CONSTRUCTAL LAW CONFERENCE – CLC2024
Design in Nature and Evolution
Romanian Academy, Bucharest, 10–11 October 2024

Proceedings of CLC2024
pp. 109–112

ROBOTIC LEGS DESIGN – CONSTRUCTAL CONSIDERATIONS

JUAN ORDONEZ[a], CAMILO ORDONEZ[b]

[a] Department of Mechanical Engineering, FAMU-FSU College of Engineering, Energy and Sustainability Center, Center for Advanced Power Systems, Florida State University, Tallahassee, FL, 32310
[b] Department of Mechanical Engineering, FAMU-FSU College of Engineering, Energy and Sustainability Center, Center Intelligent Systems, Control, and Robotics, Florida State University, Tallahassee, FL, 32310
Correspondence: jordonez@fsu.edu, cordonez@eng.famu.fsu.edu

In robotic navigation, wheels are highly efficient for engineered surfaces. However, they need to be more efficient when navigating over rough terrains. Evolution has resulted in limbed creatures that are highly adapted to extreme terrains. This paper explores the design of robotic legs for sagittal motion in uneven terrains. The paper builds upon locomotion theories to identify geometrical features of the leg and groups of legs as a function of terrain features. The predictions of the analytical models are compared against observations in the animal kingdom. The study is focused on legs that can be represented with two degrees of freedom, connected via revolute joints. It uses simplified terrain features and reduced-order dynamic models to evaluate the cost of transport associated with the different legs and terrain configurations. Examples of animal and animal-inspired robots to which the model applies include crabs, horses, and some insects. The model is expected to help design more agile robots and be extensible to designing human prostheses and end effectors for robotic manipulators.

Keywords: Robotic locomotion; Animal design; Locomotion.

1. INTRODUCTION

The evolution of animal locomotion is a primary example of the constructal law in action. Locomotion modalities have evolved to support more efficient ocean, air, and land motion. Several features of the locomotion strategies and the geometry of the structures that support that locomotion have evolved to facilitate access to the flows of animal mass on the planet. This paper builds upon the constructal theory of animal locomotion [1] to explore features of legs that can guide the designs of mobile robots.

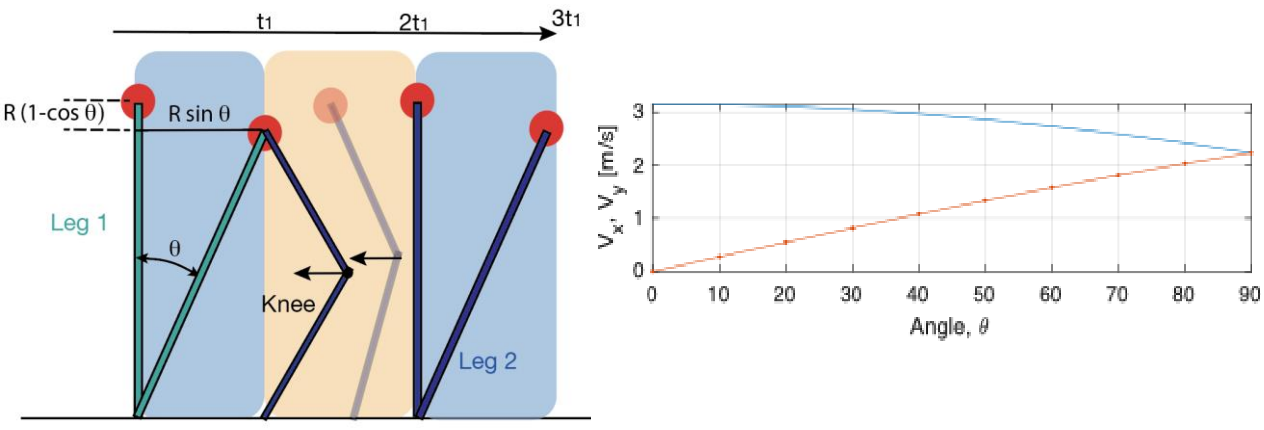
2. MATERIALS AND METHODS

In [1], Bejan shows how, by treating land animal locomotion as a rolling body, it is possible to predict inclination angles that favor forward motion, the benefit of two legs, and time scales for the swinging between them. The average forward speed, V_x, during the time interval t_1 while the body free falls in a rotational motion to the angle, θ, (see Fig. 1) is given by

$$V_x = \frac{R\sin(\theta)}{\left[2\left(\frac{R}{g}\right)(1-\cos(\theta))\right]^{1/2}}\,;\, V_y = \frac{R\,(1-\cos(\theta))}{\left[2\left(\frac{R}{g}\right)(1-\cos(\theta))\right]^{1/2}}, \quad (1)$$

where the duration of the time interval of free fall is the denominator in eq. (1). The variation of V_x and V_y with the angle of rotational motion is shown in Fig. 1.

