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NOVEL 2X2 FULL-RATE SPACE-TIME BLOCK CODE WITH IMPROVED PERFORMANCE AT SMALL SEPARATION BETWEEN COLLOCATED ANTENNAS

EDUARD VALENTIN TOMA[1], ANCA MIHAELA RUXANDRA TOMA[1], CORNELIU EUGEN STERIAN[1], MATTHIAS UWE PÄTZOLD[2]

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In this paper, we introduce a novel full-rate 2X2 space-time block code and compare its FER and BER performance against that of the Golden code by way of computer simulations using a mathematical model that takes the distance between collocated antennas as a parameter. For $d = 1\ \lambda$, where λ is the wavelength of the carrier frequency, the superiority of the new code is obvious. For $d = 2\ \lambda$, the Golden code is better, but the new space-time block code still works rather well. While we do not provide here plots for $d = \lambda/2$, we know from our experiments that our new code shows only a gracious degradation as compared to the case $d = 1\ \lambda$. This recommends the new code for applications where a small distance between collocated antennas is unavoidable. The main originality of the new code is using two signal constellations instead of a single one, as detailed in the body of our work. In so doing, we also greatly simplified the issue of determining the separators and the shapers, which are rather complex in the case of the Golden code.

1. INTRODUCTION

It is an established fact that the performance of digital mobile links is much improved by using antenna diversity at one end or at both ends of the wireless channel. Antenna diversity at the transmit side is also known as space-time coding or multiple-input multiple-output (MIMO). As our contribution is a novel full rate 2X2 space-time block code, we first briefly overview the state-of-the-art in this area of endeavor. An ingenious, simple but remarkably effective scheme was designed by Alamouti [1]. As recalled in Section II, it is a half-rate 2X2 space-time block code. The famous Golden code was introduced in [2] and further described in [3]. It is a full rate 2X2 space-time block code and as such, it promises to transmit a double quantity of information in the same channel use as compared to Alamouti's. However, for a fair comparison, the Golden code would transmit, for instance, four 16QAM 2D symbols in two consecutive channels used with two transmit antennas, while the Alamouti's scheme would transmit two 256QAM 2D symbols. Yet, as two youngers of us proved by way of computer simulations, the half-rate performs better than the full rate space-time block code [4]. The Alamouti code has also another strong advantage: its transmit matrix is available in at least two versions, which makes it adequate to be used in order to design so-called super-orthogonal space-time trellis codes [5,6,7]. The same property of the Alamouti matrix was cleverly exploited to create opportunistic channels in [7] and [8].

Recently, we witnessed a flurry of new contributions to the field as follows. In [10], the performance of space-time block codes combined with discrete multi-tone modulation as applied in a large core step-index POF link is examined theoretically. In [11], the authors use the random matrix theory and an approximation function for the probability density function for the largest eigenvalue of a Wishart matrix to obtain a new design criterion for space-time block codes. In [12], the authors use deep learning to reduce the training overhead which scales with the number of antennas, users, and subcarriers. In [13], the authors proposed a cooperative strategy based on STBC and CMIMO, which is referred to as space-time to inherit the advantages from both STBC and CMIMO. The paper [14] analyses simulations using orthogonal space-time block codes in Rayleigh fading channels to evaluate the performance of MIMO systems.

In this paper, we first define with our notations the Alamouti code and the Golden code in section II. In section III, we introduce our novel full rate full diversity 2X2 space-time block code and describe the signal constellation, which is the main ingredient. In Section IV, we report the results of our simulations performed with our code and the Golden code in the same conditions for two distances between collocated antennas: $d = 1\lambda$ and $d = 2\lambda$, where λ is the wavelength of the carrier frequency. The plots of BER and FER performance are analyzed and commented upon. By anticipating a bit, we mention just here that the Golden code is hardly usable for $d = 1\lambda$, but it is the best for $d = 2\lambda$. Our new code instead works acceptably well for $d = 2\lambda$ and shows a gracious degradation of the performance for $d = 1\lambda$. A concluding section is closing our paper.

