



ENHANCING THE TRANSIENT PERFORMANCES AND STABILITY OF THREE-TANK LIQUID LEVEL USING A MODIFIED PID CONTROLLER

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Managing liquid levels in industrial tanks is crucial, especially for precise component mixing. Traditional PID controllers, though widely used, often exhibit slow settling times and excessive overshoot, which can affect system performance. This study proposes a fractionalized order PID (FrOPID) controller optimized using the Modified Artificial Hummingbird Algorithm (MAHA) to enhance stability and response in a three-tank system. The controller's effectiveness is evaluated under varying valve coefficient (K_v) and tank cross-sectional area conditions. A comparative analysis with advanced metaheuristic-optimized PID controllers confirms the superiority of the MAHA/FrOPID in terms of accuracy, response speed, and robustness, making it a highly efficient solution for liquid level control.

1. INTRODUCTION

Control engineering, a field dedicated to designing systems that maintain the desired performance of a process or device, frequently relies on PID controllers to ensure stability and optimal functionality. The PID controller, known for its simplicity and robustness, is one of the most used controllers in industry. It remains the preferred choice in over 90% of industrial control systems [1,2].

Recent studies have demonstrated that fractional-order PID (FOPID) controllers [7–10] and fractionalized-order PID (FrOPID) controllers [3–6] offer superior performance compared to standard PID controllers, particularly in systems with a greater number of time-varying variables or nonlinear dynamics. Developing the FrOPID controller requires the use of fractional calculus, an extension of classical calculus that deals with integrals of non-integer orders. In terms of tuning, FrOPID controllers provide greater flexibility and have the potential to deliver improved control performance.

As a result, the internal parameters of a FrOPID controller, denoted by, P, I^α, D are influenced by numerous factors in every plant. One of the advantages of using a FrOPID controller is its ability to deliver reliable performance across a wide range of operating conditions by adjusting the distinct values of K_p, K_i, K_d and α .

Many research studies have explored various optimization methods to determine optimal parameter settings for controlling different systems. For example, intelligent techniques have been used to tune the PID controller for liquid-level tank systems [11], genetic algorithms have been applied for PID tuning in double-tank liquid-level control [12], hybrid GA-PSO algorithms have been employed for three-tank systems [13], particle swarm optimization (PSO) [14], improved PSO-fuzzy PID [15], genetic algorithms [16], grey wolf optimization (GWO) [17], salp swarm algorithm (SSA) [18], prairie dog optimization (PDO) [19], tree-seed optimization (TSO) [20], African vulture optimization algorithm (AVOA) [21], fractional-order sliding mode controller (FOSMC) optimized using the Harris Hawks optimization (HHO/FOSMC) algorithm [22], a hybrid differential evolution PSO with an aging leader and challengers (ALC-PSODE)

[23] and Nelder Mead optimization (NMO) [24].

In this study, the authors aimed to implement an innovative optimization approach to design a proposed FrOPID controller for a three-tank level control system, with the objective of developing a more advanced and efficient control method. The artificial hummingbird algorithm (AHA) [25] was employed as a novel optimization technique, and its performance was further enhanced using a modified version. Extensive comparisons with other algorithms demonstrated the proposed controller's effectiveness. A detailed analysis of the transient and frequency responses, combined with a robustness evaluation, demonstrated the superior performance of the proposed MAHA-based modified PID (MAHA/FrOPID) controller compared with alternative methods.

The following summarizes the contribution of this paper:

- This paper introduces the MAHA-based modified PID controller, marking its first documented application in three-tank liquid level control using this specific optimization approach.
- A comparative analysis is conducted between the proposed MAHA/FrOPID controller and state-of-the-art algorithms from the literature.
- Transient response and frequency analyses are performed to evaluate the controller's dynamic performance.
- Robustness assessment is carried out under variations in valve coefficient (K_v) and tank cross-sectional area (A_1, A_2, A_3), demonstrating its adaptability and stability across varying conditions.
- The results confirm that MAHA/FrOPID outperforms existing methods, achieving superior accuracy in overshoot, rise time, and settling time, demonstrating its effectiveness for precision liquid-level control.

