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INFORMATIONAL PERSPECTIVE ON RAMIFICATIONS IN CONSTRUCTAL DESIGN

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What insights can informational analysis provide regarding the ramifications predicted by constructal design? The first step in answering this research question is to consider symmetric and asymmetric flow architectures. In the symmetric case, radial multi-branching shows diminishing returns of diversity (measured by the system's *informature*) in line with high thermofluid performance. In asymmetric flow architectures, assuming an evolution toward maximum complexity (as defined in information theory), patterns emerge like sap distribution in leaves.

Keywords: Diversity; Constructal design; Information theory; *Informature*; Complexity.

1. INTRODUCTION

In Bejan [1], the optimal solutions were converted to references. This conceptual change not only avoids the mistake of considering constructal design as an alternative optimization tool, but also emphasizes the freedom to change and evolve as a result of considering as much as 1% imperfection in the design goal of a physical system. Flow architectures that are free to change lead to diverse possible solutions, introducing a non-deterministic element in their design. In fact, Gosselin and Bejan [2] showed that even asymmetries in radial tree architectures can facilitate access to liquid flows, increasing the number of possible configurations and implying a higher diversity and complexity of flow architectures. This study sought insights retrieved by quantifying the information contained in ramified flow structures.

2. BASIC ELEMENTS OF INFORMATIONAL ANALYSIS

In 1948, Shannon [3] developed a formulation that quantifies the amount of information in any system containing several possible solutions. Although John von Neumann induced Shannon to name his formulation "entropy" due to its similarity to statistical mechanics [4], Denbigh [5] remarked it as a disservice to science because functions with the same formal structure do not necessarily represent the same. Instead, the Shannon's formulation in Eq. (1) is indeed a "measure of information" because it refers to the entire probability distribution, although some interpret it as average uncertainty or *indeterminacy*.

$$H = -K \sum_i p_i \log_2(p_i). \quad (1)$$

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To simplify and clarify the language of informational analysis, I introduce two new words: *informature* and *infotropy*.

Informature $H_{I,b}$ is only part of Shannon's formulation, which measures the amount of information obtained from the knowledge of the probability distribution:

$$H_{I,b} = -K_b \sum_i p_i \log_2(p_i), \quad (1)$$

where K_b is a constant that can change the logarithmic base and define the informature units. For example, $K_b = 1$ leads to $H_{I,2}$ measured in *bits*, whereas $K_b = \ln(2)$ leads to $H_{I,e}$ measured in *nats*. *Infotropy* is a neologism that synthesizes information (*info*-) and transformation (from the Greek *trope*) to signify an informature "contextualized" by a K_c scale parameter, as suggested by Tribus and McIrvine [4], corresponding to Shannon's formulation. Thus, $H_b = K_c \times H_{I,b}$ is an *infotropy*, *i.e.*, a "contextualized" *informature*. For example, in a gas where there are as many possible arrangements as W molecules, the probability of each configuration is $p_i = 1/W$, and if one defines $K_c = k_B$ with k_B as Boltzmann's constant and $K_b = \ln(2)$, the resulting *infotropy* is the well-known thermodynamic entropy at the microscopic level of reality $H_e = S = k_B \ln(W)$. If one considers the relative information, also known as the Kullback-Leibler information distance or information gain that measures the distance between the maximum informature, $H_{max,b} = K_b \cdot \log_2(N_k)$, where all possible configurations (N_k) are equally probable, and the actual informature, the result for the relative information becomes $D = H_{max,b} - H_{I,b}$. According to Feldman and Crutchfield [6], one can quantify the complexity of a finite-size system as the product of its informature and the relative information as $C = H_{I,b} \times D$, where is zero for systems without information ($H_{I,b} = 0$) or fully *indeterminate* systems (D = 0). For example, the dimensionless complexity $C_n = \frac{C}{H_{max,b}^2} = H_n(1 - H_n)$ with $H_n = H_{I,2}/H_{max,2}$ allows the comparison of different flow structures. In this study, an informational analysis of the informature of multi-branching radial symmetric and asymmetric flow structures provides insights into the evolutionary direction.

