2}-a_{6}^{2}}}=0 \\
\frac{D_{c y} \cdot\left(D_{c x}^{2}-2 \cdot a_{6}^{2}-\sqrt{D_{c x}^{2}-a_{5}^{2}} \cdot \sqrt{D_{c x}^{2}-a_{6}^{2}}\right)}{}=0 .
\end{array}\right.
$$

These coefficients, which can be called "cross-section diameter-step width coefficients", can be calculated with the (8) equation for a circular core cross-section ( $D_{c x}=D_{c y}=D_{c}$ ).

$$
\begin{equation*}
c_{i}=\frac{a_{i}}{D_{c}} \tag{8}
\end{equation*}
$$

In the case of an oblong core cross-section, $D_{c x} \neq D_{c y}$, two categories of coefficients, $c_{\mathrm{xi}}$ and $c_{\mathrm{yi}}$, corresponding to the two diameters of the ellipse, can be computed using:

$$
\begin{align*}
& c_{x i}=\frac{a_{i}}{D_{c x}}  \tag{9.1}\\
& c_{y i}=\frac{b_{i}}{D_{c y}} \tag{9.2}
\end{align*}
$$

where: $c_{x i}$ - cross-section diameter-step width coefficients, corresponding to the $D_{c x}$ diameter; $c_{y i}$ - cross-section diameter-step width coefficients, corresponding to the $D_{c y}$ diameter.
Applying the proposed algorithm, for $n=6$ steps and using the Maple 12 software, were obtained:
a) for circular core cross-section ( $D_{c x}=D_{c y}=D_{c}$ ), the $c_{\mathrm{i}}$ coefficients in Table 1, the columns numbered with (1). Also, in Table 1, the same coefficients from the literature [16] are presented in the columns numbered (2). It can be seen that almost the same values were obtained, with very small differences at the third decimal place;
b) for the oblong core cross-section ( $D_{c x} \neq D_{c y}$ ), the values of the two coefficient categories, $c_{\mathrm{xi}}$, and $c_{\mathrm{yi}}$, are in Table II.

From Table 1 and Table 2, it can be noticed that the values of the $c_{\mathrm{i}}$ coefficients associated with the circular cross-section core are identical to the values of the $c_{x i}$ coefficients related to the oblong cross-section core. The computed coefficients, $c_{x i}$, and $c_{y i}$, allow the estimation of steps' sizes for any value of the two ellipse diameters without solving eq. (7) and (6.1)-(6.6), see Fig 3. In the case of a circular cross-section, the $c_{\mathrm{yi}}$ coefficients, corresponding to the $D_{c y}$ diameter, could be used in calculating the step height, $b_{i}$. In this way, the procedure for sizing the core circular cross-section is simplified, and it is no longer necessary to use (4.1)-(4.6) equations or to apply
the Pythagorean theorem, as recommended in the classical design literature $[15,16]$.

Table 1
"Cross-section diameter-step width" coefficients, for circular core section, (1) computed and (2) existing in the literature

| 3 steps |  |  | 4 steps |  | 5 steps |  | 6 steps |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (1) | (2) | (1) | (2) | (1) | (2) | $(1)$ | $(2)$ |  |
|  | $[16]$ |  | $[16]$ |  | $[16]$ |  | $[16]$ |  |
| 0.906 | 0.905 | 0.933 | 0.935 | 0.949 | 0.950 | 0.960 | 0.960 |  |
| 0.707 | 0.707 | 0.795 | 0.800 | 0.846 | 0.847 | 0.878 | 0.885 |  |
| 0.424 | 0.424 | 0.606 | 0.600 | 0.707 | 0.707 | 0.770 | 0.775 |  |
|  |  | 0.359 | 0.355 | 0.534 | 0.532 | 0.638 | 0.631 |  |
|  |  |  |  | 0.314 | 0.312 | 0.479 | 0.465 |  |
|  |  |  |  |  |  | 0.280 | 0.280 |  |

(1) the values of the cross-section diameter-step width coefficients $\left(c_{\mathrm{i}}\right)$ determined using the proposed algorithm and Maple 12;
(2) $c_{i}$ coefficient values in the literature $[14,16]$.


