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SOLUTIONS OF NEWTONIAN GRAVITATIONAL WAVES AND GRAVITATIONAL POYNTING VECTOR

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Solutions to equations producing Newtonian gravitational waves are being presented. The derivations of these equations emerging directly from the second Newton law and mass conservation applied to a continuous mass distribution, *e.g.*, a compressible fluid or equivalent, have already been shown to lead to a form identical to the Maxwell equations for electromagnetism subject to a specific condition. Consequently, Newtonian gravitational waves are Lorentz invariant when the speed of wave propagation equals the speed of light. The resulting equations can derive a gravitational Poynting vector in complete analogy to the electromagnetic Poynting vector. A stationary solution exists for a spherical mass, creating the gravitational field. The evolution of a gravitational wave emerges and is presented as a linear, neutrally stable solution around this stationary one.

Keywords: Gravitational waves; Continuous mass; Newtonian dynamics; Lorenz force; Poynting vector; Maxwell equations.

1. INTRODUCTION

Gravitational waves were shown by Vadasz [1] to derive directly from Newtonian dynamics for a continuous mass distribution, *e.g.* compressible fluids or equivalent. It was shown that the equations governing a continuous mass distribution, *i.e.* the inviscid Navier-Stokes equations for a general variable gravitational field $\mathbf{g}(t,\mathbf{x})$, are equivalent to a form identical to Maxwell equations from electromagnetism, subject to a specified condition. The consequence of this equivalence is the creation of gravity waves that propagate at finite speed.

The latter implies that Newtonian gravitation is not "spooky action at a distance" but is like electromagnetic waves propagating at finite speed despite the apparent form appearing in the integrated field formula. Since gravitational waves were so far derived only from Einstein's general relativity theory, it becomes appealing to check if there is a connection between the Newtonian waves presented in this paper and the general relativity type of waves, at least in a specific limit of overlapping validity, *i.e.*, as a flat-space approximation.

Two specific stationary solutions of the gravitational field and mass density distribution within the mass-creating field and the possible waves associated with these solutions are presented.

2. GOVERNING EQUATIONS FROM NAVIER-STOKES TO MAXWELL EQUATIONS

The equations governing the gravitational field effect and its dynamics for a distributed mass (compressible fluid or equivalent) are the inviscid and compressible Navier-Stokes equations

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{v})=0, \tag{1}$$

$$\frac{\partial\mathbf{v}}{\partial t}=-\nabla\left[v_o^2(\ln\rho)+\left(\frac{\mathbf{v}\cdot\mathbf{v}}{2}\right)\right]+\mathbf{v}\times(\nabla\times\mathbf{v})+\mathbf{g}, \tag{2}$$

where $\rho(t,\boldsymbol{x})$ is the mass density, $\mathbf{v}(t,\boldsymbol{x})$ is the velocity, $\mathbf{g}(t,\boldsymbol{x})$ is the gravitational field, t is time and $\boldsymbol{x}$ is the position vector. The term $\rho\mathbf{g}$ represents the intrinsic gravitational forces impressed between differential mass elements due to the distributed mass within the domain occupied by this mass. By using a linear approximation for the constitutive relationship between pressure and density in the form $P=P_o+v_o^2(\rho-\rho_o)$ where P_o and ρ_o are reference values of pressure and mass density, respectively, and $v_o=\sqrt{\frac{\partial P}{\partial\rho}}=\frac{1}{\sqrt{\rho_o\beta_P}}$ is the constant speed of propagation of the pressure wave (speed of sound as a special case, but affecting the wave propagation of the gravitational field g beyond the region containing the mass). These equations convert into a form identical

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to Maxwell equations subject to an extended Beltrami condition namely $\nabla \times (\mathbf{v} \times \boldsymbol{\xi}) = 0$ where $\boldsymbol{\xi}$ is the counter-vorticity vector. The solutions to the Newtonian gravitational field equations can be obtained from equations (1), and (2) in a form like Maxwell's equations presented in the form

$$\frac{1}{4\pi G}\nabla \cdot \mathbf{g} = -\rho; \frac{\partial g}{\partial t} = v_o^2 \nabla \times \boldsymbol{\xi} + 4\pi G\rho\mathbf{v}; \quad \frac{\partial \boldsymbol{\xi}}{\partial t} = -\nabla \times \mathbf{g}; \quad \nabla \cdot \boldsymbol{\xi} = 0. \tag{3}$$

