EXPERIMENTAL ASSESSMENT OF CONSUMER UNBALANCE AND NONLINEARITY EFFECTS IN THREE-PHASE NETWORKS WITHOUT NEUTRAL CONDUCTOR

IONUT-MARIUS MINDREANU¹, RADU-MIRCEA CIUCEANU², IOSIF VASILE NEMOIANU^{2,*}

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Optimizing the performance and quality of electrical energy is essential in the context of the increasing complexity of modern power systems. This article delves into the power effects specific to distorted and non-symmetrical regimes of three-phase power systems without neutral. We assess the effects in strongly distorted and unbalanced three-phase circuits and the negative impact of the unwanted additional power flow. Two unequally distorting single-phase devices are introduced in series with the elements of the phases of two 3-phase consumers, then on two phases of the same consumer. The effects inflicted by the nonlinear elements on the active and reactive power exchange are evaluated from the perspective offered by the Tugulea power theory. The latter introduces the concepts of symmetry, non-symmetry, and residual active/reactive powers to account for the unbalance and distortion of three-phase loads. We propose an experimental platform modeling a simplified power system. The powers mentioned above are computed using dedicated software. The results are summarized and commented on, aiming at a quantitative countering and billing of the unwanted energy transfer associated with the powers specific to Tugulea theory.

1. INTRODUCTION

In recent years, the ever-present power electronics associated with introducing alternative energy sources [1-4] have prompted novel approaches to managing electrical grids [5-8]. The additional strain has been put along with the classical equipment traditionally affecting power quality, *e.g.*, electrical machines, electric arc furnaces, welding equipment, and many more. Hence, unbalanced and distorted regimes represent a major challenge in managing electrical systems, energy efficiency, stability, and delivered power quality [9 - 12]. Harmonic distortions and phase imbalances can lead to significant energy losses and additional stress on critical components of electrical networks [12-14].

This paper aims to implement the theoretical concepts developed in the referenced articles through an experimental platform, whose acquired data are fed to software developed to assess the nonlinearity and phase imbalance effects inflicted by such three-phase consumers on the grid from a quantitative point of view. The theory of choice is the Țugulea power theory as the appropriate framework aiming to investigate and quantitatively demonstrate the energetic effects of non-symmetrical and unbalanced regimes [15]. The findings will contribute to the ongoing advancement of electro-energy technologies, promoting more efficient and sustainable management of energy resources [15–17].

From a theoretical perspective of Budeanu's power theory definitions, and implicitly according to Ţugulea power theory, the following expressions are considered for a single-phase consumer [9–13]:

$$P = U_0 I_0 + \sum_{k=1}^{\infty} U_k I_k \cos \varphi_k, \qquad (1)$$

$$Q = \sum_{k=1}^{\infty} U_k I_k \sin \varphi_k, \qquad (2)$$

$$S = \sqrt{(\sum_{k=0}^{\infty} U_k^2)(\sum_{k=0}^{\infty} I_k^2)},$$
 (3)

$$D = \sqrt{S^2 - P^2 - Q^2},$$
 (4)
$$PF = \frac{P}{S},$$
 (5)

where: P[W] – active power; Q[var] – reactive power; S[VA] – apparent power, D[vad] – distortion power; PF – power factor; $U_k[V]$ – harmonic rms voltage value; $I_k[A]$ – harmonic rms current value; k – harmonic order. Budeanu considers similar definitions, and Țugulea approaches for three-phase consumers.

It is known that unbalanced electrical loads are sources of negative sequence and zero sequence power [9-13]. In general, the active powers reinjected into the network on the negative and zero sequences by unbalanced loads represent additional losses [18-20]. An important aspect is the method of measuring and billing active energy resulting from the interpretation of the Tugulea power theory. In brief, the theory proves that a distorting/unbalanced three-phase load injects active power (non-symmetry and residual) into the grid and is absorbed by the linear and balanced loads. As a result, the 'legit' consumers are billed more than they should, whereas the 'harmful' loads are billed less [19-25]. This inequity is essentially highlighted by the Tugulea power theory, which is also valid for the reactive power QB (Budeanu definition), with an appropriate interpretation resulting from the two kinds of reactive power (inductive vs. capacitive).