Fig. 3 - Coefficients relating the step sizes to the two diameters of the oblong core cross-section

Table 2
"Cross-section diameter-step width" coefficients computed for oblong core section

| 3 steps |  | 4 steps |  | 5 steps |  | 6 steps |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $c_{\mathrm{xi}}$ | $c_{\mathrm{yi}}$ | $c_{\mathrm{xi}}$ | $c_{\mathrm{yi}}$ | $c_{\mathrm{xi}}$ | $c_{\mathrm{yi}}$ | $c_{\mathrm{xi}}$ | $c_{\mathrm{yi}}$ |
| 0.906 | 0.212 | 0.934 | 0.180 | 0.950 | 0.157 | 0.960 | 0.140 |
| 0.708 | 0.142 | 0.796 | 0.124 | 0.846 | 0.110 | 0.878 | 0.099 |
| 0.424 | 0.099 | 0.606 | 0.094 | 0.710 | 0.087 | 0.770 | 0.080 |
|  |  | 0.360 | 0.069 | 0.534 | 0.069 | 0.638 | 0.066 |
|  |  |  |  | 0.314 | 0.052 | 0.478 | 0.054 |
|  |  |  |  |  |  | 0.280 | 0.041 |

## 3. IMPACT ANALYSIS OF OBLONG CORE CROSSSECTION ON POWER TRANSFORMER CHARACTERISTICS. CASE STUDY

The impact analysis of changing the core cross-section from circular to oblong, on the core dimensions (height, length, and width) and on the transformer performance (Joule losses, no-load losses, efficiency), applying the algorithm proposed in section 2, has been performed for a three-phase oil-cooled power transformer with rated data in Table 3.

Table 3

|  | The power transformer data |  |
| :--- | :--- | :--- |
| Symbol | Value | Quantity |
| $S_{\mathrm{n}}$ | 250 kVA | Rated power |
| $V_{\mathrm{ln}}$ | 20 kV | Rated primary voltage |
| $V_{2 \mathrm{n}}$ | 420 V | Rated secondary voltage |
| $I_{1 \mathrm{n}}$ | 7.21 A | Rated primary current |
| $I_{2 \mathrm{n}}$ | 343.67 A | Rated secondary current |
| $P_{\mathrm{NLn}}$ | 630 W | No-load losses /core losses |
| $P_{\mathrm{LLn}}$ | 4100 W | Short circuit losses |
| $U_{\mathrm{scn}}$ | $3 \%$ | Short circuit voltage |
| $f_{\mathrm{n}}$ | 50 Hz | Rated frequency |
|  | $\mathrm{Yy}-12$ | Vector symbol |
|  | Al | Conductive material |
| $g$ | 0.35 mm | Thickness of the thin layer |

The electromagnetic sizing of the magnetic circuit and windings of the three-phase oil-cooled power transformer has been carried out using a classical analytical design algorithm by taking into account the limits for short-circuit voltage ( $\pm 5 \%$ ), Joule losses ( $\pm 10 \%$ ), and no-load losses ( $\pm 10 \%$ ) [16, 17].
For the transformer core circular cross-section, the $D_{\mathrm{c}}=160 \mathrm{~mm}$ diameter has been obtained. Starting from this value, the condition of maintaining the volt per turn value, 6 V , (therefore also the geometric area of the core crosssection) and a ratio of $1 / 2$ between the two diameters of the ellipse has been imposed for the oblong core cross-section. The following values of the two diameters: $D_{c x}=113 \mathrm{~mm}$ and $D_{c y}=226 \mathrm{~m}$ have been obtained. The core cross-section step dimensions have been computed using the coefficients obtained in Section 2 (Table 2).