The paper is organized as follows: Section 2 introduces the mathematical model of the three-tank liquid-level system. Section 3 describes the modified artificial hummingbird algorithm (MAHA). Section 4 outlines its integration with the fractional-order PID controller. Section 5 presents simulation results and comparative analysis. Finally, Section 6 concludes the study.

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2. MATHEMATICAL MODEL OF A THREE-TANK LIQUID LEVEL SYSTEM

The three-tank level system calculates the difference between the desired and measured liquid levels to control the tanks' levels. A PID controller analyzes this deviation to generate a control signal. This control signal adjusts an electric valve with an opening range of 0% to 100%. The system regulates the liquid level in the tanks by modulating the valve's flow rate [23].

Figure 1 shows a basic liquid level management system comprising three tanks, B, C, and D, along with a primary tank, E, which supplies liquid to a pump [25].

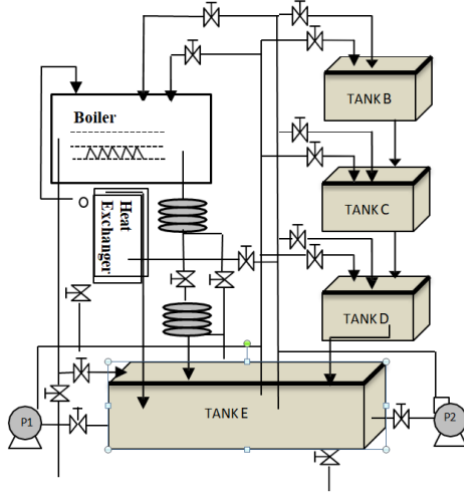


Fig. 1 –Three-tank liquid level system.

A simple structure of tank D is shown on the right side of Fig. 1. According to the mass balance:

$$q_2 - q_1 = A \left(\frac{dh}{dt} \right), \quad (1)$$

$$q_1 = k_1 \times v_0, \quad (2)$$

$$q_2 = k_2 \times \sqrt{h}. \quad (3)$$

where q_1 and q_2 represent the rates at which liquid enters and exits. The cross-sectional area of the tank is denoted by A , while h represents the liquid height within the tank. The valve's opening range is indicated by v_0 , with k_1 and k_2 representing the flow proportional coefficients of the inlet and outlet valves, respectively.

From eq. (3), the liquid level system exhibits nonlinear behavior. However, for the sake of simplicity, near the equilibrium point (V_0, h_0), eq. (3) can be linearized and approximated as:

$$\frac{q_2}{h} \approx \frac{k_2}{\sqrt{h_0}} = k_3. \quad (4)$$

Perform the Laplace transformation of eqs. (1), (2), and (4):

$$\frac{H(s)}{V_0(s)} = \frac{k}{Ts + 1} \quad (5)$$

where $k = k_1/k_3$, $T = A/k_3$. For $k_1 = 2$, $k_3 = 5$, $\tau = 0$, $A = 20$; according to [26].

For a three-tank system, the process is cascaded, meaning the total transfer function is obtained by multiplying three first-order transfer functions:

$$G_p(s) = \frac{H(s)}{Q(s)} = \frac{1}{64s^3 + 9.6s^2 + 0.48s + 0.008}. \quad (6)$$

3. MODIFIED ARTIFICIAL HUMMINGBIRD ALGORITHM (MAHA)

The modified artificial hummingbird algorithm (m-AHA) is an improved version of the artificial hummingbird algorithm (AHA), a bio-inspired optimisation method inspired by hummingbird foraging behaviour. The alterations implemented in AHA seek to enhance its efficacy, including accelerated convergence, improved global search, and the avoidance of local optima in optimization.

