CONSTRUCTAL LAW CONFERENCE – CLC2024
Design in Nature and Evolution
Romanian Academy, Bucharest, 10–11 October 2024

Proceedings of CLC2024
pp. 25–28

COMPLETE FORMULAS DEVELOPED TO CALCULATE THE ENTROPY CHANGES OF IDEAL GASES

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The entropy change of a system in a process is a key criterion to distinguish if the process is actual or ideal. The process without the entropy change is a perfect process called an isentropic process. Typical cycles involving the isentropic processes are the Otto, Ranking, and Brayton cycles. Two kinds of thermodynamic systems in the cycles are commonly encountered in engineering practices: closed and open. The equations calculating the entropy changes of ideal gases in processes are derived from the first Tds relation, the Gibbs equation, and the second Tds relation. Generally, the first Tds relation is applied to the closed systems, and the second Tds relation is applied to the open systems. There are two approaches used to analyze the entropy changes of ideal gases. They are the approximate analysis and the exact analysis, respectively. Equations relating to the Gibbs equation for the closed system in the precise analysis needed to be included in thermodynamics. This paper presents the development of the missing equations. The author describes the derivation of the complete equations to calculate the entropy changes of ideal gases and the correlated properties in the isentropic processes from the Tds relations for approximate and exact analysis. Examples illustrate the calculation using the equations for the entropy changes of air and the correlated properties in the isentropic processes.

Keywords: Entropy; Entropy change; Isentropic process; Ideal gases.

1. INTRODUCTION

Entropy is a property of a thermodynamic system. S is denoted as a total entropy, and s as a specific entropy. The entropy change of the system Δs in a process is a key criterion to distinguish if the process is actual or ideal. The process without the entropy change is an ideal process called an isentropic process. Otherwise, the process is an actual process. In engineering practice, the Otto, Ranking cycle, and Brayton cycles involving isentropic processes are presented in Figure 1.

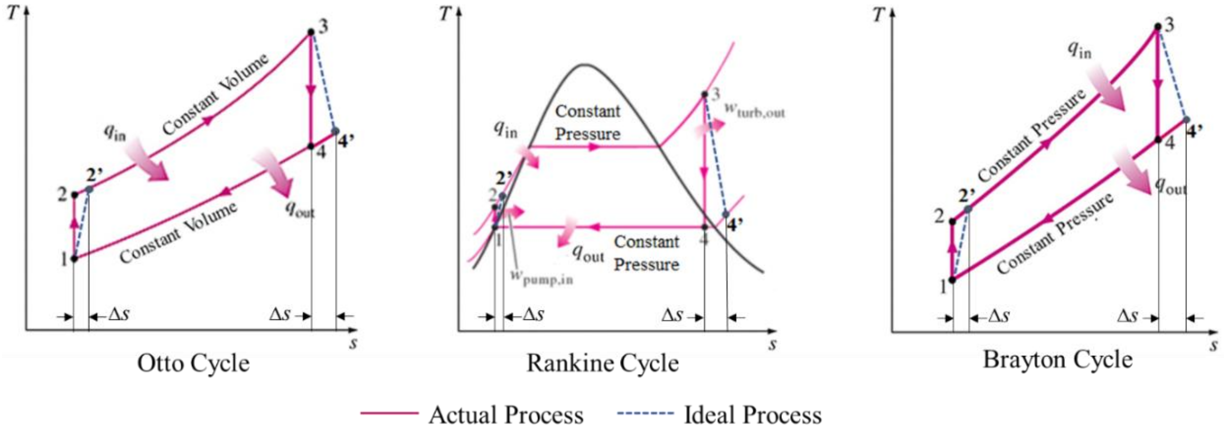

Fig. 1 – *T-s* Diagrams of Otto cycle, Ranking cycle, and Brayton cycle.

From the equipment view, the Otto cycle is a closed system, while the Rankine and Brayton cycles are open systems. In the figure, the isentropic processes are vertical, such as lines 1–2 (isentropic compression

DOI: https://doi.org/10.59277/CLC/2024.05

process) and 3–4 (isentropic expansion). Meanwhile, lines 1–2’ and 3–4’ are the actual processes. Entropy change in the processes can be described as

$$dS = \frac{\delta Q}{T} \qquad \text{(kJ/K)}. \tag{1}$$

For the Rankine cycle, the working fluid is steam. Detailed and well-organized tables of steam is available in which entropies based on temperature or pressure are entirely identified. The equations to calculate the entropy changes need to be developed for the Otto and the Brayton cycles.

