Rev. Roum. Sci. Techn.– Électrotechn. et Énerg.
Vol. **67**, *2*, pp. 161–166, Bucarest, 2022

DIGITAL FILTERS INTENDED FOR PULSE SIGNAL PERIODS

DJURDJE PERIŠIĆ[1]

Keywords: Frequency locked loop; Digital filter; Phase-locked loop; Digital circuit; Discrete linear system.

This work describes a new kind of FIR digital filter intended for the filtering of the pulse signal periods. This kind of digital filter was designed using the frequency locked loops (FLL), which are based on the time measurement and processing of the input periods only. FLL is a linear discrete system. Starting from the general form of differential equation of FLL, the transfer functions of FLL and Z transform of the FLL outputs are developed for FLL of any order. The main part of the article is devoted to describing how to design the appropriate FIR digital filter using an FLL of any order. Although the FIR digital filters and FLLs are different systems, for this purpose the theory of FIR digital filter and the corresponding MATLAB tools are used. Filtering abilities of the fifth order FLL are demonstrated. The mathematical analyzes were performed using the Z transform approach. Analysis of FLL of the fifth order was performed in the time and frequency domain. Computer simulation of FLL of the fifth order is made in the time domain to enable precise insight into its properties.

1. INTRODUCTION

The field of the Time Recursive Processing Phase and Frequency Locked Loops (TRP PLL and TRP FLL) based on the processing of the periods of input and output signals and time differences between them was recently described in refs. [3–10]. These papers described their numerous applications, but in comparison to the classical phase and frequency locked loops, some of them are completely new, such as phase and time-shifting, applications in the field of tracking and prediction, and others. Through the development of these systems, knowledge about them also increased. When the cognition of these systems reached the necessary level, the idea was imposed that this kind of FLL could be used for digital filtering of the pulse signal periods. The general form of such M-th order FLL difference equation is described in [1]. In [1,2] it was proved that, regardless of the differences between FLL and digital filters, the complete theory of FIR digital filters, as well as the corresponding MATLAB tools, can be used in the analysis and design of FLL. This discovery opened wide opportunities for the development of new theory and practice in the digital filtering of pulse signal periods, based on FLL.

In this paper, we will use the term FIR FLL (Final Impulse Response FLL) instead of the term "non-recursive FLL" used in refs. [1,2], because the article describes the usage of FLL for the special values of the parameters, *i.e.*, in the role of a Fir digital filter. This paper, among other things, describes a development methodology for FIR FLL of any order, using the theory of FIR digital filters and the appropriate MATLAB tools.

The articles and books in refs. [11–23] are used as a theoretical base, for electronics implementations, and for the development necessities.

2. TIME ANALYSES OF FLL OF M-TH ORDER

Let us borrow the derived expressions for the difference eqs. (1) and (2) for FLL of the M-th order, as well as Fig. 1, from [1]. Figure 1 is slightly changed and adapted to this paper. It represents a general case of an input signal Sin and an output signal Sop of FLL and shows the physical relations between the variables used in eqs. (1) and (2). The periods TI_k and TO_k, as well as the time difference τ_k, occur at discrete times t_k, t_{k+1}, $t_{k+2}, \ldots, t_{k+M-1}$, t_{k+M}, defined by the falling edges of the pulses of Sop in Fig. 1. Note that the variable "k", represents the discrete-time t_k when an input period is measured and taken in the calculation. According to eq. (1), there are "M" calculations of any output period with "M" system parameters b_1, $b_2 \ldots$ b_M, and "M" consecutive input periods. The number "M" represents the order of FLL, and it can be any natural number from one to infinity. The beginning of "M" calculations starts at the discrete time $t = t_k$, just like in Fig. 1, where "*k*" is usually zero, but it can be any natural number. Equation (2) comes out as natural relation between the variables in Fig. 1. The variable τ_k will serve to identify the phase relation, as well as the time relation between the input and output periods, during both the locking procedure and the stable state of FLL. Because of simplicity, discrete times in brackets of $TO(t_{k+M})$ and $TI(t_{k+M-i})$ are changed with the corresponding index marks like TO_{k+M} and TI_{k+M-i} in eq. (1). The same changes are made in Fig. 1 and in eq. (2).

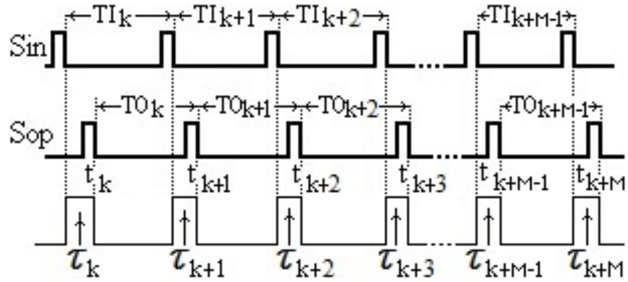

Fig. 1 – The time relations between the input and output variables of the M-th order FLL.

$$TO_{k+M} = \sum_{i=1}^{M} b_i \cdot TI_{k+M-i}\,, \tag{1}$$

$$\tau_{k+1} = \tau_k + TO_k - TI_k\,. \tag{2}$$

To perform the analyses of FLL it is necessary to determine their transfer functions, as well as the Z transforms of TO_k and τ_k. The Z transform of eqs. (1) and (2) can be derived in two ways. The first way is to develop it directly from eqs. (1) and (2). The Z transformation of an

[1] Faculty of Information Technologies, Slobomir P University, Str. Pavlovića put 76, 76300 Slobomir, Republic of Srpska, Bosnia and Herzegovina,
E-mail: djurdje@beotel.rs.