The main operational steps of the MAHA optimizer are illustrated in the flowchart and can be summarized as follows [26,27]:

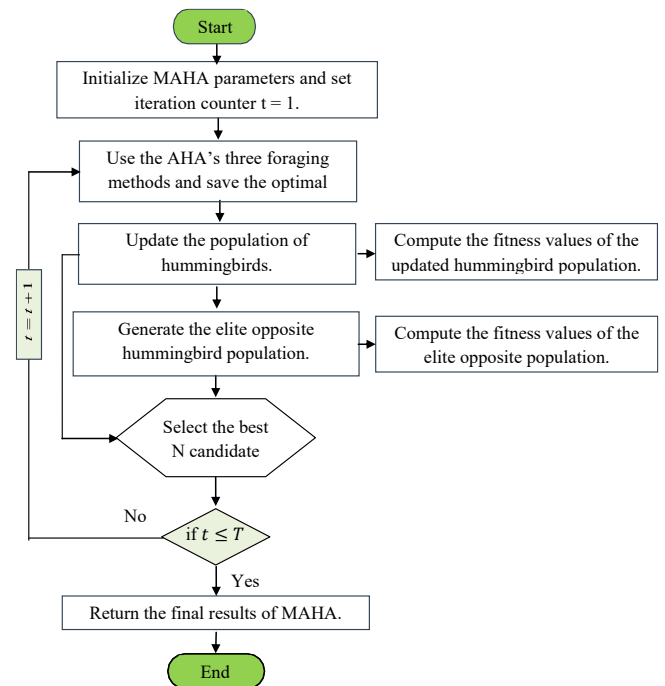


Fig. 2 –Flowchart diagram of the proposed MAHA optimizer.

4. PROPOSED DESIGN PROCEDURE AND FrOPID CONTROLLED LIQUID LEVEL SYSTEM

This paper explores the implementation of a fractional PID (FrOPID) controller within the transfer function of Equation 6 for a feedback control system managing liquid levels. The models apply the F objective function to enhance key performance indicators. The controller manages the output $Y(s)$, based on the input $R(s)$, ensuring that the system remains stable and performs as desired, even in the presence of external disturbances $D(s)$.

The parameters for the MAHA/FrOPID controller applied to the three-tank system are provided in Table 1.

Table 1
Parameters of the MAHA algorithm

Parameter	Value
Maximum number of iterations	50
Population size	3
Elite opposition ratio	0.1
Lower bounds for $[K_p, K_i, K_d]$	[0.0001, 0.0001, 0.01]
Upper bounds for $[K_p, K_i, K_d]$	[5, 5, 5]

The design of a traditional PID controller is represented by equation 7,

$$G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right). \quad (7)$$

The enhancement of fractionalization in the control system element alters the PID control rule, resulting in the fractionalization of the integral operator $1/s$ [28,29]:

$$\frac{1}{s} = \frac{1}{s^\alpha} \cdot \frac{1}{s^{1-\alpha}}. \quad (8)$$

The classical fractionalized order PID controller to be developed is represented by:

$$G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right) = \frac{1}{s^\alpha s^{1-\alpha}} \left(\frac{K_p T_i T_d s^2 + K_p T_i s + K_p}{T_i} \right); 0 < \alpha < 1. \quad (9)$$

Figure 3 shows a liquid-level control system for a three-tank configuration using a FrOPID feedback loop. In this system, $G_p(s)$ the plant and controller models are represented, respectively.

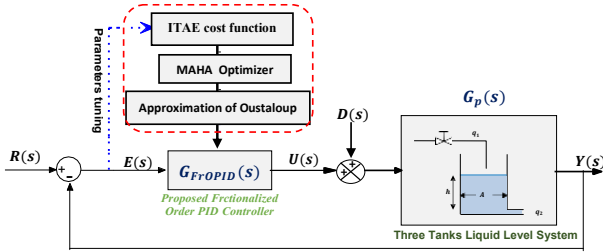


Fig. 3 – Proposed MAHA-based FrOPID controller.

In this study, ITAE is chosen as the performance criterion for the MAHA optimizer. Calculation of the error $e(t)$ involves determining the difference between the reference model and the actual model. A smaller error value indicates a closer approximation to the desired controller parameters.

$$J(K_p, K_i, K_d) = ITAE = \int_0^{t_{sim}} t |e(t)| dt. \quad (10)$$

The letter J denotes the performance criteria, while $e(t)$ represents the error signal. The simulation time (t_{sim}) was set to 10 s for this investigation. The ITAE optimization problem is non-convex and has multiple local minima; therefore, algorithms may converge to different near-optimal solutions.