2. ENTROPY CHANGE IN A PROCESS

Recalling the differential form of the conservation of energy equation for a closed stationary thermodynamic system containing a single compressible substance and undergoing an internally reversible process, $\delta Q_{\text{int rev}} - \delta W_{\text{int rev,out}} = dU$ ($W_{\text{int rev,out}}$ and U are the internally reversible work output and the internal energy), also considering $\delta Q_{\text{int rev}} = TdS$ and $\delta W_{\text{int rev,out}} = PdV$, Equation (1) can be expressed as

$$Tds = du + Pdv \qquad \text{(kJ/kg)}. \tag{2}$$

Equation (2) is called the first Tds equation, the Gibbs equation. If du is replaced by $dh = du + Pdv + vdP$ from the definition of enthalpy $h = u + Pv$, Equation (2) becomes

$$Tds = dh - vdP \qquad \text{(kJ/kg)}. \tag{3}$$

Equation (3) is known as the second Tds equation. These two Tds equations are developed with an internally reversible process. However, the results obtained are valid for both reversible and irreversible processes since entropy is a property, and the change of a property between two states is independent of the process type that the system undergoes.

Equations (2) and (3) are extremely valuable since they relate the entropy changes of a system and the changes in other properties. In general, the first Tds equation is applied to the processes of the closed systems, and the second Tds equation is applied to the processes of the open systems. Applying the relations of ideal gases, the relationship of du and dh with temperature $Pv = RT$, $du = c_v dT$, $dh = c_p dT$, and substituting them into Equation (2) and Equation (3), the differential entropy changes of an ideal gas are obtained,

$$\mathrm{d}s = c_v \frac{dT}{T} + R\,\frac{dV}{V} \qquad \mathrm{d}s = c_p \frac{dT}{T} - R\,\frac{dP}{P}.$$

Taking integration for the above two equations, the entropy changes for a closed system and an open system are,

$$s_2 - s_1 = \int_1^2 c_v\,\frac{dT}{T} + R\,\ln\frac{V_2}{V_1} \qquad \text{(kJ/kg·K)}, \tag{4}$$

$$s_2 - s_1 = \int_1^2 c_p\,\frac{dT}{T} - R\,\ln\frac{P_2}{P_1} \qquad \text{(kJ/kg·K)}. \tag{5}$$

2.1. Approximate Analysis (Constant Specific Heats)

The approximate analysis is also called a constant specific heat method. The analysis assumes constant specific heats, averages between the temperature ranges for ideal gases in Equations (4) and (5). The entropy change under the constant $c_{v,\text{avg}}$ and $c_{p,\text{avg}}$ can be determined,

$$s_1 - s_2 = c_{v,\text{avg}}\,\ln\frac{T_2}{T_1} + R\,\ln\frac{V_2}{V_1} \qquad \text{(kJ/kg·K)}, \tag{6}$$

$$s_1 - s_2 = c_{p,\mathrm{avg}} \ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1} \qquad (\mathrm{kJ/kg{\cdot}K}). \tag{7}$$

2.2. **Exact Analysis (Variable Specific Heats)**

The exact analysis is also called a variable-specific heat method. The entropy change during a process is decided by substituting the $c_v(T)$ and $c_p(T)$ relations into equations (4) and (5) and performing integrations. In practice, instead of conducting the laborious integrals each time, the entropy change from a temperature range from 0 to T is applied, which is defined as

$$s^{\mathrm{o}} = \int_0^T c_p \,\frac{dT}{T}.$$

The values of s^{o} are calculated at various temperatures and tabulated with temperature for ideal gases. Given this definition, the integral in equation (7) becomes $\int_1^2 c_p \frac{dT}{T} = s_2^0 - s_1^0$ where s_2^0 and s_1^0 are determined at the corresponding temperatures T_2 and T_1 in the thermodynamics property tables. Therefore,

$$s_2 - s_1 = s_2^0 - s_1^0 - R \ln\frac{P_2}{P_1}\,. \tag{8}$$

Using $c_v = c_p - R$,

$$\int_1^2 c_v \frac{dT}{T} = \int_1^2 (c_p - R)\frac{dT}{T} = \int_1^2 c_p \frac{dT}{T} - \int_1^2 R\,\frac{dT}{T} = s_2^0 - s_1^0 - \mathrm{R}\int_1^2 \frac{dT}{T}.$$

Applying the above expression to equation (4),

$$s_2 - s_1 = s_2^0 - s_1^0 + R\ln\left[\left(\frac{V}{T}\right)_2\left(\frac{T}{V}\right)_1\right] \qquad (\mathrm{kJ/kg{\cdot}K})\,. \tag{9}$$

The author develops equation (9) to calculate the entropy change in the exact analysis of a closed system with a volume change.