All these considerations clearly show that active/reactive power distribution on the positive sequences (symmetry powers) and the negative and zero sequences (non-symmetry powers) must be separated and accounted for distinctly, as well as the higher harmonics corresponding powers, termed residual [12, 14, 19, 20]. For this purpose, regarding the fundamental harmonic component, we consider the following definitions [9 – 14] as appropriate for a quantitative evaluation of the unbalance phenomenon under scrutiny:

$$P_{\rm s} = P^+ = 3U^+ I^+ \cos \varphi^+, \tag{6}$$

$$Q_{\rm s} = Q^+ = 3U^+ I^+ \sin \varphi^+, \qquad (7)$$

*Corresponding author: iosif.nemoianu@upb.ro.

¹ Doctoral School of Electrical Engineering, National University of Science and Technology "Politehnica" Bucharest, 313 Splaiul Independenței, 060042 Bucharest, Romania, E-mail: ionut.mindreanu@stud.electro.upb.ro.

² Department of Electrical Engineering, National University of Science and Technology "Politehnica" Bucharest, 313 Splaiul Independenței, 060042 Bucharest, Romania, E-mail: radu.ciuceanu@upb.ro.

where: $P_s[W]$ – active symmetry power; $P^+[W]$ – positive sequence active power; $U^+[V]$ – positive sequence voltage; $I^+[A]$ – positive sequence current; $\varphi^+[rad]$ – voltage-current positive sequence phase shift; $[var] - Q_s$ symmetry reactive power; $Q^+[var]$ – positive sequence reactive power.

Non-symmetry active or reactive power is defined as the sum of the active or reactive powers corresponding to the negative and zero sequence components for each three-phase load of the network [16, 21–27].



Fig. 1 – Photographic image of the experimental platform.

$$P_{\rm n} = P^{-} + P^{0} = 3U^{-}I^{-}\cos\varphi^{-} + 3U^{0}I^{0}\cos\varphi^{0} , \qquad (8)$$

$$Q_{\rm n} = Q^{-} + Q^{0} = 3U^{-}I^{-}\sin\varphi^{-} + 3U^{0}I^{0}\sin\varphi^{0}, \qquad (9)$$

where: $P_n[W]$ – non-symmetry active power; $P^-[W]$ – negative sequence active power; $P^0[W]$ – zero sequences active power; $U^-[V]$ – negative sequence voltage; $I^-[A]$ – negative sequence current; φ^- [rad] – negative sequence voltage-current phase shift; $U^0[V]$ – zero sequence voltage; I^0 [A] – zero sequence current; φ^0 [rad] – zero sequence voltagecurrent phase shift; $Q_n[var]$ – non-symmetry reactive power; $Q^-[var]$ – negative sequence reactive power; Q^0 [var] – zero sequence reactive power.

Active and reactive power are conserved globally for each harmonic in any network operating in a periodic distorted regime (by Tellegen's theorem [18]). In any network supplied with sinusoidal voltages at the terminals, the sources of higher harmonics are the distorting (nonlinear) loads [19–23]. The corresponding powers specific to the Țugulea power theory are [16]:

$$P_{\rm r} = \sum_{p=1}^{3} U_{0,p} I_{0,p} + \sum_{p=1}^{3} \sum_{k=2}^{\infty} U_{k,p} I_{k,p} \cos \varphi_{k,p}, \quad (10)$$

$$Q_{\rm r} = \sum_{p=1}^{3} \sum_{k=2}^{\infty} U_{k,p} I_{k,p} \sin \varphi_{k,p}, \qquad (11)$$

where: P_r [W]– the residual ('distortion' [12]) active power, Q_r [var]– the residual ('distortion' [12]) reactive power, p the phase index, whereas the other quantities have the same obvious significance as in (1) and (2). The term used by the author in its initial theory, 'distortion', was termed here 'residual' to avoid any possible confusion with (4).