5. SIMULATION RESULTS AND DISCUSSION

This study investigates the effectiveness of the proposed FrOPID controller for liquid level regulation in a three-tank system, optimized using the MAHA. The optimal tuning settings for the FrOPID were obtained using MAHA within a MATLAB/Simulink simulation model specifically developed for liquid-level control. The controller's performance is benchmarked against a classical PID optimized by several state-of-the-art methodologies, including the MAHA-based PID (MAHA/PID), the arithmetic optimization algorithm combined with Harris Hawks optimization (AOA-HHO/PID), the covariance matrix adaptation evolution strategy (CMA-ES/PID), the particle swarm optimization (PSO/PID), and the hybrid

differential evolution–PSO with an ageing leader and challengers strategy (ALC-PSODE/PID). The detailed parameter settings and their corresponding values are presented in Table 2.

Table 2

Proposed controller's gain parameters and other controllers are compared.

Controllers	K_p	K_i	K_d	α
MAHA/PID	0.05149	0.00101	1.39664	1
AOA-HHO/PID	0.04000	0.00050	0.42690	1
CMA-ES/PID	0.05100	0.00130	0.39140	1
PSO/PID	0.05280	0.00030	1.00000	1
ALC-PSODE/PID	0.04190	0.00090	1.00000	1
Proposed Controller	0.05149	0.00101	1.39664	0.5

The relatively small proportional gain is due to fractional-order control and the Oustaloup approximation, which alter the frequency-domain dynamics compared to classical PID.

The closed-loop transfer function is "fractionalized" in eq. (11), with the integrator's fractional order $\alpha = 0.5$ approximated using the Oustaloup technique. The approximation parameters are $\omega_b = 0.001$ rad/s, $\omega_h = 1000$ rad/s and $N = 4$. The MAHA optimizer employs unity feedback.

$$G_{MAHA-FrOPID}(s) = \frac{3s^2 + 3s + 0.9921}{s^4 + 4s^3 + 6s^2 + 4s + 1}. \quad (11)$$

5.1 TRANSIENT AND FREQUENCY STABILITY ANALYSIS

The transfer functions are employed to analyze the transient stability of the three-tank level control system using various optimization-based control techniques. The results of the analysis are presented in Figs. 4 and 5 and Tables 3 and 4, comparing the dynamic performance of different controller schemes.

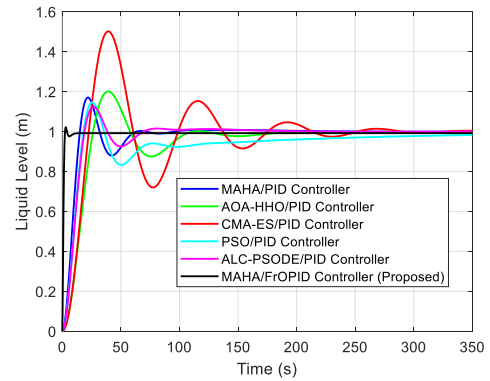


Fig. 4 – Step response of various optimizer-based controller schemes for water level.

Fig. 4 shows that the proposed MAHA/FrOPID achieves the fastest settling time with minimal overshoot and oscillations. In contrast, CMA-ES/PID and PSO/PID suffer from significant overshoot and prolonged oscillations, and AOA-HHO/PID and ALC-PSODE/PID display slower settling.

Figure 5 further confirms the superiority of MAHA/FrOPID through its well-balanced magnitude and phase responses in the Bode plots, indicating improved robustness and disturbance rejection.

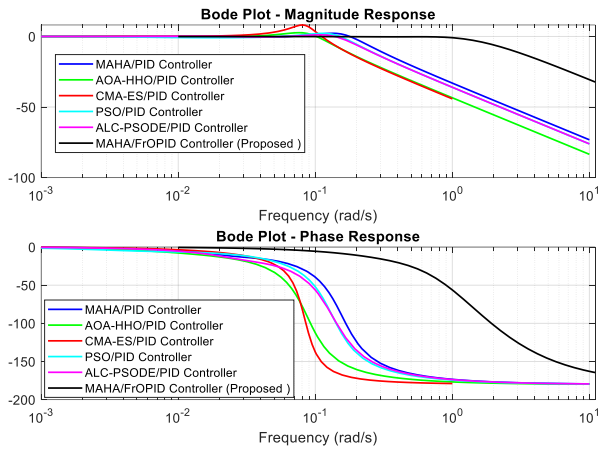


Fig. 5 – Bode plots of various optimizer-based controller schemes for water level.