M-th order FLL can also be performed from the Z transformations of multiple lower-order FLLs. We will apply the second approach, as the first would take up a lot of space. For the FLL of the second order, [2], TO(z) and τ(z) are shown in eqs. (3) and (4). For the FLL of the third order, [1], TO(z) and τ(z) are shown in eqs. (5) and (6). Based on eqs. (3), (4), (5), and (6), we can derive the Z transforms of M-th order FLL, given in eqs. (7) and (8). The shorted form of eqs. (7) and (8) are presented in eqs. (9) and (10), where $H_{TO}(z) = TO(z)/TI(z)$ and $H_{\tau}(z) = \tau(z)/TI(z)$ are the transfer function of M-th order FLL. The transfer functions are presented in eqs. (11) and (12). TO_0 and τ_0 in eqs. (3) to (10) are the initial conditions of the output periods and time differences. $R(z) = (TO_0 + z\tau_0)/(z-1)$ in eqs. (4), (6), and (8)

$$TO(z) = TI(z) \cdot (zb_1 + b_2)/z^2 + TO_0\,, \tag{3}$$

$$\tau(z) = TI(z) \cdot [-z + (b_1 - 1)]/z^2 + R(z)\,, \tag{4}$$

$$TO(z) = TI(z) \cdot (z^2 b_1 + zb_2 + b_3)/z^3 + TO_0\,, \tag{5}$$

$$, \tau(z) = TI(z) \cdot [-z^2 + z(b_1 - 1) + (b_2 + b_1 - 1)]/z^3 + R(z) \tag{6}$$

$$\begin{aligned} TO_M(z) = TI(z) \cdot (z^{M-1}b_1 + z^{M-2}b_2 + \ldots \\ \ldots + zb_{M-1} + b_M)/z^M + TO_0, \end{aligned} \tag{7}$$

$$\begin{aligned} \tau_M(z) = TI(z) \cdot [-z^{M-1} + z^{M-2}(b_1 - 1) + \\ z^{M-3}(b_2 + b_1 - 1) + \ldots + z(b_{M-2} + \ldots + b_1 - 1) + \\ (b_{M-1} + \ldots + b_1 - 1)]/z^M + R(z), \end{aligned} \tag{8}$$

$$TO_M(z) = TI(z) \cdot H_{TO_M}(z) + TO_0\,, \tag{9}$$

$$\tau_M(z) = TI(z) \cdot H_{\tau_M}(z) + (TO_0 + z\tau_0)/(z-1)\,, \tag{10}$$

$$H_{TO_M}(z) = TO_M(z)/TI(z) = \sum_{i=1}^{M} b_i \cdot z^{M-i}/z^M\,, \tag{11}$$

$$H_{\tau_M}(z) = \{-z^{M-1} + \sum_{i=1}^{M-1} z^{M-1-i} \cdot [(\sum_{j=1}^{i} b_j) - 1]\}/z^M\,. \tag{12}$$

Although eq. (12) looks complicated, it is together with eq. (11) very useful, because using them, we can easily derive Z transforms of the outputs and transfer functions of any order FLL, escaping long mathematical operations and significantly reducing the possibility to make an error. Let us now demonstrate the development of the Z transform equations for $M = 5$, *i.e.*, for FIR FLL of the fifth order (FIR FLL_5). If we enter $M = 5$ in eqs. (11) and (12), we will get the transform functions $H_{TO5}(z)$ and $H_{\tau5}(z)$ for FIR FLL_5, shown in eqs. (13) and (14). Using eqs. (9) and (10), the Z transform of the FIR FLL_5 outputs are determined and shown in eqs. (15) and (16).

$$H_{TO5}(z) = (z^4 b_1 + z^3 b_2 + z^2 b_3 + zb_4 + b_5)/z^5\,, \tag{13}$$

$$\begin{aligned} H_{\tau5}(z) = [-z^4 + z^3(b_1 - 1) + z^2(b_2 + b_1 - 1) + \\ z(b_3 + b_2 + b_1 - 1) + (b_4 + b_3 + b_2 + b_1 - 1)]/z^5, \end{aligned} \tag{14}$$