3. RESULTS

For ramifications with $b_p = 2$ and $b_p = 3$ branches, each branching level p implies the emergence of $n(p) = n_0 \cdot {b_p}^p$ channels in the flow structure. Therefore, the frequency of channels at each level p is $f(p) = n(p)/\sum_{i=0}^{p} n(i)$ with $\sum_{i=0}^{p} f(i) = 1$. The maximum informature in this simple reasoning method depends on the highest branching level: $H_{max,2} = \log_2(max\{p\})$. However, if one channel branches into several channels at the exit (n_{out}), then the flow rates at the exit $\dot{m}_i$ relative to the total flow $\dot{m}$ at the inlet provide proportions appropriate for informational analysis, $\sum_{i=1}^{n_{out}} (\dot{m}_i/\dot{m}) = 1$.

Figure 1(a) shows that 1) the informature develops toward diminishing returns with unfolding branching levels, and 2) lower informature values above bifurcations ($b_p > 2$) produce configurations with lower diversity levels. Suppose one measures the informature based on the flow rates at the exit channels ($\dot{m}_{e,i} = f_i \dot{m}$) and selects the flow structures that maximize dimensionless complexity. In that case, the results depicted in Fig. 1(b) show the emergence of a consistent pattern, such as sap distribution in leaves.

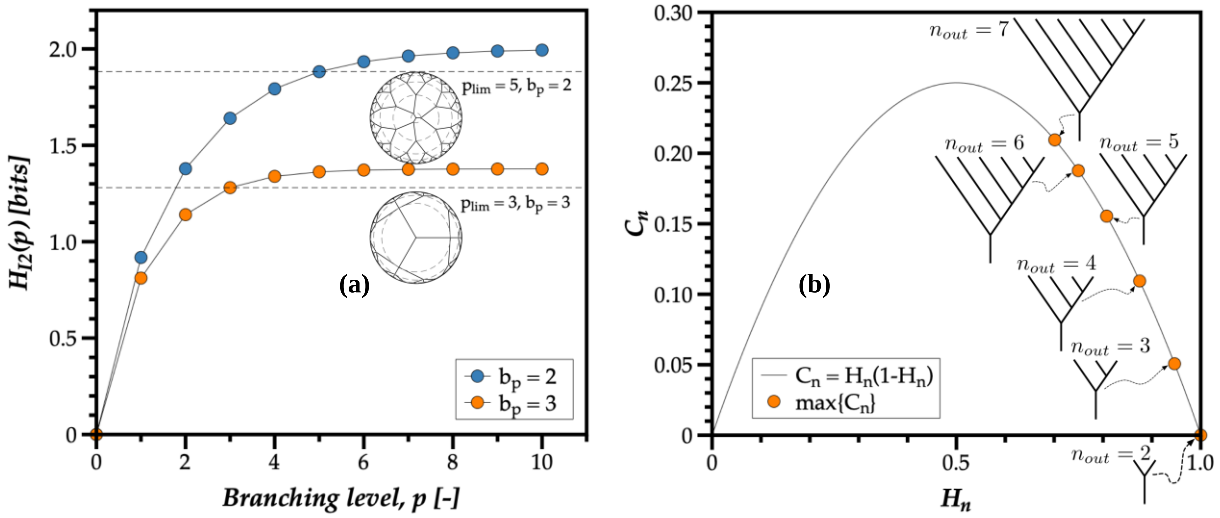

Fig. 1 – (a) *Informature* sensitivity with branching level *p*, (b) ramifications that maximize dimensionless complexity versus normalized *informature*.

4. DISCUSSION AND CONCLUSIONS

In Clemente and Panão [7, 8], the flow architectures with the highest performance considered a minimum number of $n_0 = 3$ initial channels and the miniaturization of vascularized multi-branching radial flow architectures pointed in the direction of *p* = 3 - 5 with bifurcations ($b_p = 2$). Without any consideration of the thermofluid performance, the informational insight based on the informature of the flow structure points in the same direction, considering diminishing returns. These results indicate that informature is directly proportional to the diversity of physical systems, including flow architectures, and that freedom-to-morph evolves toward higher complexity.

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