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FLUID MODES, DEEP LEARNING, AND CONSTRUCTAL LAW

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The fluid modes generated using a proper orthogonal decomposition (POD) method were predicted for a fluidized bed and a power generation turbine. The POD-based reduced-order models were solved using either a Galerkin projection or a deep learning strategy. In both cases, as the number of the fluid modes increased, the modes appeared to fragment/bifurcate, indicating that these modes follow the constructal law.

Keywords: Reduced-order model; Proper orthogonal decomposition; Machine learning; Constructal law.

1. INTRODUCTION

Inspired by the structural modes identified in structural engineering, fluid mechanicians were able to determine the fluid modes of general transport phenomena [1]. Similarly to structural analysis, the fluid modes can be used to determine the flow solution if the weightings of the modes are known. Consequently, the solution $\mathbf{u}(\mathbf{x}, t_i)$ of the transport phenomena can be approximated as

$$\mathbf{u}(\mathbf{x}, t_i) = \sum_{k=1}^{M} a_k(t_i)\phi_k(\mathbf{x}) \; i = 1, \dots, M,$$

where $\mathbf{u} \in \mathbb{R}^n$ is the state vector, $\phi_k \in \mathbb{R}^n$ is the k-th flow mode, $\mathbf{x} \in \mathbb{R}^d$ is the spatial coordinate, d is the spatial dimension $1 \leq d \leq 3, a_k \in \mathbb{R}$ is the time coefficient, the weighting of the flow mode, $t_i \in \mathbb{R}$ is time, and M is the number of snapshots. For compressible, non-reacting, three-dimensional flows $n = 5$.

2. METHODS

The minimization of the approximation error in (1) requires the minimization of the averaged least-square truncation error [2]

$$\epsilon_m =< \|\mathbf{u}(\mathbf{x}, t_i) - \sum_{k=1}^{m} a_k(t_i)\phi_k(\mathbf{x})\|^2 > \; i = 1, \dots, M,$$

where $<>$ denotes the time average and $\|\|$ is the L^2-norm. The optimum condition (2) reduces to an eigenvalue problem, which for a discrete case is

$$\mathrm{R}(\mathrm{x}, \mathrm{y})\phi(\mathrm{x}) = \lambda\phi(\mathbf{x}) \; \mathbf{x}, \mathbf{y} \in \mathbb{R}^d$$

where **R** is the autocorrelation matrix

$$\mathrm{R}(\mathrm{x}, \mathrm{y}) = \sum_{i=1}^{M} \mathbf{u}(\mathbf{x}, t_i)\mathbf{u}^T(\mathbf{y}, t_i)/M.$$

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The eigenvectors (or eigenfunctions) of the autocorrelation matrix (4) are the fluid modes, aka basis functions or proper orthogonal decomposition (POD) modes. The eigenvalues $\lambda_i, 1 \le i \le M$ of (3) determine how much relative energy is captured by the POD modes, where the relative energy is defined as $\lambda_i / \sum_{i=1}^{M} \lambda_i$.

The POD modes represent the skeleton of the solution. They can be calculated once snapshots of the solution are generated. Then the POD modes can be used to generate a reduced-order model (ROM) that allows us to predict the solution at any other time and flow conditions.

The following steps must be completed to generate a POD-based ROM: (a) generate database using a full-order model, (b) assemble autocorrelation matrix **R** and extract eigenmodes, (c) substitute approximation (1) in the governing equations and perform Galerkin projection using the POD modes, and (d) solve system of ordinary differential equations to obtain the time coefficients $a_k(t_i)$ and reconstruct the solution using (1).

Since the substitution and the Galerkin projection of step (c) are tedious, we are currently using deep learning to determine the time coefficients $a_k(t_i)$. Deep learning is a subset of machine learning methods based on neural networks, which uses multiple layers in the network. Machine learning is automated data analysis during which computer programs (or modules) are learned from data. The model (or computer program) describes the relationship between variables (or data) and properties of interest, such as the time coefficients a_k. The model is learned using training data, such as the flow snapshots, by using a learning algorithm that automatically adjusts the model's parameters to agree with the data. The cornerstones of machine learning are (i) data, (ii) model, and (iii) learning algorithm [3].

3. **RESULTS**

Whether the time coefficients are calculated using Galerkin projection or deep learning, the reconstructed solution uses the POD modes. The relative energy of the modes, which indicates the influence of the POD modes on the solution, typically decays rapidly, as shown in Fig. 1. This implies that only a reduced number of modes, m, needs to be kept in the approximation. Furthermore, often the energy of the modes vs. number of POD modes [4] comes in pairs, that is, modes i and $i+1$ have similar energy, while modes $i+2$ and $i+3$ have similar energy but significantly less than modes i and $i+1$.

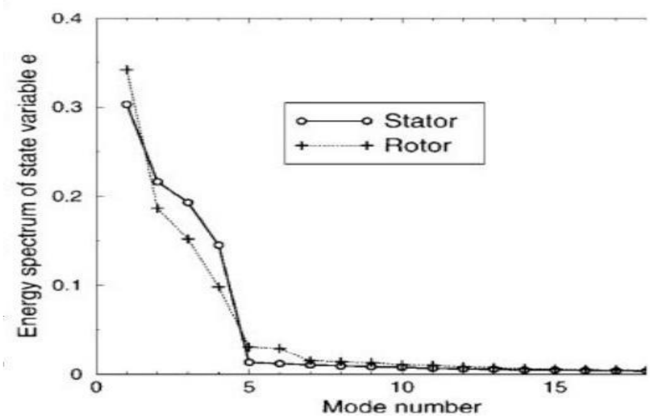

Fig. 1 – Cumulative energy (1) vs. number of modes [4].

