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TURBULENT MULTI-BRANCHING RADIAL SYMMETRIC FLOW STRUCTURES

MIGUEL R. CLEMENTE

University of Coimbra, ADAI, Department of Mechanical Engineering
Correspondence: miguel.clemente@dem.uc.pt

This study builds upon previous research that developed optimized tree-shaped flow networks within disc-shaped bodies for enhanced thermal management, extending these principles to explore turbulent flow regimes. Previously, a multi-branching approach combined with strategic hydraulic diameter settings was utilized to balance thermal and fluid performance in laminar flow conditions, successfully minimizing flow resistance. These efforts revealed that symmetric tree designs could significantly boost hydraulic and thermal efficiencies via a constrained minimization approach. In the current work, the focus shifts to examining the implications of transitioning these optimized designs from laminar to turbulent flow conditions. By comparing performance metrics between the two regimes, this study aims to elucidate the effects of flow regime transitions on network efficiency and effectiveness. The results enhance understanding of the fundamental dynamics governing flow networks and guide the development of more robust designs capable of operating across a broader range of flow conditions.

Keywords: Constructal Law; Radial branching; Bifurcation; Flow structure; Turbulent flow.

1. INTRODUCTION

Recent advancements in thermal management systems, particularly those incorporating disc-shaped bodies, have demonstrated the instrumental role of flow networks designed based on constructal theory. Previous investigations delineated an optimized framework for tree-shaped flow networks, predominantly focusing on laminar flow regimes. Expanding upon these foundational results, the current study delves into the dynamics of turbulent flow within these optimized tree-shaped networks. Turbulent flow, known for its enhanced mixing and heat transfer capabilities, is pivotal in scenarios where thermal loads surpass the handling capacity of laminar flows. The research aims to critically assess the adaptability of previously optimized geometric configurations under turbulent conditions, exploring the impact on fluid distribution and system efficacy. By comparing results from the laminar regime with the turbulent regime, a detailed examination of the interplay between flow regime transitions and network performance is provided.

2. METHODS

This study continues the exploration of tree-shaped flow networks, particularly extending the analysis to turbulent flow regimes by employing the foundational equations that defined the optimized network geometries in previous research. The objective is to evaluate how these geometric configurations perform under turbulent conditions. Following the Multi-Branching Generalization method found in Clemente and Pañao [1], and adapting it to turbulent regime conditions, the core geometric and flow equations are presented in Eqs. 1, providing the framework for this investigation.

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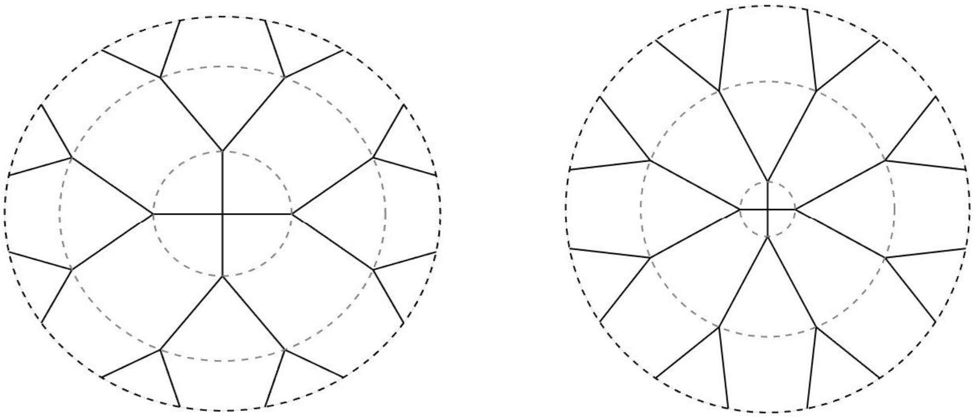
$$d_i = d_0 2^{-11i/27}, \quad (1a)$$

$$R_{G_t} = 2^6 n_0^{-\frac{7}{4}} \pi^{-\frac{7}{4}} d_0^{-\frac{19}{4}} g, \quad (1b)$$