$$TO_5(z) = TI(z) \cdot H_{TO5}(z) + TO_0\,, \tag{15}$$

$$\tau_5(z) = TI(z) \cdot H_{\tau5}(z) + (TO_0 + z\tau_0)/(z-1)\,. \tag{16}$$

In order to investigate the properties of FIR FLL_5, let us suppose that the step input is $TI(k) = TI = \text{const.}$ Substituting the Z transform of $TI(k)$ i.e. $TI(z) = TI \cdot z/(z-1)$ into eq. (15) and using the final value theorem, it is possible to find the final value of the output period $TO_{5\infty}$, which FIR FLL_5 reaches in the stable state. We can calculate $TO_{5\infty} = \lim TO_5(k)$ if $k \to \infty$, using $TO_5(z)$. This is shown in eq. (17). It comes out from eq. (17), that $TO_{5\infty} = TI$ if eq. (18) is satisfied. FIR FLL_5 possesses the properties either of a FLL or of a PLL, if eq. (18) is satisfied. To make decision, it is necessary to determine the behaviour of time difference τ_5. Substituting now $TI(z)$ into eq. (16) and using the final value theorem, it is possible to find the final value of the time difference $\tau_{5\infty} = \lim \tau_5(k)$ if $k \to \infty$, using $\tau_5(z)$. This is shown in eq. (19). Equation (19) also confirms that FIR FLL_5 possesses the properties of a FLL, since $\tau_{5\infty}$ depends on the initial conditions. It comes out that the system does not possess the properties of a PLL

$$\begin{aligned} TO_{5\infty} &= \lim[(z-1)TO_5(z)]_{z\to1} \\ &= TI(b_1 + b_2 + b_3 + b_4 + b_5), \end{aligned} \tag{17}$$

$$b_1 + b_2 + b_3 + b_4 + b_5 = 1\,, \tag{18}$$

$$\begin{aligned} \tau_{5\infty} &= \lim[(z-1) \cdot \tau_5(z)]_{z\to1} \\ &= TI(b_4 + 2b_3 + 3b_2 + 4b_1 - 5) + TO_0 + \tau_0. \end{aligned} \tag{19}$$

All reached math results can be confirmed by the simulations in the time domain, realized by MATLAB tools. The results derived from math must agree with the simulated ones. Besides that, the simulations are to enable better insight into the procedures and physical meaning of the variables described. All discrete values in simulations were merged to form continuous curves. Note that all variables in the following diagram were presented in time units. The time unit can be, μsec, msec, or any other, but assuming the same time units for all time variables TI, TO and τ, it was more suitable to use just "time unit" or abbreviated "t.u." in the text. It was more convenient to omit the indication "t.u." in the diagrams.

The simulations of $TO_5(k)$ and $\tau_5(k)$ are realized using eqs. (1) and (2), for $M = 5$. They are shown in Fig. 2 for the step input $TI_k = 10$ t.u. The presentation for three cases with different parameters b_1, b_2, b_3, b_4, b_5, initial conditions and final values, are shown in Fig. 2. In the cases 1 and 2, the system parameters satisfy eq. (18) and therefore, FIR FLL_5 is a stable system, and the output periods reached the input periods. According to eq. (19), $\tau_{5\infty 1} = TI(b_4 + 2b_3 + 3b_2) + 4b_1 - 5) + TO_0 + \tau_0 = 10(0.3 + 2\cdot 0.2 + 3\cdot 0.2 + 4\cdot 0.1 - 5) + 4 - 4 = -33$ t.u.

This result agrees with the simulated $\tau_{1\infty}$ in Fig. 2 (the label 5, which indicates the order of the loop, is omitted in Fig. 2). However, in case 2, it was intentionally chosen that the parameters additionally satisfy the condition $b_4 + 2b_3 + 3b_2 + 4b_1 - 5 = 0 + 2\cdot 0.8 + 3\cdot 0.6 + 4\cdot 0.4 - 5 = 0$

According to eq. (19), $\tau_{5\infty 2} = TO_0 + \tau_0 = 8 + 0 = 8$ t.u. This result agrees with the simulated $\tau_{2\infty}$, shown in Fig. 2. We can conclude that there are two types of FLL, regarding time differences $\tau_{1\infty}$ and $\tau_{2\infty}$. In the first case, when the FLL is in a stable state, the time difference $\tau_{1\infty}$ depends on the initial conditions, the input period, and the system parameters. In the second case, $\tau_{2\infty}$ depends only on the initial conditions TO_0 and τ_0. Both are interesting for practice, depending on the type of application.

At last, in case 3, it was intentionally chosen an unstable system. The sum of chosen parameters is $b_1 + b_2 + b_3 + b_4 + b_5 = 0.2 + 0.2 + 0.2 + 0.7 + 0.2 = 1.5$.

The sum does not satisfy eq. (18). According to eq. (17), $TO_{5\infty} = TI(b_1 + b_2 + b_3 + b_4 + b_5) = 10\cdot 1.5 = 15$ t.u. This result agrees with $TO_{3\infty}$ in Fig. 2. However, the corresponding time difference $\tau_{3\infty}$ is not constant after five steps, like in cases 1 and 2. Time difference $\tau_{3\infty}$ tends to infinity in Fig. 2. That means, there is no time compatibility between the input and output periods, even after 5 steps, when FLL should reach the stable state.

The simulation results completely agree with the calculated ones. This proves the correctness of all previous mathematical descriptions.