2. THE ALAMOUTI CODE AND THE GOLDEN CODE

We assume that the following string of 2D symbols is to be transmitted over a wireless channel: $s_0, s_1, ..., s_{2n}, s_{2n+1}, ...\ s0$. A single antenna will do, but to improve the performance by introducing antenna diversity, we group together two consecutive 2D symbols and s_{2n+1}, and use a second transmit antenna. Then, the Alamouti transmit matrix is as follows:

$$\mathbf{M}_n = \begin{pmatrix} s_{2n} & -a \cdot s_{2n+1}^* \\ s_{2n+1} & a \cdot s_{2n}^* \end{pmatrix}. \qquad (1)$$

The 2X2 matrix $\mathbf{M_n}$ should be understood as follows: the 2D symbols s_{2n} and s_{2n+1} are transmitted in two consecutive channels using $2n$ and $2n$+1, while the second antenna transmits the entries of the right column. As usual, the star designates complex conjugation, and a is a complex-valued quantity. Note that the two columns (as well as the two lines) are orthogonal. This excellent property allows the decision to be made independently for

[1] University Politehnica of Bucharest, eduard_valentin.toma@stud.trans.upb.ro, anca_mihaela.toma@stud.trans.upb.ro, steriancorneliu@hotmail.com
[2] University of Agder, matthias.paetzold@uia.no

the two symbols at the receiver. We assume that the receiver is equipped with two antennas such that, after quadrature-amplitude demodulation, it disposes of the complex-valued quantities $r1_{2n}$ and $r2_{2n}$ in a first channel use $2n$ and $r1_{2n+1}$ and $r2_{2n+1}$ in a second channel use $2n$+1. We denote by h_{ij} the path gain from transmit antenna j to receive antenna i and by $z1_{2n}, z2_{2n}, z1_{2n+1}$ and $z2_{2n+1}$ the noise terms experienced by the antenna 1 and antenna 2 in two consecutive channels using $2n$ and $2n$+1. Thus, we can write with baseband notation the following equations governing the transmission:

$$r1_{2n} = h_{11} \cdot s_{2n} - a \cdot h_{12} \cdot s_{2n+1}^* + z1_{2n}\,, \tag{2}$$

$$r1_{2n+1} = h_{11} \cdot s_{2n+1} + a \cdot h_{12} \cdot s_{2n}^* + z1_{2n+1}\,, \tag{3}$$

$$r2_{2n} = h_{21} \cdot s_{2n} - a \cdot h_{22} \cdot s_{2n+1}^* + z2_{2n}\,, \tag{4}$$

$$r2_{2n+1} = h_{21} \cdot s_{2n+1} + a \cdot h_{22} \cdot s_{2n}^* + z2_{2n+1}\,. \tag{5}$$

As usual, we assume that, by periodically measuring the wireless channel, the receiver acquires perfect channel state information (CSI), that is, the perfect knowledge of the path gains h_{11}, h_{12}, h_{21} and h_{22}. The CSI is not available at the transmit site. The fading is assumed as constant between two consecutive channel measurements, which defines a *frame*.

The receiver, then, makes the following independent maximum likelihood decisions:

$$\hat{s}_{2n} = \frac{\left(h_{11}^* \cdot r1_{2n} + h_{21}^* \cdot r2_{2n} + a\left(h_{12} \cdot r1_{2n+1}^* + h_{22} \cdot r2_{2n+1}^*\right)\right)}{\delta} \tag{6}$$

$$\hat{s}_{2n+1} = \frac{\left(h_{11}^* \cdot r1_{2n+1} + h_{21}^* \cdot r2_{2n+1} - a\left(h_{12} \cdot r1_{2n}^* + h_{22} \cdot r2_{2n}^*\right)\right)}{\delta} \tag{7}$$