The transformer oblong core cross-section required a few modifications to the classical analytical design algorithm. As an example, for the Joule losses calculation ( $\mathrm{P}_{\mathrm{LL}}$ ), it has been necessary to recalculate the average winding length of the low voltage coils ( $l_{\text {avij }}$ ) and the average winding length of the high voltage coils ( $l_{\text {avri }}$ ), considering the (10) and (11) equations for the ellipse perimeter. The dimensions of the two windings, high and low voltage (conductors, number of turns, height), and the dimensions of all oil ducts determined for the circular transformer core cross-section have been maintained (Fig 4).

$$
\begin{equation*}
l_{a v r j}=2 \cdot \pi \cdot \sqrt{\frac{R_{x a v j}^{2}+R_{y a v j j}^{2}}{2}} \tag{10}
\end{equation*}
$$

where: $R_{x a v r j}$ - the average radius of the low voltage coil on the $O x$ axis, calculated with (10.1):

$$
\begin{equation*}
R_{x a v r j}=\frac{D_{c x}}{2}+a_{0 j}+\frac{a_{j}}{2} \tag{10.1}
\end{equation*}
$$

$R_{\text {yavrj }}$ - the average radius of the low voltage coil on the $O y$ axis, calculated with (10.2):

$$
\begin{equation*}
R_{y a v r j}=\frac{D_{c y}}{2}+a_{0 j}+\frac{a_{j}}{2} \tag{10.2}
\end{equation*}
$$

and:

$$
\begin{equation*}
l_{a v r i}=2 \cdot \pi \cdot \sqrt{\frac{R_{\text {xavri }}^{2}+R_{y a v r i}^{2}}{2}} \tag{11}
\end{equation*}
$$

where: $R_{x a v r i}$ - the average radius of the high voltage coil on the $O x$, axis was calculated with (11.1):

$$
\begin{equation*}
R_{x a v r i}=\frac{D_{c x}}{2}+a_{0 j}+a_{j}+a_{j i}+\frac{a_{i}}{2} \tag{11.1}
\end{equation*}
$$

$R_{\text {yavi }}$ - the average radius of the high voltage coil on $O y$, was calculated with (11.2):

$$
\begin{equation*}
R_{y a v i}=\frac{D_{c y}}{2}+a_{0 j}+a_{j}+a_{j i}+\frac{a_{i}}{2} \tag{11.2}
\end{equation*}
$$

where: $a_{0 j}$ - insulation thickness (oil duct) between the core and the low-voltage winding; $a_{j i}$ - insulation thickness (oil duct) between low-voltage and high-voltage windings; $a_{\mathrm{j}}-$ the low-voltage winding thickness; $a_{i}-$ the high-voltage winding thickness.


Fig. 4 -The average winding length calculation of a low-voltage and high-voltage turn.

Tables 4, 5, and 6 compare the results obtained for the two design solutions, circular core cross-section, and oblong core cross-section, of the transformer. Figure 5 is a qualitative comparative view of the core cross-section for the two design solutions.

Table 4 shows that similar values were obtained for the geometric core area, $A_{\text {cg }}$, and for the flux density in the core, $B_{\mathrm{c}}$. For the net core area, the 0.95 value of the filling factor, $k_{\mathrm{Fe}}$, has been considered. It also resulted in:

- a $7.7 \%$ reduction in core weight, $M_{\text {core }}$;
- a $7.4 \%$ increase in winding weight, $M_{\mathrm{Al}}$.


Fig. 5 - Qualitative comparison of transformer core cross-section: a) circular section ( $D_{\mathrm{cx}}=D_{\mathrm{cy}}=160 \mathrm{~mm}$ ); b) oblong section ( $D_{\mathrm{cx}}=113 \mathrm{~mm}, D_{\mathrm{cy}}=226 \mathrm{~mm}$ ) (Maple 12 output).

## Table 4

Parameters obtained for the two transformer design solutions, circular cross-section, and oblong cross-section

| cross-section, and oblong cross-section |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $A_{c g}$ <br> $\left[\mathrm{~mm}^{2}\right]$ | $B_{\mathrm{c}}$ <br> $[\mathrm{T}]$ | $M_{\text {core }}$ <br> $[\mathrm{kg}]$ | $M_{\mathrm{Al}}$ <br> $[\mathrm{kg}]$ |
| Circular cross-section <br> $\left(D_{\mathrm{c}}=D_{\mathrm{cx}}=D_{\mathrm{cy}}=160 \mathrm{~mm}\right)$ <br> Oblong cross-section <br> $D_{\mathrm{cx}}(113 \mathrm{~mm})<D_{\mathrm{cy}}(226 \mathrm{~mm})$ | 18555 | 1.548 | 523 | 88 |

Table 5 shows the effects of changing the core crosssection type on the transformer core dimensions: the core
height, $H_{\text {core }}$, the core length, $L_{\text {core }}$, and the core width, $I_{\text {core }}$. Thus, for the design solution with an oblong core crosssection, the result is:

- an $8.8 \%$ reduction in the magnetic core height, $H_{\text {core }}$;
- an $18.7 \%$ reduction in the magnetic core length, $L_{\text {core }}$;
- a $41.3 \%$ increase in magnetic core width, $I_{\text {core }}$.