3. STATIONERY AND WAVE SOLUTIONS

The solution inside the spherical domain contains the distributed mass assuming spherical symmetry, *i.e.*, $r \in [0, r_o]$ is due to the intrinsic gravitational forces impressed between differential mass elements due to the distributed mass. For this case there is a possible equilibrium mass density distribution and the corresponding equilibrium gravitational field distribution that can be obtained from (3) and (2) subject to $\mathbf{v} = 0$. Substituting $\mathbf{v} = 0$ yields

$$v_o^2 \nabla(\nabla \cdot \mathbf{g}) - \mathbf{g}(\nabla \cdot \mathbf{g}) = 0. \tag{4}$$

Assuming spherical symmetry, this equation becomes

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r^2}\frac{\mathrm{d}(r^2 g_r)}{\mathrm{d}r}\right] - \frac{g_r}{v_o^2 r^2}\frac{\mathrm{d}(r^2 g_r)}{\mathrm{d}r} = 0, \tag{5}$$

$$g_r = -\frac{2v_o^2}{r}; \ \rho = \frac{v_o^2}{2\pi G}\frac{1}{r^2}, \tag{6}$$

$$\mathbf{g} = -\frac{m_o G}{r_o r}\hat{\mathbf{e}}_r; \ \rho = \frac{m_o}{4\pi r_o}\frac{1}{r^2}. \tag{7}$$

where $\int_0^{\tilde{V}_o} \rho \, \mathrm{d}\tilde{V} = m_o$.

This equilibrium solution might or might not be stable. To investigate the stability of this equilibrium, one needs to consider the equations when $\mathbf{v} \neq 0$. Preliminary results from a linear stability analysis of this equilibrium indicate that the basic equilibrium solution might be neutrally stable with oscillations subject to some conditions, *i.e.*, $\mathbf{g} = \mathbf{g}_o + \boldsymbol{\varepsilon}\mathbf{g}^{(l)}(t, r, \varphi)$ and $\mathbf{v} = \mathbf{v}_o + \varepsilon \mathbf{v}^{(I)} = v_\varphi(t, r, \varphi)\hat{\mathbf{e}}_\varphi$ with $\mathbf{v}_o = 0$ because of the existence of the stationary solution.

The wave equations for waves propagating in the radial direction and having azimuthal polarization, *i.e.*, $\mathbf{g} = \mathbf{g}_o + \varepsilon \mathbf{g}^{(l)}(t, r, \varphi) = -\left(\frac{Gm_o}{r_o r}\right)\hat{\mathbf{e}}_r + \varepsilon g^{(l)}(t, r, \varphi)\hat{\mathbf{e}}_r$, $\mathbf{v}^{(t)} = v_\varphi(t, r, \varphi)\hat{\mathbf{e}}_\varphi$ based on the equation (3) have the form

$$\left[\frac{\partial^2}{\partial t^2} - \nabla^2 - \frac{2}{r^2}\right]\nabla\psi^{(1)} - \frac{2}{r}\hat{\boldsymbol{e}}_r\left[\frac{\partial^2}{\partial t^2} - \nabla^2\right]\psi^{(1)} = 0, \tag{8}$$

$$\left[\frac{\partial^2}{\partial t^2} - \nabla^2 - \frac{2}{r^2}\right]\mathbf{v}^{(1)} = 0, \tag{9}$$

$$\rho^{(1)} = \frac{1}{8\pi Ma_o^2}\left[\frac{\partial^2}{\partial t^2} - \nabla^2\right]\psi^{(1)}, \tag{10}$$

$$\mathbf{g}^{(1)} = \frac{\partial \boldsymbol{v}^{(1)}}{\partial t} + \nabla\psi^{(1)}, \tag{11}$$

where $Ma_o^2 = \frac{Gm_o}{2v_o^2 r_o}$ is the square of the gravitational Mach number and ψ is a potential. Waves solutions to these equations will be presented. In addition, a gravitational Poynting vector emerges and is defined in the form

$$\mathbf{S}_g = \frac{v_o^2}{4\pi G}\mathbf{g} \times \boldsymbol{\xi}, \tag{12}$$

representing the power due to the gravitational field per unit area.

REFERENCES

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