DOI: https://doi.org/10.59277/CLC/2024.28



(a) (b)

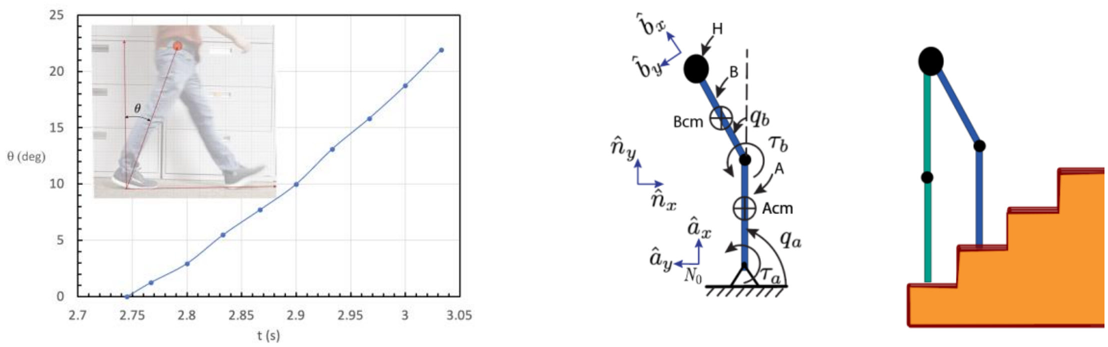
Fig. 1 (a) Up to t_1, leg 1 supports rotational 'free-fall' motion, at that point leg 2 absorbs impact and provides the torque necessary to lift the body again. (b) Variation of vertical and horizontal speeds in the free-fall rotational model of locomotion. The horizontal speed is maintained near the maximum if $\theta \lesssim 30°$.

3. **EXPERIMENTAL RESULTS:**

A motion tracking system was used to monitor the angles during the sagittal walking stance of an individual with and without obstacles. Figure 2a illustrates the tracking of the angle under normal walking speed with no obstacles. It can be observed that the rotational angle 22º is close to that predicted by constructal theory (≲30º) [1]. Figure 2b shows schematically key variables involved in two-link multibody dynamic systems used to study stair climbing.

4. **CLIMBING UPSTAIRS:**

The model assumes that the stance leg is composed of two massive thin rods A, B with masses m_A and m_B. The remaining body mass is modelled as a particle H with mass m_H, and situated at the hip. Actuation is provided via ankle (τ_a) and knee (τ_b) torques. Additionally, we assume that each joint has linear viscous damping characterized by a damping coefficient b. The dynamic model is developed using a Newton/Euler formulation.



(a) (b)

Fig. 2 – (a) Measurement of rotational motion angle limit at which trigger to second leg occurs under normal walking stance. (b) Model schematic – stairs climbing.

Let the state of the system be $x = [q_a\ \dot{q}_a\ q_b\ \dot{q}_b]$, and its equations of motion in state space form, $\dot{x} = f(x,u)$. The system control input is $u = [\tau_a\ \tau_b]$. To determine the effort required to climb one step, we employ trajectory optimization:

$$\min_{x,u,T} \int_0^T \tau_a^2(t) + \tau_b^2(t)\,)dt,$$

$$s.t.\quad \dot{x} = f(x,u);\quad x(0) = x_0;\ x(T) = x_f,$$

$$q_{a,min} \leq q_a \leq q_{a,max};\ q_{b,min} \leq q_b \leq q_{b,max},$$

where the initial and final states are $x_0 = [q_{a,0}\ 0\ q_{b,0}\ 0]$ and $x_f = [\frac{\pi}{2}\ 0\ 0\ 0]$.

