# MAXIMUM BENEFITS FROM THE USE OF *T*-SHAPED TREE FLOW GEOMETRY WITH RECTANGULAR SHAPE OF THE CHANNELS: PERFORMANCE EVALUATION

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The study defines the conditions and constraints that must be obeyed to obtain the maximum benefit from the use of a heat exchanger with a *T*-shaped tree flow configuration and rectangular shape of the channels. The flow is laminar, and fully developed with constant physical properties. The boundary condition is the fixed temperature of the channel wall. The variations of the heat flow ratio  $q_t^+$  and augmentation entropy generation number  $N_{sa}$  with the dimensionless mass flow rate M, shape factor  $\mathcal{X}$ , ratio B, and complexity n have been investigated. Comparisons of the thermal performance with other heat exchanger configurations like serpentine for two cases  $q_t^+ = 1$  and  $q_t^+ > 1$  have been made. The requirement for  $N_{sa} \leq 1$  has been implemented as a constraint instead of one of fixed pumping power. The performance evaluation criterion  $N_s^+$  has been introduced as a general criterion to define the most beneficial complexity and working parameters.

Keywords: T-shaped tree; Rectangular shape; Maximum benefits; Performance evaluation criteria.

#### 1. INTRODUCTION

Dendritic heat exchangers arose at the beginning of this century as a new direction for developing heat exchanger architectures [1]. They revealed a greater potential to reduce the pressure drop and improve the temperature distribution homogeneity and thermal efficiency than the traditional parallel channel network [2]. Since then, many experimental and numerical studies have been reported [3].

The results developed, and the optimization criteria applied reveal that the tree-shaped design heat exchangers are the following (fourth) generation heat exchangers using the natural heat transfer enhancement technique, pursuing the same objectives arising from the first and second laws of thermodynamics [4].

This paper demonstrates the usefulness of two criteria, like those used in [4]: augmentation entropy generation number  $N_{sa}$  and  $q_t^+$ . The objective of the first case is to evaluate the maximum reduction of the driving temperature difference,  $\Delta T_m^* < 1$ , whereas the objective of the second case is to assess the maximum increase of the heat duty,  $q_t^+ > 1$ .

### 2. ANALYSIS OF THE HEAT EXCHANGER CONFIGURATION

Figure 1 shows tree-shaped streams distributed over a rectangular area. The flow is laminar and fully developed with constant physical properties. The constraints are fixed heat transfer surface area of the channels  $A_w$  in the allocated area A. The boundary condition in the wall is  $T_w = const$ .

The lengths are obeyed the rule  $L_i = 2^{i/3} L_o$  and  $D_{h,i} = 2^{i/3} D_{h,o}$ . The total heat flow in dimensionless form is

$$\tilde{q}_{n} = \frac{\dot{q}_{n}}{k_{f} A^{1/2} T_{in}} = M \left( T^{*} - 1 \right) \left\{ \left[ 1 - \exp \left( -\chi N u \frac{2^{(3n-7)/6}}{M} \right) \right] + S_{1} \right\},$$
(1)

$$S_{1} = \sum_{i=1}^{n} \left\{ \exp\left(-\sum_{k=0}^{k=i-1} \chi N u \frac{2^{(3n-4k-7)/6}}{M}\right) \left[1 - \exp\left(-\chi N u \frac{2^{(3n-4i-7)/6}}{M}\right)\right] \right\}.$$
 (2)



Fig. 1 – The flow of tree-shaped streams is distributed over a rectangular area.

The overall entropy generation for a tree-shaped heat exchanger in dimensionless form is

$$\tilde{S}_{gen} = M \left(T^* - 1\right)^2 \left\{ \exp\left(-\chi N u \frac{2^{(3n-7)/6}}{M}\right) \left[1 - \exp\left(-\chi N u \frac{2^{(3n-7)/6}}{M}\right)\right] + S_2 \right\} + \chi^3 \times B \times Po \times 2^{(3n-17)/6} \times \left[2^{-(n+1)/3} - 1\right]^4 M^2$$
(3)

$$S_{2} = \sum_{i=1}^{n} \left\{ \begin{bmatrix} \exp\left(-\sum_{k=0}^{k=i-1} \chi N u \frac{2^{(3n-4k-7)/6}}{M}\right) \end{bmatrix}^{2} \exp\left(-\chi N u \frac{2^{(3n-4i-7)/6}}{M}\right) \times \\ \times \left[1 - \exp\left(-\chi N u \frac{2^{(3n-4i-7)/6}}{M}\right) \end{bmatrix}^{2} \right\},$$
(4)

$$B = \frac{A^{3} \nu k_{f}}{A_{w}^{4} T_{w} \rho c_{p}^{2} \left(2^{-1/3} - 1\right)^{4}}.$$
(5)

#### 3. RESULTS AND DISCUSSION

Figures 2 and 3 show the variation of the augmentation entropy generation number  $N_{sa} = \tilde{S}_{gen,n} / \tilde{S}_{gen,n=0}$ and the ratio of heat flows  $q_t^+ = \tilde{q}_{t,n} / q_{t,n=0}$  with the dimensionless mass flow rate M and the complex B for several levels n = 1, 2, 4 and  $T^* = 1.4$ . The results revealed several important characteristics: (i)  $N_{sa}$  strongly depends on shape factor  $\chi = p / D_h$  and complexity n. As seen from Fig. 2, for  $\chi = 6.25$  only for  $n \le 4$ ,  $N_{sa} \le 1$ , whereas for  $\chi = 10.125$ , only for  $n \le 2$   $N_{sa} \le 1$ .

The parts of the curves  $N_{sa}$  vs. M where  $N_{sa} > 1$  have been removed from Fig. 2. (ii) The variation of  $q_t^+$  with M shows two regions with different behavior of  $q_t^+$ , namely  $q_t^+ = 1$  for small values M < 4, and  $q_t^+ > 1$  for M > 4. In the range M < 4, the benefit is the decrease in the driving temperature difference defined by the value of  $N_{sa} < 1$ . For instance, if  $\chi = 6.25$ , the most significant benefit is obtained for M = 4 and n = 2, whereas if  $\chi = 10.125$ , the most significant benefit is obtained for M = 1.

In the second range M > 4, the objective is the maximum increase in the heat flow that can be achieved, not a minimum entropy generation. The maximum  $q_t^+$  is achieved for that value of M for which  $N_{sa} = 1$ . As seen from Fig. 3, the smaller  $\chi$ , the more significant the benefit is.



Fig. 2 – The variation of  $N_{sa}$  with M; (a)  $\chi = 6.25$ , (b)  $\chi = 10.125$ .

With this study we try to encourage the use of these two criteria for assessing the benefits that treeshaped heat exchangers can bring about in comparison with conventional heat exchangers with different structures like serpentine or banks of parallel tubes.



Fig. 3 – The variation of  $q_t^+$  with *M*; (a)  $\chi = 6.25$ , (b)  $\chi = 10.125$ .

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