where

$$\delta = |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2. \tag{8}$$

The Golden code is a full rate full diversity 2X2 space-time block code, that is, it transmits four 2D symbols v_{2n}, w_{2n}, v_{2n+1} and w_{2n+1} in two consecutive channel uses using two transmit antennas and two receive antennas. The transmit antenna of the Golden code is as follows:

$$\mathbf{G}_n = \frac{1}{\sqrt{5}}\begin{bmatrix} \alpha(v_{2n} + \theta w_{2n}) & i\bar{\alpha}(v_{2n+1} + \bar{\theta} w_{2n+1}) \\ \alpha(v_{2n+1} + \theta w_{2n+1}) & \bar{\alpha}(v_{2n} + \bar{\theta} w_{2n}) \end{bmatrix}, \tag{9}$$

where $\theta = \frac{1+\sqrt{5}}{2}, \bar{\theta} = \frac{1-\sqrt{5}}{2}, \alpha = 1 + \mathrm{i}\,\bar{\theta}, \bar{\alpha} = 1 + \mathrm{i}\,\theta$ and $\mathrm{i} = \sqrt{-1}$

We may call θ and $\bar{\theta}$ as *separators* for the obvious reason. The parameters $\alpha/\sqrt{5}$ and $\bar{\alpha}/\sqrt{5}$ are *shapers*; their role is to make the average norm of the entries of the matrix (9) equal to the average norm of the 2D signal constellation used for modulation.

Such a code is known to be decodable by the sphere decoding method. Nevertheless, we preferred in this paper to use brute force, the case at hand being too simple. We introduce our novel space-time 2X2 block code in the next section.

3. DESIGNING THE NEW CODE

In designing a new space-time block code, we largely benefited from the experience gained by simulations with the Alamouti and Golden codes. At first glance, it seems that the Golden code is superior since it is the full rate; unfortunately, it is not so. This is since the separation between the 2D symbols transmitted together, like v_{2n} and w_{2n}, is penalized by worsening the performance. For instance, and for a fair comparison, if a Golden transmit matrix transports four 16QAM 2D symbols, an Alamouti matrix should transmit two 256QAM 2D symbols. A signal constellation with 256 signal points is not very comfortable and yet the simulations witness the superiority of the Alamouti over the Golden code. And this is not the whole story: in our study of the influence of the distance between collocated antennas, we discovered that the performance of the Golden code if the distance d between collocated antennas equals λ or is smaller, where λ is the wavelength of the carrier frequency, is rather poor. On the contrary, Alamouti performs rather acceptable even for $d = \lambda/2$. We explain this behavior by the orthogonality of the columns of the Alamouti matrix. With this insight in our minds, we tried to combine the merits of the two codes. We also realized that, for a better separation between the 2D symbols transmitted together, we should use two signal constellations as different as possible. Let 16QAM be one of them. It is well known that the sixteen signal points of this constellation may be considered as belonging to a co-set of a 2D lattice. The idea has dawned on us to construct a new 16 points signal constellation using the deep holes of the said lattice. This is not difficult. But since point 0, the origin of the coordinate axes is a deep hole also, we excluded it. This can be seen in Fig. 1, where the black points belong to the familiar 16QAM signal constellation, while the clear points belong to another sixteen points signal constellation, named here just for convenience as 16CONST.

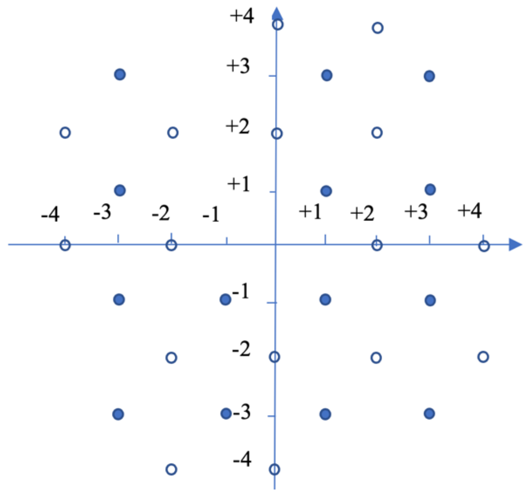

Fig. 1 – The two signal constellations used in the illustrative example. Black dots represent the familiar 16QAM signal constellation. The sixteen clear dots form the second signal constellation, named here 16CONST.