$$g = \sum_{i=0}^{p} 2^{5i/27} L_i. \quad (1c)$$

Equations 1 correspond to Blasius flow, valid for a Reynolds number in the range of $4 \times 10^3 \leq \text{Re} \leq 10^5$ [2,3], with the equations valid for scenarios involving smooth circular channels, in a dichotomous branching structure where $b = 2$. Equation (1a) presents the diameter of a channel d, at level i of the structure, as a function of the diameter of the first channel ($i = 0$). Equation (1b) defines the total flow geometric resistance, where n_0 is the number of initial channels, and g (Eq. 1c) is the objective function to minimize, with L_i representing the length of the channel. The volume of the structure is the same for all structures, maintained constant at 0.1 m^3.

The subsequent section provides a comparative analysis of the results obtained from turbulent versus laminar flow regimes. This comparison aims to highlight significant differences and insights into the performance and efficiency of tree-shaped flow networks under varied flow conditions.



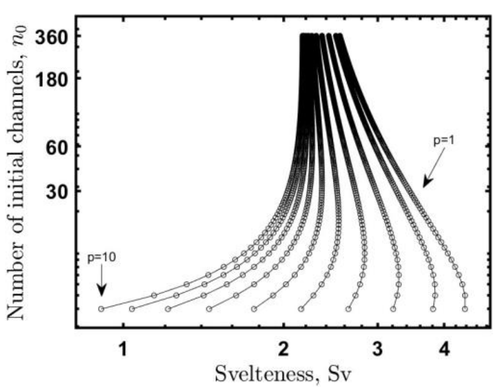
(a) Laminar regime - Hagen-Poiseuille flow. (b) Turbulent regime - Blasius flow.

Fig. 1 – Laminar and turbulent radial symmetric flow structures with $b = 2$, $p = 2$, and $n_0 = 4$.

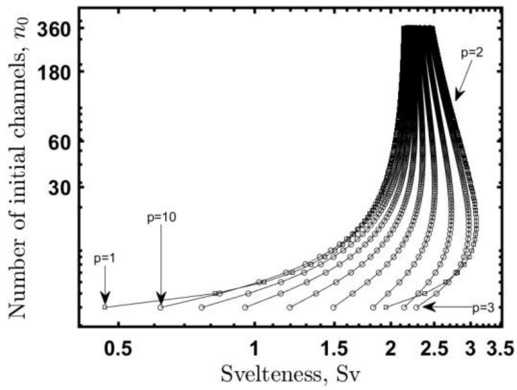
3. **RESULTS AND DISCUSSION**

A significant distinction between turbulent and laminar flow structures is observed in the configuration of initial channels. Specifically, turbulent flow architectures do not incorporate three initial channels ($n_0 = 3$) due to the resultant negative length in the first channel. To facilitate an effective comparison, laminar structures configured with $n_0 = 3$ have been excluded from this analysis.

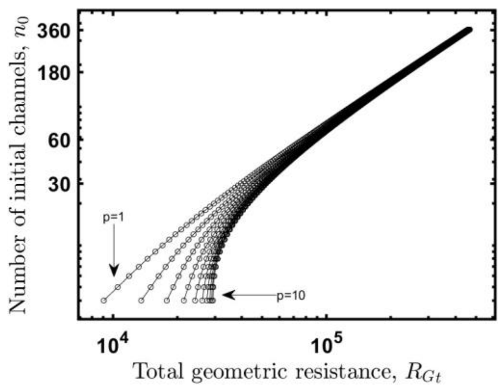
Turbulent structures exhibit lower Svelteness values than their laminar counterparts, yet they demonstrate marginally reduced geometric resistance. Figure 3 illustrates the evolution of Svelteness (Sv) and total geometric resistance (RGt), presenting results specifically at the lower boundary of the Blasius flow, where $\text{Re} = 4 \times 10^3$. In the laminar regime, the evolution of Svelteness is consistent, whereas in the turbulent regime, it stabilizes starting from $p = 3$. This stabilization in the turbulent regime is attributed to much shorter initial channels possessing larger diameters. Like the laminar regime, an increase in p leads to a general decrease in Sv, indicating that deeper branching diminishes Svelteness. However, the differences in geometric resistance are minimal, with the turbulent regime exhibiting slightly more favorable outcomes.