In most cases, the active power associated with the zero sequence is minimal or absent, as zero sequence components generally do not contribute to active power distribution. However, under significant imbalances or nonlinear loads, these components can impact system losses and overall efficiency [15, 16, 22 - 24]. Note that the initial theory did not include the DC components in (10), which were later introduced to account for the abovementioned aspects.

A nonlinear load is an electrical device where the relationship between the current and the applied voltage is not linear. This type of load does not follow Ohm's linear law and introduces harmonic distortions into the system because the current and voltage do not vary proportionally [16, 26, 27].

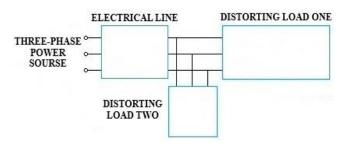


Fig. 2 – Synthetic block diagram illustrating the first conducted experiment.

As mentioned before, the experimental study, based on the laboratory platform shown in Fig. 1, is based on the fundamental research of Professor Andrei Țugulea, whose work has been essential in developing the theory of distorted regimes on electromagnetic systems and electrical circuits, identifying energy losses caused by these effects.

2. EXPERIMENTAL CIRCUITS

Two models were proposed for simulation to highlight the power flow in the circuit. The first, having the synthetic block diagram shown in Fig. 2, is an experimental circuit without a neutral conductor (see details in Fig. 3), with a nonlinear element mounted on the first phase of the distorting three-phase load one and a second nonlinear element, with the same characteristics as the first nonlinear element but a different conduction angle, mounted on the third phase of the distorting load two.

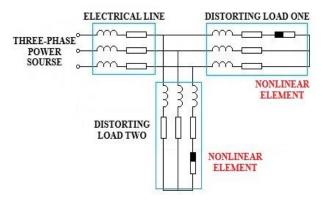


Fig. 3 – Circuit diagram comprising two distorting three-phase loads; the two unequally distorting nonlinear elements are mounted on phase one of the first load and phase three of the second load (Model One).

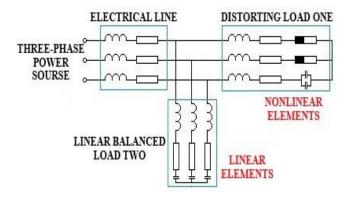


Fig. 4 – Electrical schematic with multiple nonlinear elements mounted on the first and second phases of distorting load one (Model Two).

A slightly different experimental circuit is proposed for the second model (Fig. 4). For distorting load one, two nonlinear elements with the same characteristics and different conduction angles are mounted in the circuit on the first and second phases, and two different capacitors are mounted in parallel on the three phases. Three capacitors with the same characteristics are mounted for distorting load two, one on each phase.

The same symmetrical power supply configuration was maintained during the simulation of the two experimental circuits, and the loads for the two consumers were modified. For the second model, more nonlinear elements were introduced into the circuit to emphasize the phenomenon of unwanted power circulation within the three-phase network.

3. CASE STUDIES – MEASUREMENTS, RESULTS, AND DISCUSSION

The experiments mainly used four Janitza UMG 509-PRO network analyzers to simultaneously acquire voltage and current waveforms at various relevant points in the network, shown in the upper-left region of Fig. 1. For illustration purposes (screen captures of voltage or current waveforms and the corresponding harmonic content histograms), we used a Chauvin Arnoux C.A 8336 network analyzer appears in the center-left region of Fig. 1. All five multifunctional network analyzers are used for monitoring and analyzing the quality of electrical energy.

3.1. CIRCUIT MODEL ONE

The waveforms of the currents become increasingly distorted as the data acquisition approaches the nonlinear elements' terminals. The power line current waveforms and their harmonic histograms are shown in Fig. 5.