Supporting these observations, Tables 3 and 4 summarize the time- and frequency-domain metrics.

Table 3
Transient Performance Metrics.

Controllers	T_r (s)	T_s (s)	T_p (s)	OS (%)
MAHA/PID	10.188	57.779	22.363	16.977
AOA-HHO/PID	17.793	160.11	39.279	20.116
CMA-ES/PID	15.013	238.66	38.279	49.991
PSO/PID	12.218	320.55	26.322	14.434
ALC-PSODE/PID	12.826	64.219	26.398	12.459
Proposed Controller	1.4321	3.607	2.98	3.0257

Table 4
Frequency Performance Metrics.

Controllers	G_m (dB)	ϕ_m (°)	B_w (Hz)
MAHA/PID	Inf	69.992	0.20939
AOA-HHO/PID	Inf	64.636	0.1183
CMA-ES/PID	Inf	29.143	0.12315
PSO/PID	Inf	70.906	0.17719
ALC-PSODE/PID	Inf	83.32	0.17088
Proposed Controller	Inf	Inf	1.4757

The proposed controller achieves the lowest rise time (1.4321 s), the shortest settling time (3.607 s), and the minimal overshoot (3.0257%), while also maintaining an infinite gain margin and the highest bandwidth (1.4757 Hz). Collectively, these results demonstrate that MAHA/FrOPID consistently outperforms the other controllers in both transient and frequency performance, ensuring enhanced stability, robustness, and precision in three-tank liquid level regulation.

5.2 ROBUSTNESS ANALYSIS

5.2.1 VALVE COEFFICIENT (K_v) VARIATIONS

The study examines how variations in the valve flow gain (K_v) affect the performance of different controllers in tank level regulation. Testing $K_v = 0.5$ and $K_v = 2$ simulated reduced and increased flow capacities, respectively, with significant impacts on rise time, settling time, and overshoot. This analysis demonstrates the controller's robustness to practical valve variations, ensuring reliable operation under different conditions. Importantly, no fixed nominal test at K_v was performed; instead, the focus was on a range of flow gains to better reflect real-world scenarios.

Figures 6 and 7 present step response comparisons of the proposed controller and various PID controllers under different valve coefficient (K_v) conditions: $K_v = 0.5$ (Fig. 6) and $K_v = 2$ (Fig. 7).

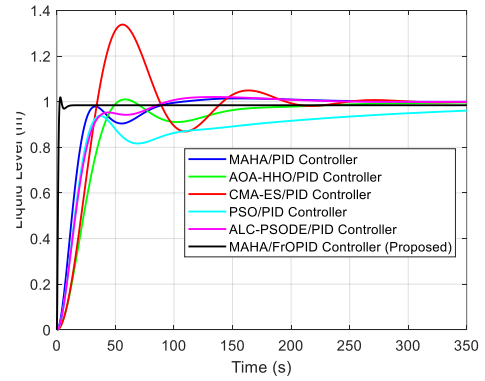


Fig. 6 – Step response of various optimizer-based controller schemes for water level with K_v variation ($K_v = 0.5$).

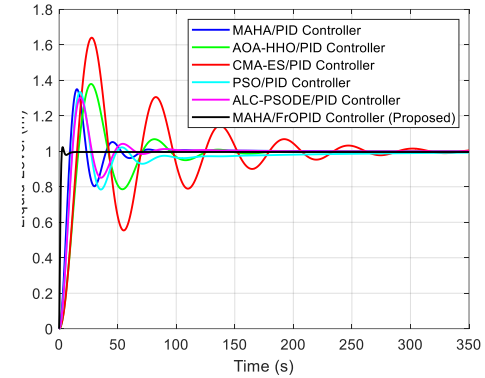


Fig. 7 – Step response of various optimizer-based controller schemes for water level with K_v variation ($K_v = 2$).