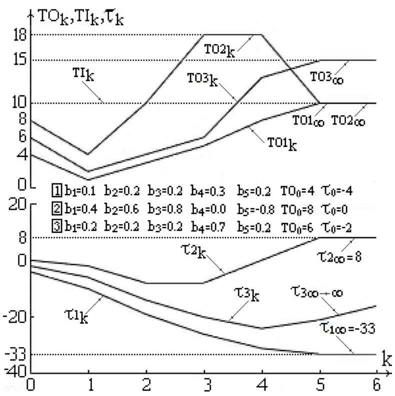

Fig. 2 – Transition and stable states of FLL_5 for the step input and for three cases of system parameters and the initial conditions.

3. DESIGN OF FIR FLL DIGITAL FILTER USING THEORY OF CLASSIC DIGITAL FILTER

3.1 FIR FLL FILTER OF THE FIFTH ORDER

In section 1 we developed the Z transform of the general form of the transfer functions of M-th order FLL, eqs. (11, 12). Using these equations, for M is equal to 5, we easily obtained eqs. (13, 14), which represent the Z transforms of the FLL_5 transfer functions $H_{TO5}(z)$ and $H_{\tau 5}(z)$. Using these two transfer functions we can discover how will the changes in the input period TI reflect on the FLL_5 output variables in the frequency domain, *i.e.*, on the output period TO and time difference τ. In other words, we can analyze the filtering properties of two digital filters, *i.e.*, two outputs belonging to FLL_5. Note that TO and τ are time variables. Period TO is always present in digital form at an output of any FLL and τ can be also easily calculated in digital form. They also appear inside pulse signals (semi-digital forms). Period TO is inside of output TO and τ is presented by a pulse width of a τ-pulse signal, which is generated by τ generator as a constituent part of an FLL, refs. [4–7, 9]. If we can, using an FLL, an input frequency spectrum of TI change into a completely different frequency spectrum of TO, in a way like digital filters do, then we can say that FLL functions like a digital filter, but is intended for the filtering of pulse signal periods. In other words, the output periods TO represent the filtered period TI. In the physical sense, it means that the variations inside of periods TO are reduced or eliminated in comparison with those insides of TI. We will now show this filtering of periods by an example of FIR FLL_5, which functions as a digital filter.

In [1], besides the comparison regarding the similarities and differences between classic digital filters and FLLs, the difference equations of these two systems of any order are also compared. The comparisons and analyzes in [1] showed that the complete theory of FIR digital filters, as well as MATLAB tools dedicated to FIR digital filters, can be used in the analysis and design of FIR FLL digital filters, considering the determined differences. In this article, we will rely on the previous conclusions from [1]. To demonstrate how we can use the existing FIR digital filter theory; we will first design a low pass FIR digital filter of the fourth order (N is equal to 4). Let us suppose that the filter is defined by the cutoff frequency fg is equal to 2000 Hz and sampling frequency fs is 14000 Hz. If we choose triangle windowing, using MATLAB command "fir1", we can get vector "b_d" of the filter coefficients as b_d is fir1(N, fn, triang (N+1)), where the normalized cutoff frequency fn is fg/(fs/2). This command gives the next coefficients for FIR digital filters: b_{0d} = 0.0620, b_{1d} = 0.2314, b_{2d} = 0.4132, b_{3d} = 0. 2314, b_{4d} = 0.0620, where the suffix "d" signifies that these coefficients belong to the digital filter. If we use any other kind of windowing, supported by MATLAB, the coefficients would not be the same. The transfer function, for the digital filter $H_{d4}(z)$ can be presented as $b_{0d} + b_{1d}\cdot z^{-1} + b_{2d}\cdot z^{-2} + b_{3d}\cdot z^{-3} + b_{4d}\cdot z^{-4}$. It can be given in another form, as in eq. (20). How can we use the coefficients of the presented digital filter to adapt FLL_5 to make digital filtering of the input periods? If we compare the transfer functions H_{TO5} and H_{d4}, we will note that both consist of five parameters or coefficients. We can simply adopt the calculated coefficients instead of the parameters and use them in H_{TO5} in a way that $b_1 = b_{0d}$, $b_2 = b_{1d}$, $b_3 = b_{2d}$, $b_4 = b_{3d}$ and $b_5 = b_{4d}$. If we enter the proposed parameters into eq. (13), $H_{TO5}(z)$ changes into eq. (21). The sum of the

$$H_{d4}(z) = (z^4 b_{0d} + z^3 b_{1d} + z^2 b_{2d} + z b_{3d} + b_{4d})/z^4 , \qquad (20)$$

$$H_{TO5}(z) = (z^4 b_{0d} + z^3 b_{1d} + z^2 b_{2d} + z b_{3d} + b_{4d})/z^5. \quad (21)$$

chosen parameters of FLL: b_{0d}, b_{1d}, b_{2d}, b_{3d} and b_{4d} is equal to one. It satisfies eq. (18). It means that the FLL_5 is a stable system. $H_{TO5}(z)$ possesses the same zeros as $H_{d4}(z)$. The difference between $H_{d4}(z)$ and $H_{TO5}(z)$, in eqs. (20) and (21), are in their denominators. Namely, their relation can be expressed as $H_{TO5}(z) = H_{d4}(z) \cdot z^{-1}$. This means that the magnitudes of the frequency responses of $H_{TO5}(z)$ and $H_{d4}(z)$ will be the same. But due to the one-step delay, which refers to factor "z^{-1}", FLL_5 will introduce an additional delay of $-\pi$ rad on the output signal, in relation to the phase that which digital filter makes on its output signal, for half of the sample rate. Based on transfer functions $H_{d4}(z)$ and $H_{TO5}(z)$, shown in eqs. (20) and (21) and considering the MATLAB rules for the definition of vector "b", we can define vectors b_{d4} and b_{TO5}, shown in eqs. (22) and (23).