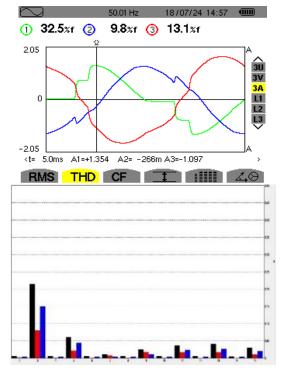


Fig. 5 – a) Waveforms of the currents flowing along the electrical line;
 b) current harmonics before the connection point of the two distorting three-phase consumers.

Concerning the three-phase distorting consumer one, the current's total harmonic distortion (THD) corresponding to phase one, where the nonlinear element is present, is greater than that of phases two and three of the same consumer, where only linear elements are connected, as illustrated in Fig. 6.

Similarly, for the three-phase distorting consumer two, phase three, containing the nonlinear element, exhibits higher THD values compared to phases one and two, containing only linear elements, as shown in Fig. 7.

The current waveforms for the three phases have different distortion factors depending on where they are measured. The more distorted the nonlinear element, the greater the distortion factor for the two disturbing consumers.

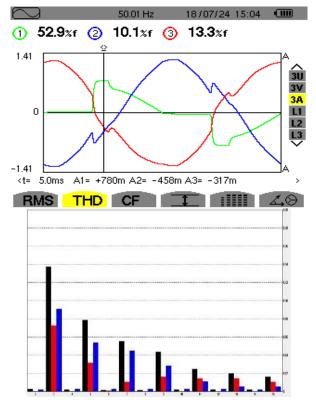


Fig. 6 – a) Waveforms of the currents for the three phases; b) Current harmonics at the supply terminals of distorting consumer one.

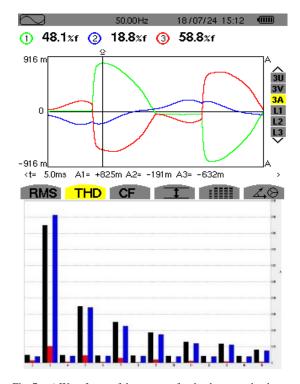


Fig. 7 - a) Waveforms of the currents for the three supply phases; b) current harmonics at the supply terminals of distorting consumer two.

The current waveform for the second phase, for both distorting loads one and two, is the least distorted, as there is no nonlinear element mounted on the second phase for either consumer, showing that the harmonics associated with the second phase have the lowest amplitude. The active and reactive power balance verification is illustrated in Table 1, showing minimal measurement and computation errors, thus experimentally reconfirming the validity of the Țugulea power theory.

 Table 1

 Symmetry, non-symmetry, residual, and total active and reactive powers corresponding to circuit model one; power balance verification

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	$P_{\rm s}$ [W]	$P_{\rm n}$ [W]	$P_{\rm r}$ [W]	P_{total} [W]
Power line	157.29	18.84	5.35	181.48
Con. 1	158.15	-16.5	-2.09	139.54
Con. 2	55.68	-2.47	-3.26	49.94
Sum [W]	371.12	-0.13	-0.01	370.97
3 ~ source	370.68			
Error [%]	-0.07	0.83	0.74	
	$Q_{\rm s}$ [var]	$Q_{\rm n}$ [var]	$Q_{\rm r}$ [var]	$Q_{\text{total}}[\text{var}]$
Power line				
I Ower mile	2.09	0.81	1.14	4.05
	2.09 43.08	0.81 -10.98	1.14 1.96	4.05 34.05
Con. 1 Con. 2		0.00-		
Con. 1	43.08	-10.98	1.96	34.05
Con. 1 Con. 2	43.08 39.55	-10.98 9.74	1.96 -3.23	34.05 46.06

Note that the passive sign convention was used at the terminals of each phase, meaning that the negative sign values correspond to generated (delivered) power. The balanced, non-distorting line receives symmetrical, non-symmetry, and residual active power. In a distorting load, one receives symmetrical active power but absorbs non-symmetry and residual active power from the system.