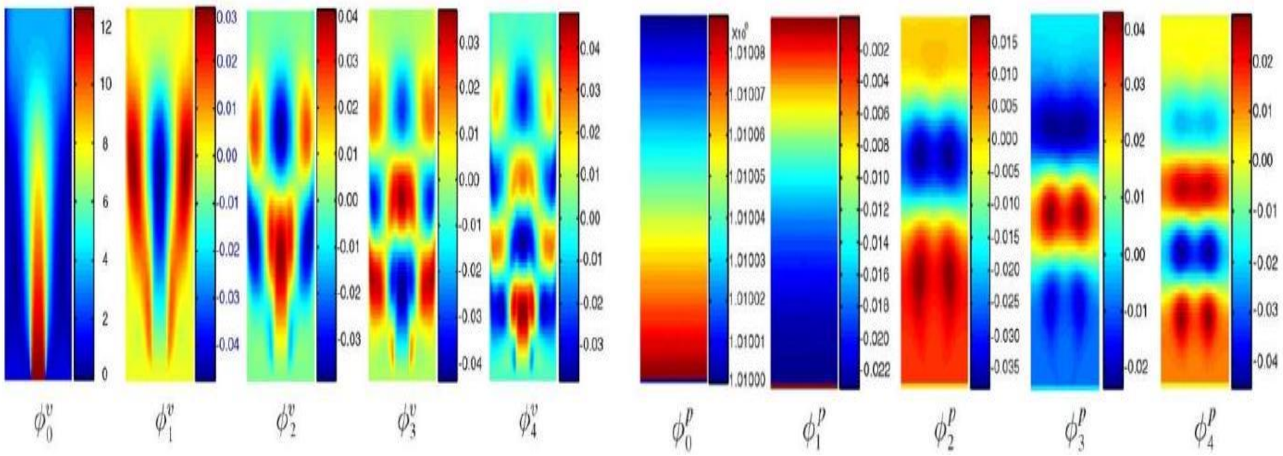

Fig. 2 – First five POD modes of v velocity (left) and pressure (right) [5].

Figure 2 shows the POD modes of the velocities and pressure in a fluidized bed [5]. It is apparent that as the mode number increases, the fragmentation of the contour plots increases. The vertical velocity shown in Fig. 2 is fragmented first in the x direction while shifting from mode ϕ_0^v to ϕ_1^v. Subsequently, the fragmentation happens in y direction while advancing from mode ϕ_1^v to ϕ_4^v.

A similar trend of fragmentation/bifurcation is evident in Fig. 3, which shows the POD modes of the unsteady flow generated by the rotor-stator interaction in a turbine. In this case, the fragmentation of the contour plots is more pronounced than that shown in Fig. 2, as multiple flow features are present in the turbine flow.

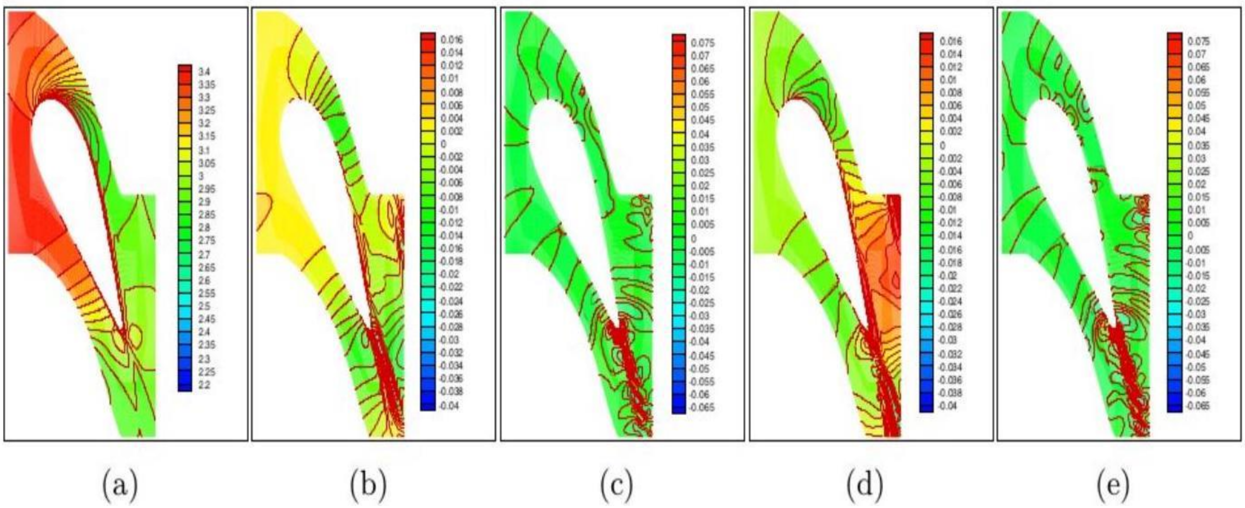

Fig. 3 – Five most relevant POD modes of energy [4].

3. **CONCLUSIONS**

The POD modes of transport phenomena equations are the building blocks for the solutions of these conservation equations. As the mode number increases, the modes appear to fragment/bifurcate, indicating that these modes follow the contructal law.

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