$$b_{d4} = [b_{0d}\ b_{1d}\ b_{2d}\ b_{3d}\ b_{4d}], \quad (22)$$

$$b_{TO5} = [0\, b_1\, b_2\, b_3\, b_4\, b_5] = [0\, b_{0d}\, b_{1d}\, b_{2d}\, b_{3d}\, b_{4d}] = [0\ b_{d4}]. \quad (23)$$

Based on the results obtained, we can define relation between any order transfer function of FLL and the transfer function of the digital filter, whose coefficients are used as parameters of FLL. To cover all zeros of the digital filter, an FLL order must be higher for one. If the digital filter is of (M-1) order, FLL must be of M-th order. The relation of their transfer functions is presented in eq. (24). The second important conclusion relates to vectors b_{TOM} and $b_{d(M-1)}$ of the transfer functions respectively H_{TO} of M-th order and H_d of (M-1)-th order. These vectors are used in commands, devoted to the design of digital filters. Their relation is shown in eq. (25)

$$H_{TO_M}(z) = H_{d_{M-1}}(z) \cdot z^{-1}, \quad (24)$$

$$b_{TO_M} = [0 \quad b_{d_{M-1}}]. \quad (25)$$

3.2 FILTERING ABILITIES OF FIR FLL_5

After described design procedure, we can present the filtering abilities of FIR FLL_5 and compare it with the designed digital filter of the fourth order. To do that we will use the tools of FIR digital filters. Using commands "freqz (b_{TO5}, 1, 1024, fs)" and "freqz (b_{d4}, 1, 1024, fs)", the frequency responses of $H_{TO5}(z)$ and $H_{d4}(z)$ are determined and presented in Fig. 3, for the half of the sample rate. The magnitudes of the digital filter and the FLL are identical. Both phases are linear, but for half of the sample rate, the phase of FIR FLL_5 is -540° and the phase of the digital filter is -360°. The phases in which two systems are introduced into the output signals differ for expected -180°. Let us now present the effects of FIR FLL_5 filtering in the time domain. For this purpose, the input period $TI(k+1) = 10+S_1(k)+S_2(k)$ t.u. was fed into the input of FIR FLL_5, where $S_1(k) = 6 \cdot \sin[2\pi/fs \cdot f_1 \cdot k]$ and $S_2(k) = 6 \cdot \sin[2\pi/f_s \cdot f_2 \cdot k]$. The values of frequencies f_1 and f_2 are f_1 = 1500 Hz and f_2 = 4500 Hz. Since f_s= 14000 Hz, it means that S_1 is sampled by 14000/1500 ~ 9.33 samples per period, Fig. 4a. Signal S_2 is sampled with 14000/45000 ~ 3.11 samples per period, Fig. 4b.

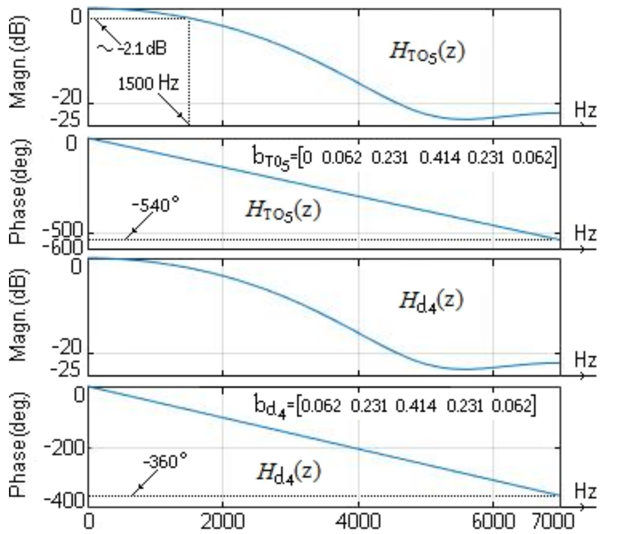

Fig. 3 – Magnitudes and phases of the frequency responses of $H_{TO5}(z)$ and $H_{d4}(z)$.