Since we excluded the origin of the coordinates as signal points, the average norm of 16CONST is slightly larger than the average norm of 16QAM. We will fix this easily. In (1), put a = 1 and -2.

$$s_{2n} = \left(v_{2n} + \sqrt{\frac{5}{6}} w_{2n}\right) / \sqrt{2}\,, \tag{10}$$

$$s_{2n+1} = \left(v_{2n+1} + \sqrt{\frac{5}{6}} w_{2n+1}\right) / \sqrt{2}\,. \tag{11}$$

Note that the two terms within the brackets have the same average norm. The term $\sqrt{2}$ from the denominator is a shaper: its role is to keep the average norm of s_{2n} equal to the average norm of the 16QAM signal constellation. However, the coefficient $\sqrt{5/6}$ is not a separator properly: the separation is made by using different signal constellations, 16QAM for v_{2n} and 16CONST for w_{2n}. This explains the remarkably simple form of the transmit matrix of the new code. It is an extension of the half-rate Alamouti code, being a full rate space-time block code. The main novelty of our new code is using two different signal constellations instead of one, as in the classical design of full rate space-time block codes. In the next section, we present our experimental results.

4. NUMERICAL RESULTS

We used a space-time channel simulator written in MATLAB based on the geometrical one-ring scattering model. It allows to set the distance between collocated antennas at different values: λ/2, λ, 2λ, 3λ and larger, where λ is the wavelength of the carrier frequency. We mention that we used an August 2015 version of MATLAB as licensed to University POLITEHNICA with number 38860. In our experiments, we made the following assumptions:

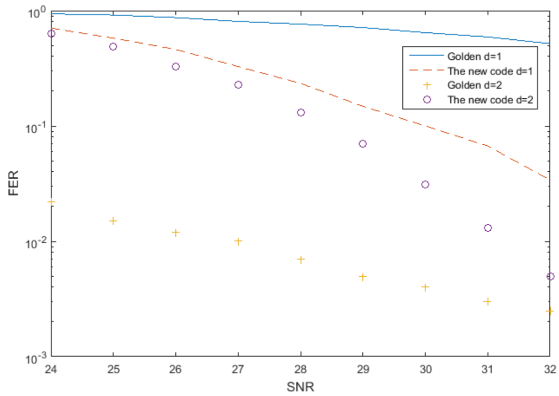

Fig. 2 – The FER comparison of the new code with Golden code.

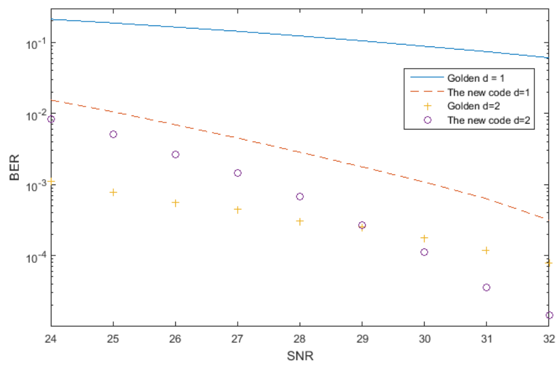

Fig. 3. – The BER comparison of the new code with Golden code.

1. The receiver acquires perfect channel state information (CSI). CSI is not available to the transmitter.
2. The fading is constant between two consecutive channel measurements, which defines a *frame*.

