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OPTIMIZING FORMATION FLIGHT *VIA* THE CONSTRUCTAL LAW

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This paper explores 2 cases of flyers in formation: birds and aircraft, both seeking to save energy by flying in the wake of preceding flyers. The V-formation design occurs naturally with birds and is used in practice as a fundamental aircraft formation – aiming to boost lift/reduce induced drag felt by the flyers. The extent of the energy savings depends on the design configuration of the formation. The Constructal Law is applied to this formation flight problem to analyze how the distribution of drag among flyers in a V-formation is related to the optimal formation flight configuration. An analytical model that predicts this optimal configuration is developed and expressed regarding the fundamental formation parameters (velocity, number of aircraft, wingspan, weight, and air density).

Model results show that there is not a fixed optimal V-formation design for all formation systems. Instead, the optimal configuration adapts as parameters change. The optimal configuration becomes narrower as the flyers' speed and spacing increase. As the number of flyers increases, the optimal configuration approaches a finite maximum angle. Trends predicted by the model are substantiated by observations from nature, including those of birds, boats, and other wake-generating flow systems. In line with predictions of the Constructal Law, induced drag along the optimal V-formation is distributed as uniformly as possible among flyers (depending on the longitudinal spacing). The model suggests the formation flight system constantly evolves towards a state that maximizes collective access to energy savings by changing the formation design accordingly.

Keywords: Formation flight; Birds; Fighter jets; Aircraft; Constructal Law.

1. INTRODUCTION

Flyers rarely fly alone. Fighter jets have a "wingman" to assist in their mission and provide aerodynamic benefits. Birds have a natural advantage: they can "feel" what is most accessible as they fly. Thus, they will naturally fly in the optimal configuration (the configuration that uses the least energy). Birds naturally fly in a V-formation (figure 1a), and fighter jets use a nearly identical formation in practice called "fingertip" (Figure 1b) [1,2].

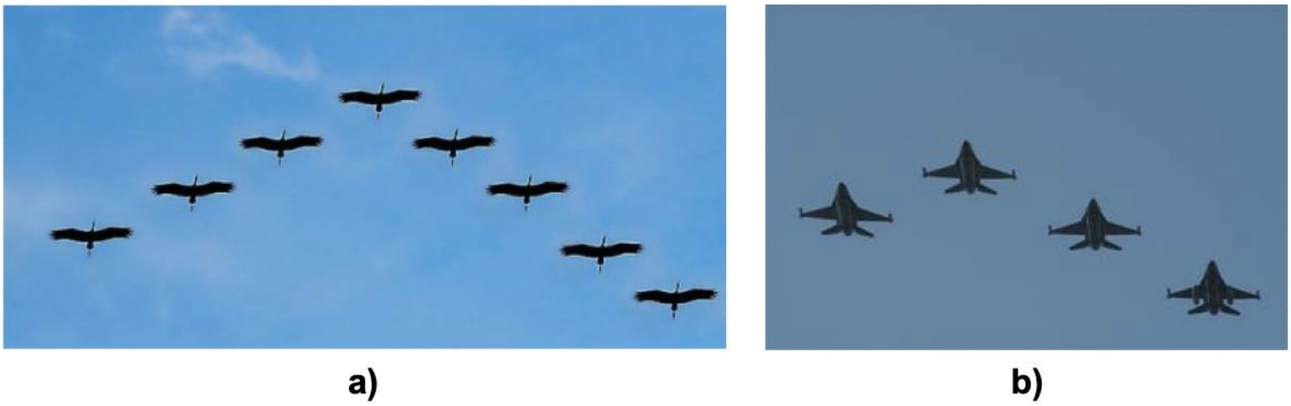

Fig. 1 – a) Flock of geese flying in a V-formation, credit: Birding World [3];
b) fighter jet aircraft flying in a V-formation, credit: 10th AF Public Site [4]

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The V-formation employs a concept known as "vortex surfing," trailing flyers benefit from the updraft of wingtip vortices trailing from the preceding flyers – boosting lift, reducing drag, and making flight easier [5,6]. Wieselsberger first suggested that the ideal V-formation among birds would provide an equal distribution of drag to maximize the collective energy savings of the flock [7]. For a fixed-wing flyer to maximize these benefits, it must be positioned at the ideal angle behind the leader. This paper models the V-formation design, defined by the V-angle α as shown in Fig. 2, to maximize energy savings and provide insight into predicting the natural optimal design for formation flyers.

This paper applies fundamental physics principles to this question, specifically the Constructal Law, which states that a flow system persists when it evolves to provide easier flow access to its currents [8]. The Constructal Law suits the formation flight problem well, as its application can lead to the prediction of nature and can be employed to predict the V-formation design at which birds fly. In this case, the formation flight configuration is a flow system of aerodynamic lift in which the induced drag is imperfection. The Constructal Law predicts that the optimal design is the configuration for which the imperfection is most evenly distributed across the system or the induced drag is equally partitioned among flyers in formation.