From Figs. 6 and 7, it is evident that the proposed MAHA/FrOPID Controller consistently provides superior performance across both K_v levels. It exhibits minimal overshoot, fast response times, and strong stability, indicating exceptional resilience to variations in valve coefficients. In contrast, other controllers such as CMA-ES and PID display noticeable performance degradation, especially at higher K_v values, with increased overshoot and prolonged settling times, reflecting reduced adaptability to dynamic system changes.

Tables 5 and 6 provide detailed comparison of transient performance metrics for various controllers with valve coefficient $K_v = 0.5$ and $K_v = 2$, focusing on several key parameters: rise time (T_r), settling time (T_s), peak time (T_p) and overshoot (OS%).

Table 5

Transient and frequency performance metrics for valve coefficient variations $K_v = 0.5$.

Controllers	T_r (s)	T_s (s)	T_p (s)	OS (%)
MAHA/PID	19.169	86.532	155.86	1.4256
AOA-HHO/PID	32.169	209.06	57.293	0.9711
CMA-ES/PID	23.146	185.12	56.279	33.800
PSO/PID	24.295	483.88	1100.2	0.0000
ALC-PSODE/PID	26.098	143.63	133.62	2.0492
Proposed Controller	1.4112	3.7569	2.9600	3.5660

Table 6

Transient and frequency performance metrics for valve coefficient variations $K_v = 2$.

Controllers	T_r (s)	T_s (s)	T_p (s)	OS (%)
MAHA/PID	6.2766	66.134	15.312	34.982
AOA-HHO/PID	11.119	120.61	26.589	37.799
CMA-ES/PID	10.307	278.26	27.208	63.844
PSO/PID	7.4604	191.93	17.929	33.400
ALC-PSODE/PID	7.6799	60.830	18.430	30.135
Proposed Controller	1.4429	3.5194	2.9	2.757

Tables 5 and 6 confirm that the proposed MAHA/FrOPID controller outperforms the others with faster rise and settling times and minimal overshoot. Unlike alternatives that show slower responses or higher overshoot, it ensures both efficiency and robustness, making it well-suited for level control under varying valve coefficients.

5.2.2 VARIATIONSTANK CROSS-SECTIONAL AREA (A1, A2, A3) VARIATIONS

To investigate the practical implications of tank size on control performance, experiments were conducted with tanks having cross-sectional areas scaled by factors $A = 0.5$, and 2. These tests evaluated how changing the tank volume influences the dynamic response of liquid levels under different controllers.

Figs 8 and 9 present step responses of various PID controllers and the proposed FrOPID controller for a liquid-level control system under different tank cross-sectional areas: $A = 0.5$ (Fig. 8) and $A = 2$ (Fig. 9).

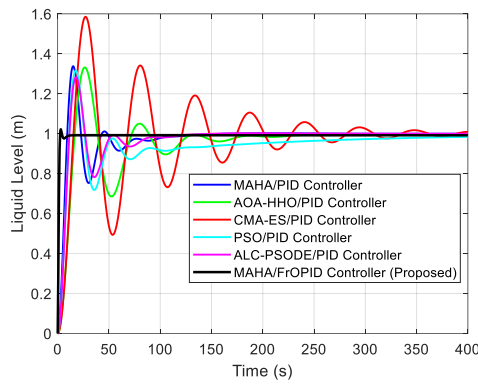


Fig. 8 –Step response of various optimizer-based controller schemes for water level under cross-sectional areas, ($A = 0.5$).

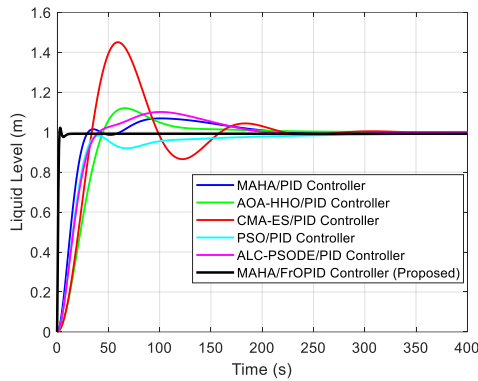


Fig. 9 –Step response of various optimizer-based controller schemes for water level under cross-sectional areas, ($A = 2$).

As shown in the Figs. For 8 and 9, the proposed MAHA/FrOPID controller demonstrates significantly better performance across both cross-sectional areas than other PID-based controllers. It offers faster stabilization, minimal or no overshoot, and no steady-state error, making it a robust choice for liquid-level control systems regardless of tank size.