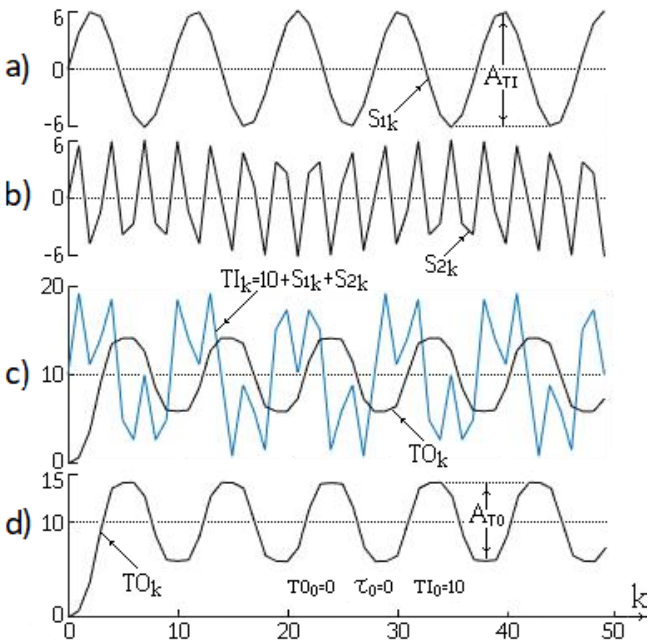

Fig. 4 – Presentations of the initial conditions and signals in the time domain: a. $S_1(k)$, b. $S_2(k)$, c. TI(k) and TO(k), d. TO(k) and the initial conditions.

Due to fact that the numbers of samples per period are not integers, S_{1k} and S_{2k} are deformed sinusoidal signals. This is especially true for S_{2k}, which has a needle shape and deformation in amplitude, creating a wide range of higher frequency components in the frequency domain. The input TI(k+1), as the sum of 10 t.u, S_{1k}, and S_{2k}, as well as TO(k) are shown in Fig. 4c. The separated TO(k) is shown in Fig. 4d. The initial conditions are TO_0 = 0 t.u., τ_0 = 0 t.u. and TI_0 = 10 t.u. The described filtering of FIR FLL_5, shown in Fig. 4, is presented in the time domain for the 50 steps. The FIR FLL_5 decreased the amplitude of S_{1k}, but it mainly preserved the basic harmonic of the input signal S_{1k}, because the frequency f_1=1500 Hz is less than the cutoff frequency f_c = 2000 Hz. However, signal S_{2k} disappeared at the output, because f_2 = 4500 Hz belongs to the stop band of FIR FLL_5.

A completed insight into the filtration process can be obtained if we present its effects in the frequency domain. Using commands "fft" and "stem", frequency spectrums of TI_k and TO_k are presented in the whole sample rate, in Fig. 5. As it was expected, the frequency component at 4500 Hz, corresponding to S_2, is suppressed, just as in Fig. 4d in the time domain. According to the results of the computer listing, shown in Fig. 5, the frequency component at 1500 Hz, corresponding to S_{1k}, is attenuated for $TO_{f1500}/TI_{f1500} = 33110/42000 = 0.788$ or for 20 log (0.788) = -2.065 dB. A similar result can be found at the magnitude of $H_{TO}(z)$ in Fig. 3. If we magnify Fig. 3, using the proportionality, it could be determined roughly, that the attenuation at 1500 Hz is about -2.1 dB. It is expected that the same attenuation can be determined in the time domain using Fig. 4. If we magnify Fig. 4 and measure A_{TO} and A_{TI}, it can be found that $A_{TO}/A_{TI} = 47/70 = 0.671$, which in relation to $TO_{f1500}/TI_{f1500} = 0.788$ represents a lower value than it is expected. This is since TO_k, in Fig. 4d, does not have an identical form as S_{1k} in Fig. 4a. This deformation is a consequence of the fact that TO_k, besides the components of S_{1k}, also contains attenuated components of signal S_{2k}, whose frequencies are higher than 1500 Hz. But note that these components, according to the FIR FLL_5 characteristics, shown in Fig. 3, have a higher attenuation than the component at 1500 Hz. To prove this claim, Fig. 6 shows TO_k for the case when $S_2 = 0$ t.u., *i.e.*, for $TI_k = 10+S_{1k}$. From the picture it can be found that now $A_{TO}/A_{TI} = 59/76 = 0.776$, which corresponds to the previously determined ratio $TO_{f1500}/TI_{f1500} = 0.788$. This is at the same time the proof for the previous claim. The attenuation of FIR FLL_5 at 1500 Hz can be also determined using the linear magnitude response of FIR FLL_5, shown in Fig. 7a. Using the proportionality of the magnified Fig. 7a, it was determined that the magnitude at 1500 Hz is approximately 0.783, which agrees with all previous results, concerning the attenuation of FIR FLL_5 at 1500 Hz. At last, note that zero component TO_{f0} in the output spectrum is equal to zero component TI_{f0} in the input spectrum in Fig. 5. This agrees with Fig. 7a, because the ratio of output and input signal is equal to one at 0 Hz. This result is also confirmed in Fig. 3, in which the attenuation at zero frequency is 0 dB. A comparison between linear magnitudes of FIR FLL_5, FIR FLL_{11} and FIR FLL_{20}, for the same previously defined conditions, is shown in Fig. 7b. As it was expected, the higher FIR FLL order provides the better filtering quality.