3. **ISENTROPIC PROCESS**

3.1 **Approximate Analysis (Constant Specific Heats)**

An isentropic process means no entropy change in the process. By setting the entropy change to zero in equations (6) and (7),

$$0 = c_{v,\mathrm{avg}} \ln\frac{T_2}{T_1} + R \ln\frac{V_2}{V_1}, \qquad 0 = c_{p,\mathrm{avg}} \ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1},$$

and rearranging above equations, following relations are obtained,

$$\left(\frac{T_2}{T_1}\right)_{s\,=\,\mathrm{const.}} = \left(\frac{V_1}{V_2}\right)^{k-1}, \tag{10}$$

$$\left(\frac{T_2}{T_1}\right)_{s\,=\,\mathrm{const.}} = \left(\frac{P_1}{P_2}\right)^{(k-1)/k}, \tag{11}$$

$$\left(\frac{P_2}{P_1}\right)_{s\,=\,\mathrm{const.}} = \left(\frac{V_1}{V_2}\right)^{k}\,. \tag{12}$$

Equations (10), (11), and (12) are called the first, the second, and the third isentropic relations of ideal gases under the constant specific heats.

3.2. **Exact Analysis (Variable Specific Heats)**

Equations (8) and (9) developed in Section 2.2 should be used when the approximate analysis is inappropriate for the entropy change in the isentropic process. By setting the entropy change to zero in these two equations,

$$0 = s_2^0 - s_1^0 - R\,\ln\frac{P_2}{P_1}, \qquad 0 = s_2^0 - s_1^0 + R\,\ln[(\frac{V}{T})_2(\frac{T}{V})_1]\,,$$

and rearranging the first equation above,

$$s_2^0 - s_1^0 = R\,\ln\frac{P_2}{P_1} \rightarrow \frac{P_2}{P_1} = \exp\frac{s_2^0 - s_1^0}{R} \rightarrow \frac{P_2}{P_1} = \frac{\exp\,(s_2^0/R)}{\exp\,(s_1^0/R)},$$

The quantity $\exp(s^0/R)$ is defined as the relative pressure P_r. Therefore,

$$(\frac{P_2}{P_1})_{s\,=\,\mathrm{const.}} = \frac{P_{r2}}{P_{r1}}\,. \tag{13}$$

P_r is a dimensionless quantity and a function of temperature only. The value of P_r is tabulated in the property tables of thermodynamics. Likewise, rearranging the second equation above, the following equations are developed:

$$s_2^0 - s_1^0 = -R\,\ln[(\frac{V}{T})_2(\frac{T}{V})_1] \rightarrow (\frac{T}{V})_2(\frac{V}{T})_1 = \exp\frac{s_2^0 - s_1^0}{R} \rightarrow (\frac{T}{V})_2(\frac{V}{T})_1 = \frac{\exp\,(s_2^0/R)}{\exp\,(s_1^0/R)}.$$

Then, $\frac{V_2}{V_1} = \frac{T_2}{T_1}\frac{P_{r1}}{P_{r2}} \longrightarrow \frac{V_2}{V_1} = \frac{T_2/P_{r2}}{T_1/P_{r1}}.$

The quantity T/P_r is only a function of temperature. It is defined as the relative specific volume V_r. So,

$$(\frac{V_2}{V_1})_{s\,=\,\mathrm{const.}} = \frac{V_{r2}}{V_{r1}}\,, \tag{14}$$

the value of V_r is tabulated in the property tables in thermodynamics.

4. **SUMMARY**

The author systematically describes the entropy changes of ideal gases and develops the missing equations for the closed system in the exact analysis. Complete Equations (6) – (14) in the approximate and exact analyses to calculate the entropy changes and the correlated properties in the isentropic processes for both the closed and opened systems are presented in the paper.

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