Tables 7 and 8 present quantitative transient performance data for several controllers used in a liquid-level control system under two distinct cross-sectional areas, ($A = 0.5$) and ($A = 2.0$), focusing on several key parameters: rise time (T_r), settling time (T_s), peak time (T_p), and overshoot ($OS\%$).

Table 7

Transient performance metrics under cross-sectional areas ($A = 0.5$).

Controllers	T_r (s)	T_s (s)	T_p (s)	OS (%)
MAHA/PID	6.3782	101.28	14.886	33.78
AOA-HHO/PID	11.311	173.9	24.954	32.456
CMA-ES/PID	10.354	325.61	26.601	58.399
PSO/PID	7.5916	355.12	17.723	31.642
ALC-PSODE/PID	7.7993	88.343	17.417	28.221
Proposed Controller	1.4321	3.607	2.98	3.0257

Table 8

Transient performance metrics under cross-sectional areas ($A = 2$).

Controllers	T_r (s)	T_s (s)	T_p (s)	OS (%)
MAHA/PID	18.662	182.46	100.72	6.9064
AOA-HHO/PID	30.036	126.72	66.351	12.004
CMA-ES/PID	22.615	206.79	58.581	45.017
PSO/PID	22.939	215.85	596.7	0.0000
ALC-PSODE/PID	24.372	178.13	100.98	10.121
Proposed Controller	1.4321	3.607	2.98	3.0257

It can be seen from Tables 7 and 8 that the proposed MAHA/FrOPID controller demonstrates superior performance in both cross-sectional areas, with faster rise, settling, and peak times, minimal overshoot (indicating better control precision), and no signs of instability.

6. CONCLUSIONS

This study presents a Fractionalized Order PID (FrOPID) controller, using the MAHA algorithm, for regulating liquid levels in a three-tank system. The FrOPID controller improves upon the conventional PID method by incorporating a fractional-integral term, offering greater flexibility and precision in the control strategy. To demonstrate the effectiveness of the proposed MAHA/FrOPID controller, comprehensive comparisons were conducted with contemporary, top-performing methods that also employ the PID controller for regulating the three-tank liquid level system.

We carried out thorough assessments of the system, focusing on transient response, frequency response characteristics, and robustness studies, while accounting for variations in valve coefficient (K_v) and cross-sectional areas (A). These investigations demonstrate the superior efficacy of the MAHA/FrOPID controller compared to alternative techniques. The transient response analysis highlights the system's behavior during the initial control phase, emphasizing the speed and stability of the MAHA/FrOPID controller in achieving the specified liquid level setpoints. Furthermore, the frequency response analysis evaluates the controller's ability to respond to varying input frequencies, providing valuable insights into its performance under dynamic conditions. The robustness analysis assesses the impact of valve variations and cross-sectional areas on the controller's effectiveness in regulating tank levels. Our findings consistently demonstrate the MAHA optimizer's exceptional ability to achieve optimal control performance in the liquid level system. Its ability to efficiently tune PID controller parameters surpasses that of alternative approaches, underscoring its superiority in providing stable, precise liquid level regulation.

For future work, we acknowledge the value of exploring adaptive fractional-order controllers, such as adaptive F-PID systems, which have the potential to enhance control performance further by better handling the dynamic behavior of the three-tank system. Incorporating derivative action within an adaptive fractional control framework

could offer additional robustness and stability benefits. Moreover, experimental validation of the proposed MAHA/FrOPID controller on a physical three-tank setup is essential to verify its practical applicability and performance in real-world conditions.

CREDIT AUTHORSHIP CONTRIBUTION STATEMENT

Abdelhakim Idir: Formal analysis, literature review, data acquisition, data organization, implementation, writing – original manuscript preparation.
Mokhtar Nesri: Supervision, data consistency analysis, and verification.
Khaled Belhouchet: reviewing and editing the manuscript, literature review.
Sifelislam Guedida: Conceptualization, software development, writing – manuscript review and editing, validation, and investigation.
Laurent Canale: Supervision, validation, critical revision of the manuscript, and final approval.

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