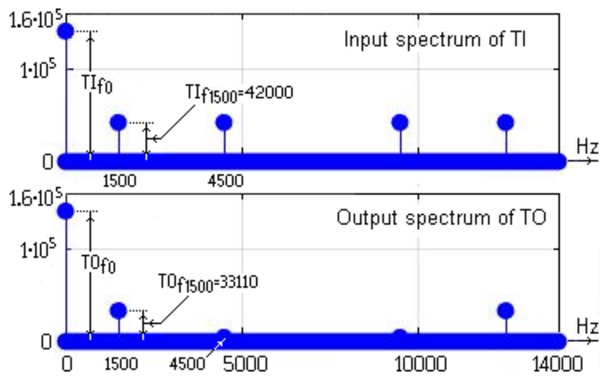

Fig. 5 – The input spectrum of TI and the output spectrum of TO.

Based on the given description, we can now describe the design procedure of FIR FLL filter of any order.

The first step is to make any kind of classic FIR digital filter, which will determine b_{dM-1}. The characteristic of this filter should cover the needs for the filtering of pulse signal periods by a FIR FLL filter. The second step is to determine the transfer function H_{TOM} and vector b_{TOM}, according to eqs. (24) and (25). Using H_{TOM} and vector b_{TOM}, all frequency analyses of FIR FLL_M filter can be now performed just in a way like with the classic digital filter, *i.e.*, by Matlab tools for FIR digital filters. If we need to make the frequency analysis of time difference τ, it is necessary to determine the transfer function $H_{\tau M}(z)$, using eq. (12) and new values of parameters "b_M" corresponding to $H_{\tau M}(z)$, [*e.g.*, $H_{\tau 5}(z)$, eq. (14), for M = 5]. New values of parameters "b_M" are to be calculated using the coefficients b_{dM-1} of the digital filter, changing $b_1 = b_{0d}$, $b_2 = b_{1d}$, and so on. After that, it is necessary to determine vector $b_{\tau M} = [0\ b_M]$. The further procedure of analyses of the output τ_k is identical to demonstrated analyzes of the output TO_k.

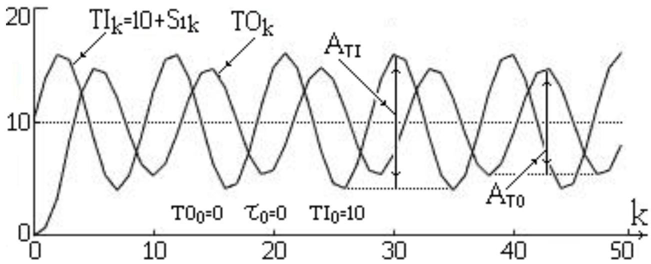

Fig. 6 – The output TO_k preserved the form of TI_k. Relation $A_{TO}/A_{TI} = 0.776$ is very close to $TO_{f1500}/TI_{f1500} = 0.788$.

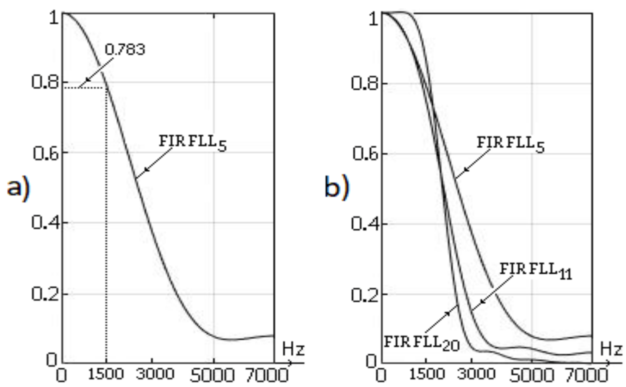

Fig. 7 – Linear magnitude of frequency responses for the half of sample rate: a. FIR FLL_5, b. FIR FLL_5, FIR FLL_{11} and FIR FLL_{20}.

4. CONCLUSIONS

This article describes the basic theory and development approach to a new kind of FIR digital filter, intended for the filtering of pulse signal periods. This kind of FIR digital filter is based on the recently described theory and design of FLL. Unlike the classic amplitude FIR digital filters, these digital filters process the periods, *i.e.*, time instead of amplitude. For the special values of the system parameters, FLL functions as a digital filter for pulse signal periods. The article describes the methodology, procedures, math, simulation support and analyzes in time and frequency domains, providing development of any order FIR FLL digital filter with any filter requirements. This is the first article in the literature describing the general development approach to FIR FLL digital filter of any order.

Although classic FIR digital filters and FLLs are two different systems, their different equations have a similar structural form, due to which the theory of FIR digital filters and the appropriate MATLAB tools, were used in the analysis and development of FIR FLL digital filters. Thanks to this fact, the process of developing new FIR FLL digital filters has been enormously shortened and almost reduced to the procedure of developing classic FIR digital filters. In this usage, care must be taken of the difference between these two systems as well as the correct physical interpretation of the results obtained.

This article opened the wide possibilities for the usage of FIR FLL digital filters in electronics, telecommunications, control, and measurements which use the different forms of periodic and non-periodic pulse signals. There is an obvious need to filter them in some of the applications. This kind of digital filter does not require A/D and D/A converters. Instead of them, the measurement of time is used, which provides more precise, simpler, and cheaper electronic solutions.

The authors are aware of the complexity of the presented material and therefore they made significant efforts to connect in logical whole all segments of different presentations and analyses like math, simulation, time presentations of signals, frequency responses of transfer functions, and frequency presentations of signals for FIR FLL of the fifth order. This helped, not only to prove the correctness of all presented materials but to facilitate the understanding of the physical process described.