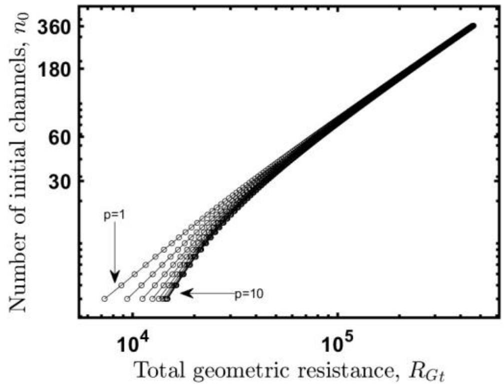
(a) Svelteness in Hagen-Poiseuille flow.



(b) Svelteness in Blasius flow.



(c) Total geometric resistance in Hagen-Poiseuille flow.



(d) Total geometric resistance in Blasius flow.

Fig. 3 – Evolution of Svelteness and total geometric resistance with branching level p, and several initial channels n_0 for laminar and turbulent regimes.

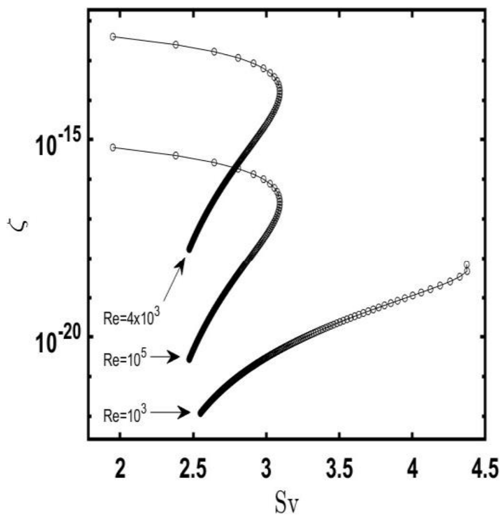


Fig. 4 – Evolution of Svelteness and performance with the Reynolds number, for configurations with $b = 2$ and $p = 2$.

Figure 4 illustrates the evolution of Svelteness (Sv) and its impact on thermal fluid performance (ζ) across different Reynolds numbers. It presents a compelling visualization of how geometric optimizations translate into performance enhancements in fluid flow systems. The figure showcases three distinct set of architectures by varying the Reynolds numbers, Re = 10^3, Re = 4 ×10^3, and Re = 10^5. Each set displays a unique trajectory of performance as a function of Sv. In the laminar regime, performance increases proportionally with Sv in the direction of less initial channels. This behavior contrasts markedly with the turbulent regime, where thermal fluid performance starts at a higher baseline and maintains a gradual ascent until a maximum Sv, indicating a sustained benefit from increased Svelteness even at higher values. This discrepancy underscores the influence of flow characteristics – dictated by Reynolds numbers – on the effectiveness of geometric enhancements.

Transitioning from the laminar regime to the turbulent regime leads to notable performance improvements. However, there’s a clear disadvantage in further increasing turbulence. In the turbulent regime, increasing the number of initial channels initially leads to a more prominent effect in increasing Sv, while performance decreases slightly until it reaches a plateau. This behavior is indicative of a moderate flow regime where geometric optimizations are effective but have limited benefits.

These insights are critical for the design of flow networks, emphasizing the need to tailor geometric configurations based on anticipated operational flow conditions to optimize thermal fluid performance effectively.

4. **CONCLUSION**

This investigation into the evolution of Svelteness and its impact on performance in tree-shaped flow networks across different regimes provides crucial insights into the interplay between flow characteristics and network geometry. These findings underscore the critical role of geometric design in enhancing flow efficiency and can significantly influence the development of more efficient thermal management systems. By emphasizing the need to consider the flow regime during the design phase, this study highlights how tailored design strategies can optimize performance.

Furthermore, the detailed comparison across various Reynolds numbers clearly shows that although increasing Svelteness generally boosts performance, the extent and nature of these improvements are profoundly influenced by the specific fluid dynamics of each flow regime. This nuanced understanding allows for the design of flow networks that are not only optimized for efficiency but also customized to the operational conditions expected in practical applications.

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