Distorting load two receives symmetrical active power from the system. It returns non-symmetry and residual active power to the system, resulting in unwanted active power circulation within and between the distorting loads, as qualitatively depicted in the Sankey diagram shown in Fig. 8.

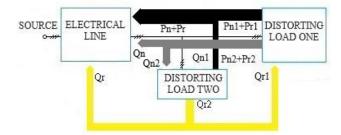


Fig. 8 - Active and reactive powers' flow (Model One).

There is also a reactive power flow between the components of the experimental circuit. As mentioned, the negative sign indicates that a certain distorting consumer injects reactive power into the system. Conversely, a positive sign for the non-symmetry and residual reactive powers indicates that the respective consumer absorbs the abovementioned powers. In contrast, distorting load two receives both symmetrical and non-symmetry reactive power and delivers residual reactive power to the system.

3.2. CIRCUIT MODEL TWO

For the second proposed experimental setup, where multiple nonlinear elements are mounted on phases one and two of distorting consumer one, the waveforms are much more distorted than the first proposed model, and the THD valuer is significantly more pronounced. Additionally, the conduction angles for the nonlinear elements differ from those in the first case studied.

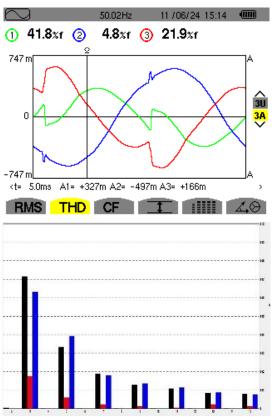


Fig. 9 – a) Waveforms of the currents flowing along the electrical line; b) current harmonics before the connection point of the two three-phase consumers.

The waveform data are acquired from the same point locations as in the first experiment. However, even from the connection point of the two consumers, a higher distortion factor can be noticed in Fig. 9 for the first and third phases. The results for the two consumers are shown in Figs. 10 and 11, respectively.

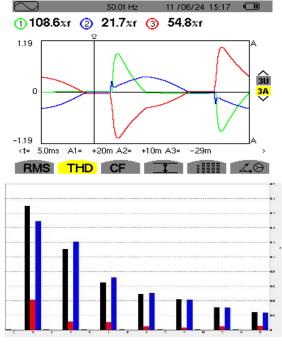
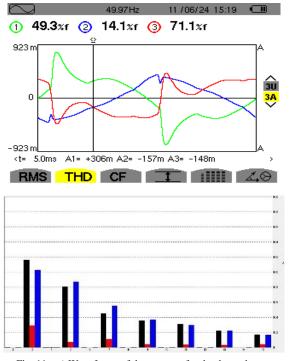
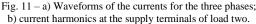


Fig. 10 - a) Waveforms of the currents for the three phases; b) current harmonics at the supply terminals of load one.





The waveform for the current absorbed by the loads on the first supply phase of the two consumers is the most distorted. The nonlinear elements mounted in the circuit negatively affect the power circulation behavior.

Distorted consumer one transfers non-symmetry active power and residual active power to the circuit, while linear consumer two transfers non-symmetry active power but receives residual active power, as summarized in Table 2.

From the power circulation diagram of the second model, distorting consumer one transfers non-symmetry and residual power, while linear consumer two receives residual power. That proves the possibility of using a device to record the residual active power for fair billing. As nonlinear elements are introduced into the circuit, the power circulation between the system's components becomes increasingly complex, but keeping track of them through appropriate metering and billing may result in an incentive to less pollute (inject) non-symmetry and residual powers into the grid.