We formulate a nonlinear program (NLP) via direct collocation [2] using smooth and exact derivatives of the objective function and all constraints using the MATLAB COALESCE framework [3] and generate a solution using the large-scale NLP solver, IPOPT [4]. Figure 3 shows trajectory optimization results to climb one step of height of 12 cm. The ankle and knee torque magnitudes are consistent with torque values estimated via motion capture in [5]. Assuming that the overall objective is to climb upstairs for a total height of 3 m and that every step results in a horizontal displacement of 0.3 m, it is then possible to estimate the total effort to climb up as a function of step height. Table 1 presents the results and indicates that for this cost function and the model being used, the optimal step height is 12 cm.

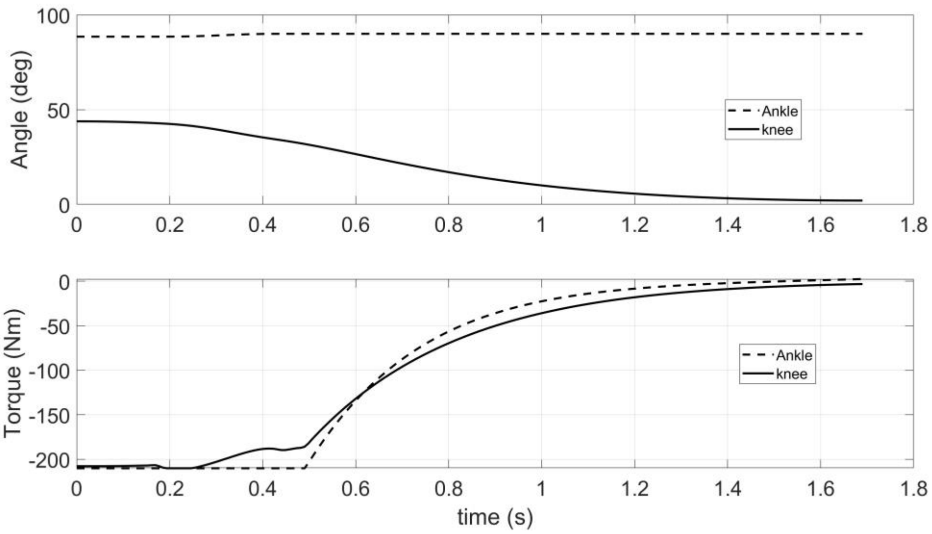


Fig. 3 – Angles and torque profiles to minimize climb effort (step height 12 cm).

Table 1

Effort required for climbing upstairs. The calculation assumes that with each step, the horizontal displacement is 30 cm

Step height (cm)	6	8	10	12	14	16
Effort per step ($\times 10^4$ Nm^2 s)	4.04	4.51	4.91	5.24	6.65	14.47
Number of steps needed to climb 3 m	50	37.5	30	25	21.42	18.75
Effort to climb 3m ($\times 10^4$ Nm^2 s)	202	169.12	147.3	131	142.44	271.31

5. **DISCUSSION AND CONCLUSIONS**

For walking on horizontal surfaces, the experimental value for the rotational angle 22^o (Fig. 2 (a)) is close to that predicted by constructal theory (~30^o) [1]. For obstacle climbing, the dynamic model optimization results suggest that the optimal step height for human-scale legs is close to 12 cm, which is reflected in modern staircases. This model and optimization approach can be used to size robot legs for robot navigation when there is information on the typical obstacle height.

REFERENCES

1. Bejan A., The constructal-law origin of the wheel, size, and skeleton in animal design, *American Journal of Physics*, **78**, *7*, pp. 692–699 (2010).
2. Kelly M., An introduction to trajectory optimization: How to do your own direct collocation, *SIAM Review*, **59**, *4*, pp. 849–904 (2017).
3. Jones M., Optimal control of an underactuated bipedal robot, Master Thesis, Oregon State University, 2014.
4. Wachter A., Biegler L., On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming, *Math. Prog*., **106**, *1*, pp. 25–57 (2006).
5. Serbest K. Cilli M., Eldogan O., Biomechanical effects of daily physical activities on the lower limb, *Acta Orthopaedica et Traumatologica Turcica*, **49**, *1*, 85–90 (2015).