For the realization of the FIR FLL digital filter, it is necessary to use a microprocessor to perform numerous calculations. Note that almost all functions of the FLL parts, described in [3–10], can be realized using microprocessors, providing that the described principles of hardware control of FLL functioning are respected.

The results of this article represent the base for the further possible applications of both FLLs and FIR FLL digital filters in different fields. However, the most probable and useful next step is the development of the IIR digital filters, based on the processing of the input and output periods.

ACKNOWLEDGEMENTS

This article was supported by the Ministry of Science and Technology of the Republic of Serbia within the project TR 32047.

Received on 18 April 2021.

REFERENCES

1. Dj. M Perisic, "*Frequency locked loops of the third and higher-order*", Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., (In revision).
2. Dj. M. Perisic, V. Petrovic, B. Kovacevic "*Frequency locked loop based on the time nonrecursive processing*", Engineering, Technology & Applied Science Research, **8**, *5*, pp. 3450-3455 (2018).
3. Dj. M. Perisic, A. Zoric, Ž. Gavric, N. Danilovic "*Digital circuit for the averaging of the pulse periods*", Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **63**, *3*, pp. 300-305 (2018).
4. Dj. M. Perisic, M. Bojovic, "*Application of time recursive processing for the development of the time/phase shifter*", Engineering, Technology & Applied Science Research, **7**, *3*, pp. 1582-1587, (2017).
5. Dj. M. Perisic, M. Perisic, D. Mitic, M. Vasic "*Time recursive frequency locked loop for the tracking applications*", Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **62**, *2*, pp. 195-203, Bucarest, (2015).
6. Dj. M. Perisic, A. Zoric, M. Perisic, V. Arsenovic, Lj. Lazic, *Recursive PLL based on the measurement and processing of time*, Electronics and Electrical Engineering, 20, 5, pp. 33-36, (2014).
7. Dj. M. Perisic, A. Zoric, M. Perisic, D. Mitic, *Analysis and application of FLL based on the processing of the input and output periods*. Automatika, **57**, *1*, pp. 230–238 (2016).
8. Dj.M Perisic, M. Bojovic, *Multipurpose time recursive PLL,* Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **61**, *3*, pp. 283-288, Bucarest, (2016).
9. Dj.M Perisic, M. Perisic, S. Rankov, *Phase shifter based on a recursive phase locked loop of the second order,* Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **59**, *4,* pp. 391–400, (2014).
10. Dj.M. Perisic, A. Zoric, Dj. Babic, Dj.Dj. Perisic, *Decoding and prediction of energy state in consumption control*, Rev. Roum. Sci. Techn. – Electrotechn. et Energ., **58**, *3*, pp. 263–272, Bucarest, (2013).
11. D. Jovcic, *Phase locked loop system for FACTS*, IEEE Transaction on Power System, **18**, pp. 2185-2192 (2003).
12. A.S. N. Mokhtar, B.B.I. Reaz, M. Maruffuzaman, M.A.M. Ali, *Inverse Park transformation using cordic and phase-locked loop,* Rev. Roum. Sci.Techn.-Electrotechn. et Energy., **57**, *4*, pp. 422-431 (2012).
13. C. C. Chung, *An all-digital phase-locked loop for high-speed clock generation*, IEEE Journal of Solid-State Circuits, **38**, *2*, pp. 347-359 (2003).
14. F. Amrane, A. Chaiba, B.E. Babes, S. Mekhilef, *Design and implementation of high-performance field-oriented control for grid-connected doubly fed induction generator via hysteresis rotor current controller,* Rev. Roum. Sci. Techn. Et Energy., **61**, *4*, pp. 319–324 (2016).
15. M. Büyük, M. İnci, M. Tümay, *Performance comparison of voltage sag/swell detection methods implemented in custom power devices,* Rev. Roum. Sci. Techn. – Electrotechn. et Energ., **62**, *2*, pp. 129–133 (2017).
16. L. Joonsuk, B. Kim, *A low noise fast-lock phase-locked loop with adaptive bandwidth control-Solid-State Circuit*, IEEE Journal, **35**, *8,* pp. 1137-1145 (2000).
17. D. Abramovitch, *Phase-locked loops: a control centric tutorial*, American Control Conference-2002, Proceedings of 2002, **1**, pp. 1-15 (2002).
18. R. Vich, *Z Transform Theory and Application (Mathematics and Applications)*, Ed. Springer (1987-first edition).
19. G. Bianchi, *Phase-Locked Loop Synthesizer Simulation*, Nc-Hill, Inc. New York, US, (2005).
20. B.D. Talbot, *Frequency Acquisition Techniques for PLL,* Wiley-IEEE Press (2012).
21. C.B. Fledderman, *Introduction to Electrical and Computer Engineering*, Prentis Hall, (2002).
22. M. Gardner, *Phase lock techniques*, Hoboken, Wiley-Interscience (2005).
23. S. Winder, *Analog and Digital Filter Design (second edition)*, Copyright©2002 Elsevier Inc. (2002).