 Table 2

 Symmetry, non-symmetry, residual, and total active and reactive powers corresponding to circuit model two: power balance verification

prresponding to circuit model two; power balance verification							
	$P_{\rm s}$ [W]	$P_{\rm n}$ [W]	$P_{\rm r}$ [W]	P _{total} [W]			
Power line	81.50	13.18	3.69	98.38			
Con. 1	64.66	-10.5	-10.12	44.02			
Con. 2	75.84	-2.35	6.18	79.68			
S [W]	222	0.32	-0.24	222.09			
3 ~ source	221.62						
Error [%]	-0.2	2.48	2.43				
	$Q_{\rm s}$ [var]	Q_n [var]	$Q_{\rm r}$ [var]	$Q_{\text{total}}[\text{var}]$			
Power line	-47.8	7.38	-1.24	-41.67			
Con. 1	-86.76	-14.69	-1.18	-102.64			
Con. 2	83.24	7.47	2.42	93.14			
Sum[var]	-51.32	0.15	-0.005	-51.1			
$3 \sim \text{source}$	-49.93						
Error [%]	-2.48	-1.07	-0.22				

In the case of reactive power, the experimental circuit shows that distorting consumer one transfers non-symmetry reactive and residual reactive power to the system, whereas linear consumer two receives both non-symmetry and residual reactive power, as qualitatively illustrated in Fig. 12.

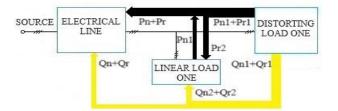


Fig. 12 – Active and reactive powers' flow (MODEL TWO).

The data in the tables is obtained by applying the Fast Fourier Transform (FFT) using Matlab, enabling us to analyze the waveforms and the waveforms and the power flow both through graphical representations and detailed tables for better clarity and interpretation.

4. CONCLUSIONS

Using conservative power quantities proves necessary and sufficient to analyze the effects of distorting and unbalanced three-phase loads in electrical distribution systems. These quantities allow for detailed tracking of power/energy circulation within the system, thus facilitating the performance assessment and optimization of the electrical network.

Introducing nonlinear elements into the circuit significantly complicates the power circulation between system components. This complexity necessitates advanced analytical approaches to fully understand and optimize power management strategies in the presence of nonlinear loads.

The two proposed test models implement the Ţugulea power theory as the appropriate framework for accomplishing the abovementioned requirement – using conservative powers accounting for both imbalance and distortion existing throughout the grid. The analysis of the two proposed models undoubtedly experimentally demonstrates the validity of the theoretical aspects most present in the literature.

Thus, firstly, we showed that the sources of non-symmetry power associated with unbalanced load equipment connected to the grid contribute to significant power flow within the system. This asymmetry in power circulation is generally associated with additional losses in the system, which can negatively affect the efficiency and stability of the electrical network. The additional losses resulting from power imbalance can cause excessive equipment heating, reduced durability and reliability, and increased operating and maintenance costs. Therefore, a thorough understanding of these losses and their proper management are essential for maintaining optimal system performance and preventing potential failures.

Secondly, we prove that nonlinear elements present in three-phase circuits inject additional electrical power into the network or absorb unnecessary electrical power, thereby overloading the system. Nonlinear elements installed in the experimental circuits produced both residual active and reactive power and active and reactive power imbalance. The second proposed model is an experimental model with multiple nonlinear elements in a single three-phase consumer, and the waveforms are much more distorted than those of the first proposed model. In that case, we highlight the adverse effects produced to the balanced and linear consumers (including here the power line) by injecting unwanted active and reactive powers components – the nonsymmetry and residual powers. The experimental results confirm the potential for utilizing a device to record residual active power, as suggested by the power circulation diagrams shown for the two circuits. This capability is crucial for accurately assessing and managing power distribution in systems with significant nonlinear and unbalanced phases consumers.

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CREDIT AUTHORSHIP CONTRIBUTION STATEMENT

Ionuț-Marius Mîndreanu: formal analysis, literature investigation, data acquisition, data organization, writing – original manuscript preparation.

Radu-Mircea Ciuceanu: measurement methodology, experimental platform design, measurement supervision, data consistency analysis, and verification.

Iosif Vasile Nemoianu: conceptualization, software development, writing – manuscript review and editing, funding, and procurement.

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