In our simulations, a frame consists of 65 transmit matrices, that is, 65X4X4 = 1040 bits. For each value of the signal-to-noise ratio, 1000 frames have been transmitted and the frame error rate (FER) and bit error rate (BER) were computed and displayed. The results are reported as plots for FER in Fig. 2 and for BER in Fig. 3. The performance of the Golden code can be seen to be poor for *d* = 1 λ, but very good for *d* = 2 λ. The performance of our new code is somewhat in between: acceptably good for *d* = 1 λ and not so bad for *d* = 2 λ. It seems to be the case to make a distinction between two operation regimes: one for short separation and the other one for larger separation between collocated antennas. We do recommend our new full rate 2X2 space-time block code for such applications in which the transmission system is working mostly in the first regime.

5. CONCLUSION

We introduced a new full rate 2X2 space-time block code and compared it by way of computer simulations using MATLAB to the Golden code. The channel model is based on the geometrical one-ring scattering model. We used the distance between collocated antennas as a parameter. The new code is superior for small antenna spacing, on the order of the wavelength of the carrier frequency. No claim of superiority is made for larger spacing between collocated antennas. Therefore, our new code could be the solution of choice when using small handheld communication devices.

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REFERENCES

[1] S.M. Alamouti, *A simple transmit diversity technique for wireless communications*, IEEE J. Select. Areas Commun., **16**, *8*, pp. 1451-1458 (1998).

[2] J.-C. Belfiore, G. Rekaya, E. Viterbo, The Golden code: a 2X2 full-rate space-time code with nonvanishing determinants," IEEE Trans. Inform. Theory, **51**, *4*, pp. 1432-1436 (2005).

[3] F. Oggier, G. Rekaya, J.-C. Belfiore, E. Viterbo, *Perfect space-time block codes*, IEEE Trans. Inform. Theory, **52**, *9*, pp. 3885-3902 (2006).

[4] E.V. Toma, A.M.R. Toma, *A comparative study of Alamouti and Golden space-time block codes*" The 13th International Conference on Communications (COMM 2020), pp. 77-80, June 8, Bucharest, Romania (2020).

[5] H. Jafarkhani, N. Seshadri, *Super-orthogonal space-time trellis codes*, IEEE Trans. Inform. Theory, **49**, pp. 937-950 (2003).

[6] C.E.D. Sterian, H. Singh, M. Pätzold, B.O. Hogstad. *Super-orthogonal space-time codes with rectangular constellations and two transmit antennas for high data rate wireless communications*," IEEE Trans. Wireless Commun., **5**, *7*, pp. 1857-1865 (2006).

[7] C.E.D. Sterian, S. Linga, H. Singh and M. Pätzold, *New super-orthogonal space-time trellis codes with 32QAM and two transmit antennas for five bits per signaling interval*," European Trans. Telecommun., **18**, *2*, pp. 205-215 (2007).

[8] K. Nikitopoulos, F. Mehran, H. Jafarkhani, *Space-time super-modulation and its application to joint medium access and rateless transmission*," IEEE Transactions on Wireless Communications, PP **99**, *1-1*, 14 pp. (2017).

[9] V. Vakilian, H. Mehrpouyan, Y. Hua, H. Jafarkhani, *High rate/low complexity space-time block codes for 2X2 reconfigurable MIMO systems*, arXiv 34 May (2015).

[10] N. Raptis, E. Grivas, E. Pikasis, and D. Syvridis, *Space-time block code-based MIMO encoding for large core step-index plastic optical fiber transmission systems*, Optics Express, **19**, *11*, pp. 10336-10350 (2011).

[11] C.A.R. Martins, M.L. Brandão Jr., E.B. da Silva, *New space-time block codes from spectral norm*, Plus ONE Collection (2019).

[12] M. B. Mashhadi, D. Gündüz, *Deep learning for massive MIMO channel state acquisition and feedback*, Journal of Indian Institute of Science **100**, *2*, pp. 369-382 (2020).

[13] H. Hai, C. Li, J. Li, Y. Pang, J. Hou, X-Q. Jiang, *Space-time block coded cooperative MIMO systems*, Sensors, 21, 1, pp. 109, (2021).

[14] C.M. Lau, *Performance of MIMO systems using space-time block codes (STBC)*, Open Journal of Applied Sciences, **11**, pp. 273-286 (2021).