Table 5
Dimensions of the transformer core for the two design solutions, circular

| cross-section and oblong cross-section |  |  |  |
| :--- | :---: | :---: | :---: |
|  | $H_{\text {core }}$ <br> $[\mathrm{mm}]$ | $L_{\text {core }}$ <br> $[\mathrm{mm}]$ | $l_{\text {core }}$ <br> $[\mathrm{mm}]$ |
| Circular cross-section <br> $\left(D_{\mathrm{c}}=D_{\mathrm{cx}}=D_{\mathrm{cy}}=160 \mathrm{~mm}\right)$ | 1056 | 755 | 154 |
| Oblong cross-section <br> $D_{\mathrm{cx}}(113 \mathrm{~mm})<D_{\mathrm{cy}}(226 \mathrm{~mm})$ | 963 | 614 | 217 |

Table VI analyzes the impact of the transition from a circular to an oblong core cross-section on the studied transformer performance. In the case of the oblong core cross-section design solution, the following was obtained:

- core losses, $P_{\mathrm{NL}}$, lower with $7.23 \%$;
- Joule losses, $P_{\mathrm{LL}}$, higher with $5.46 \%$;
- short-circuit voltage, $u_{\mathrm{sc}}$, greater with $4.16 \%$, but within the limits of $\pm 5 \%$.

Table 6
Transformer performance for the two design solutions, circular crosssection and oblong cross-section

|  | $u_{\mathrm{sc}}$ <br> $[\%]$ | $P_{\mathrm{J}}$ <br> $[\mathrm{W}]$ | $P_{\mathrm{NL}}$ <br> $[\mathrm{W}]$ | $\eta$ <br> $[\%]$ |
| :--- | :---: | :---: | :---: | :---: |
| Circular cross-section <br> $\left(D_{\mathrm{c}}=D_{\mathrm{cx}}=D_{\mathrm{cy}}=160 \mathrm{~mm}\right)$ | 2.99 | 4005 | 581 | 98.57 |
| Oblong cross-section <br> $D_{\mathrm{cx}}(113 \mathrm{~mm})<D_{\mathrm{cy}}(226 \mathrm{~mm})$ | 3.12 | 4236 | 539 | 98.53 |

The results obtained in terms of reducing the core height, core length, core weight, and no-load losses are consistent with existing examples in the literature [9].
In addition to reducing no-load losses, it is worth highlighting another valuable feature of the oblong core cross-section design solution, namely the possibility of installing these power transformers in distinctive locations with special shapes.

## 4. CONCLUSIONS

The entry into force of the second stage of EU 548/2014 Regulation, which imposes the maximum level of losses for power transformers, has led to research on reducing the losses of these devices. One identified research direction is to change the medium and high power transformer core cross-section from circular to oblong. Thus, the paper has proposed an algorithm that generalizes the procedure for sizing the stepped core cross-section of the power transformers, with the advantage of simplifying the transition from circular to oblong core cross-section. Therefore, the proposed algorithm allows determining the dimensions of the laminations stacks that the core crosssection consists of for any predetermined value of the section diameter. The novelty and the value of the paper are given by the computed $c_{\mathrm{yi}}$ coefficients values. These coefficients were not found in the literature, and by using them, the procedure for sizing the circular/oblong crosssection core is simplified.
The algorithm, applied for a three-phase oil-cooled power transformer with a 250 kVA rated power, led to a design solution with an oblong magnetic core cross-section
with the following advantages: reduction of core height, the core length, core weight, and no-load losses. As for the less favorable effects, these consist of a slight increase of the core width and Joule losses.

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