2. **THE OPTIMAL CONFIGURATION**

2.1. **Model Development**

The "optimal configuration" or "design" for formation flight maximizes energy savings by maximizing the boost in lift and corresponding reduction of induced drag. To determine the relationship between the formation flight configuration and the resulting energy savings, a theoretical model is developed for a flat-plane system of N fixed-wing flyers (in straight and level flight) with individual wingspan b, individual weight W, air density ρ, and formation velocity V_∞ in a V-formation configuration. The V-formation is parameterized with fixed lateral and longitudinal spacing coordinates x and y, respectively, as shown in Fig. 2 below.

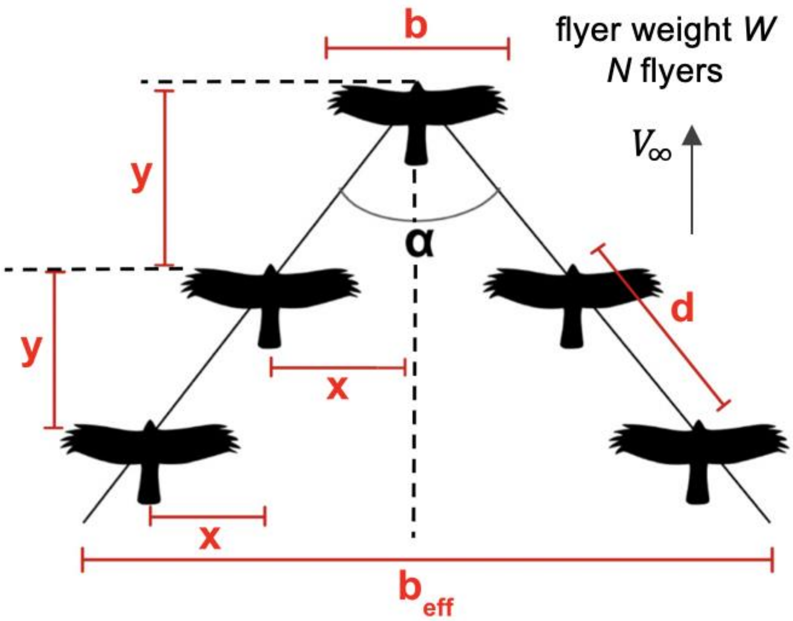

Fig. 2 – Fundamental V-formation design parameters.

Optimizing this design for maximal energy savings (minimal imperfection) yields the following model:

$$y_{opt} = \frac{2\pi\rho V_\infty^2}{NW}\left[(N-1)x_{opt}^3 + bx_{opt}^2\right]. \quad (1)$$

Since the relationship is nonlinear, no constant configuration optimizes the V-formation. Instead, the optimal configuration adapts to the variation of the formation system parameters, and the optimal flyers will evolve their formation to reflect this. To employ the model in equation 1 to find the optimal V-formation angle αopt, each (x_{opt}, y_{opt}) coordinate corresponds to a scenario-dependent nose-to-nose distance *d*, held constant in the V-formation. Given *d*, the corresponding (x_{opt}, y_{opt}) is determined from the model in equation 1, and then α_{opt} can be calculated geometrically.

2.2. **Analysis of the Optimal Design**

Analyzing the model in terms of the fundamental formation parameters provides greater insight into the model's accuracy and predictive applications to the real world. To determine trends between each parameter and the resulting optimal formation flight design, parameters *d*, V_∞, and *N* are computationally varied for an arbitrary aircraft and bird formation flight system. Beginning with nose-to-nose separation distance, the optimal V-formation becomes narrower as the flyers fly further apart. This trend aligns with experimental bird data from Williams, observing that Canada Geese fly narrower V-formations as the V-formation length increases [9]. This is likely due to an effort of the birds to distribute the induced drag as evenly as possible along the extended formation by remaining closer laterally, in line with the predictions of the Constructal Law.

Next, as V_∞ increases, the optimal configuration gets narrower. Therefore, the model suggests faster flyers fly narrower formations to maximize energy savings. Birds have also been observed to follow this trend, as bigger, faster birds tend to fly in narrower V-formations [9]. Boats display a similar phenomenon as their V-formation wakes become narrower with increasing speed.

Lastly, the optimal configuration model is evaluated for varying *N* flyers. This trend reveals an asymptotic relationship between optimal configuration and flyers. When there are few flyers ($N < 10$), the flyers should fly in a slightly wider formation as *N* increases to account for the broader divergence of the trailing wakes. However, for many flyers ($N > 10$), adding additional flyers has negligible impact on the wake geometry, leading to a constant optimal configuration angle.

3. **CONCLUSIONS**

To test the prediction of the Constructal Law of uniform distribution of drag (imperfection) in the optimal configuration, induced drag for each flyer in the formation is calculated. Results show that the induced drag force felt by the individual flyers varies very little (<6% body weight). Thus, in both cases, the flyers fly in the configuration that distributes the induced drag as evenly as possible among flyers – aligning with Constructal Law predictions. The effectiveness of this drag distribution is also primarily determined by the longitudinal spacing of flyers. However, in practice, the formation flight system can never achieve this perfect drag distribution because the induced drag consistently decreases along the V-formation, meaning the leading flyers will always work harder than the trailing flyers to maintain the V-formation. The Constructal Law also describes this inherent diversity in nature: No realistic optimal system can eliminate diversity and imperfection. Rather, its design will constantly evolve to provide better access for its currents, as does the formation flight system seeking access to flight.

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